



FACILITY FORM 802

**N66 30757**  
(ACCESSION NUMBER)

464  
(PAGES)

CR-76325  
(NASA CR OR TMX OR AD NUMBER)

\_\_\_\_\_  
(THRU)

1  
(CODE)

32  
(CATEGORY)

GPO PRICE \$ \_\_\_\_\_

CFSTI PRICE(S) \$ \_\_\_\_\_

MAIL COPY \$ 7.63

MAILING \$ 2.15



**GENERAL DYNAMICS**  
*Convair Division*



REPORT NO. GDC-DDG66-008

DATE 21 April 1966

NO. OF PAGES 292 + xxiv



GENERAL DYNAMICS | AERONAUTICS

STUDY OF STABILITY OF UNPRESSURIZED  
SHELL STRUCTURES UNDER STATIC LOADING

FINAL REPORT  
VOLUME II

Contract Number NAS8-11181  
Request Number DCN 1-5-53-01067

Prepared for the  
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama

PREPARED BY *George W. Smith*  
George W. Smith

*Frank A. Dittoe*  
Frank A. Dittoe

APPROVED BY *A. H. Hausrath*  
*eln* A. H. Hausrath  
Structures Group Engineer

APPROVED BY *D. J. Peery*  
D. J. Peery  
Chief of Structural Analysis

Advanced Structures Group (Mail Zone 587-30)  
Research, Development and Engineering Department  
General Dynamics Convair Division  
Post Office Box 1128  
San Diego, California 92112

REVISIONS

NO.	DATE	BY	CHANGE	PAGES AFFECTED

ACKNOWLEDGEMENTS

This study was conducted at General Dynamics Convair Division by G. W. Smith and F. A. Dittoe under the technical direction of Dr. A. H. Hausrath, Group Engineer, Advanced Structures Group, who acted in the capacity of program leader. Programming for the digital computer was accomplished by Mrs. L. S. Fossum and Mrs. N. L. Fraser of the Technical Programming Group, and J. R. Anderson of the Guidance and Trajectory Programming Group.

Acknowledgement is made of the encouragement and advice furnished by E. E. McClure of the Advanced Structures Group.

Appreciation is expressed to H. R. Coldwater and H. L. Billmayer of the Structures Division, Propulsion and Vehicle Engineering Laboratory, Marshall Space Flight Center, for their support of the study. In the role of NASA Technical Representative, Mr. Billmayer provided valuable assistance in the definition and achievement of the study goals.

The entire report was typed by Mrs. F. C. Jaeger of the Convair Advanced Structures Group.

ABSTRACT

30957

This final report covers the work performed by General Dynamics Convair Division under NASA Contract NAS8-11181, "Study of Stability of Unpressurized Shell Structures Under Static Loading." The primary intent of this study was to employ orthotropic shell theory to develop practical working tools for the prediction of instability in stiffened circular cylindrical shells subjected to axial compression. Emphasis is on approximate analysis techniques to be used in preliminary sizing, rough checking, and the study of trends. Methods for the more stringent requirements of final analysis are also discussed and a digital computer program is provided for such applications. In addition to considering the overall buckling strength of the stiffened shell, curves are also presented for predicting the buckling of curved isotropic skin panels such as those found between stiffening elements. The report is divided into two distinct parts. Part I furnishes the theoretical and empirical foundations for the proposed methods while Part II gives concise procedures for the practical application of these methods.

CONTENTSVOLUME I

<u>Section</u>	<u>Title</u>	<u>Page</u>
	ACKNOWLEDGEMENTS	ii
	ABSTRACT	iii
	LIST OF FIGURES	ix
	LIST OF TABLES	xiv
	LIST OF SYMBOLS	xvi
	GLOSSARY	xxv
1	INTRODUCTION	1
2	CONCLUSIONS	6
3	LIMITATIONS	13
4	RECOMMENDATIONS	15

PART ITHEORETICAL AND EMPIRICAL FOUNDATIONS

5	BUCKLING OF ISOTROPIC SKIN PANELS SUBJECTED TO EDGE COMPRESSION	24
	5.1 General	24
	5.2 Buckling Criterion	26
	5.3 Comparisons Against Test Data	32
6	COMPRESSIVE BUCKLING OF LONGITUDINALLY STIFFENED CIRCULAR CYLINDERS	42
	6.1 General	42
	6.2 Buckling Criteria	42

CONTENTS (Cont'd)

<u>Section</u>	<u>Title</u>	<u>Page</u>
	6.2.1 Almroth Extension to Thielemann Solution	42
	6.2.2 Stuhlman-DeLuzio-Almroth Solution	54
6.3	Fixity Factor for Longitudinally Stiffened Sections Between Rings	73
6.4	Comparisons Against Test Data	78
	6.4.1 Longitudinally Stiffened Circular Cylinders	78
	6.4.2 Longitudinally Stiffened Circular Cylinders with Frames, Panel Instability Mode	93
7	GENERAL INSTABILITY OF ORTHOTROPICALLY STIFFENED CIRCULAR CYLINDRICAL SHELLS UNDER AXIAL COMPRESSION	98
	7.1 General	98
	7.2 Buckling Criteria	100
	7.2.1 Thielemann Solution	100
	7.2.2 Langley Solution	110
	7.3 Comparisons Against Test Data	119
8	INTERACTION BEHAVIOR	127
	8.1 General	127
	8.2 Axial Compression and Pure Bending	129
	8.3 Axial Compression and External Pressure	133
	8.4 Axial Compression and Internal Pressure	140
	8.5 Axial Compression and Shear	144

CONTENTS (Cont'd)

<u>Section</u>	<u>Title</u>	<u>Page</u>
9	INITIAL IMPERFECTIONS	147
	REFERENCES	159
<u>VOLUME II</u>		
	ACKNOWLEDGEMENTS	ii
	ABSTRACT	iii
	LIST OF FIGURES	ix
	LIST OF TABLES	xiv
	LIST OF SYMBOLS	xvi
<u>PART II</u>		
<u>APPLICATION</u>		
10	GENERAL	164
11	BUCKLING OF ISOTROPIC SKIN PANELS SUBJECTED TO EDGE COMPRESSION	166
	11.1 Procedures	166
	11.2 Design Curves	169
12	COMPRESSIVE BUCKLING OF LONGITUDINALLY STIFFENED CIRCULAR CYLINDRICAL SHELLS	187
	12.1 Procedures	187
	12.2 Design Curves for Bare 7075-T6 Aluminum Alloy	206
13	GENERAL INSTABILITY OF ORTHOTROPICALLY STIFFENED CIRCULAR CYLINDRICAL SHELLS SUBJECTED TO AXIAL COMPRESSION	274

CONTENTS (Cont'd)

<u>Section</u>	<u>Title</u>	<u>Page</u>
	13.1 Procedures	274
	13.2 Design Curves	285
14	INTERACTION BEHAVIOR	312
	14.1 Procedures	312
	14.1.1 Axial Compression Plus Pure Bending	312
	14.1.2 Axial Compression Plus External Pressure	314
	14.1.3 Axial Compression Plus Internal Pressure	316
	14.1.4 Axial Compression Plus Shear	316
	14.2 Design Curve	318
15	INITIAL IMPERFECTIONS	320
	15.1 Procedures	320
	15.2 Design Curves	320
16	BUCKLING OF MONOCOQUE CIRCULAR CYLINDERS	323
17	SAMPLE PROBLEMS	330
18	DIGITAL COMPUTER PROGRAMS	334
	18.1 Program for Buckling of Isotropic Skin Panels Subjected to Edge Compression	334
	18.2 Program for the Compressive Buckling of Longitudinally Stiffened Circular Cylindrical Shells	354
	18.3 Programs for General Instability of Orthotropically Stiffened Circular Cylindrical Shells Subjected to Axial Compression	370

CONTENTS (Cont'd)

<u>Section</u>	<u>Title</u>	<u>Page</u>
	18.3.1 Thielemann Solution	370
	18.3.2 Langley Solution	390
	18.3.3 $C_R$ Correction Factor	409
19	REFERENCES	417
Appendix		
A	SUPPLEMENTARY OPTIONS FOR ANALYSIS OF BUCKLING OF ISOTROPIC SKIN PANELS SUBJECTED TO EDGE COMPRESSION	A-1
	A.1 OPTION 2	A-1
	A.1.1 General	A-1
	A.1.2 Buckling Curves	A-2
	A.2 OPTION 3	A-18
	A.2.1 General	A-18
	A.2.2 Buckling Curves	A-18

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1	Panel Instability	Vol. I, xx
2	General Instability	Vol. I, xxi
3	Anticlastic Bending of Beams	Vol. I, xxiv
4	Nondimensional Logarithmic Plot of Buckling Criteria for Isotropic Skin Panels	31
5	Buckling of Isotropic Skin Panels - Calculated vs. Test ( $\sigma_R$ from OPTION 1)	39
6	Buckling of Isotropic Skin Panels - Calculated vs. Test ( $\sigma_R$ from OPTION 2)	40
7	Buckling of Isotropic Skin Panels - Calculated vs. Test ( $\sigma_R$ from OPTION 3)	41
8	Equilibrium Paths for a Perfect Isotropic Circular Cylinder Subjected to Axial Compression	43
9	Thielemann Notation	45
10	Interaction of Twisting Moments in Wall of Monocoque Cylinder	57
11	Semi-Logarithmic Plot of $\tilde{N}$ vs $\alpha$ with $\eta_s$ as a Parameter	71
12	Buckling Curves for an Infinite Length Column Supported by Equally Spaced Deflectional and Rotational Springs	74
13	Alternative Method for Evaluating Rotational Spring Constant	76
14	Semi-Logarithmic Plot of $\tilde{N}$ vs. $\gamma$	103
15	Alternative Ring Loading Conditions	104
16	Semi-Logarithmic Plot of $C_R$ Correction Factor	106

LIST OF FIGURES  
(Continued)

<u>No.</u>	<u>Title</u>	<u>Page</u>
17	Semi-Logarithmic Plot of Compressive Loading Coefficient for the General Instability of Stiffened Circular Cylinders	109
18	Example Interaction Curve	127
19	Combined Axial and Bending Loading Interaction Curve for Orthotropic Cylinders	132
20	Stability Interaction Results for Combined Axial Compression and External Radial Pressure	139
21	Stability Interaction Results for Combined Axial Compression and Internal Radial Pressure	143
22	Interaction of Pure Bending and Torsion for Stiffened Cylinders	146
23	Semi-Logarithmic Plot of $\Gamma$ vs. $R/t$ for Unstiffened Isotropic Cylinders Under Axial Compression	148
24	Load-Displacement Curve for Example Monocoque Cylinder	153
25	Load-Displacement Curve for Example Orthotropic Cylinder	153
26	Geometry of Isotropic Skin Panel	166
27	Buckling of Isotropic Panels	171-182
28	$\frac{b}{R}$ vs. $\frac{R}{t}$ for $\sigma_R = 2\sigma_p$	183-186
29	Fixity Factor vs. Rotational Stiffness Parameter	192
30	Minimization Factor $\tilde{N}$ vs. $a$ and the Parameter $\tau_s$	193
31	Compressive Buckling Stress for Longitudinally Stiffened 7075-T6 Al Alloy Circular Cylinders	210-223
32	Compressive Buckling Stress for Longitudinally Stiffened 7075-T6 Al Alloy Circular Cylinders	226-241

LIST OF FIGURES  
(Continued)

<u>No.</u>	<u>Title</u>	<u>Page</u>
33	Compressive Buckling Stress for Longitudinally Stiffened 7075-T6 Al Alloy Circular Cylinders	242-257
34	Compressive Buckling Stress for Longitudinally Stiffened 7075-T6 Al Alloy Circular Cylinders	258-273
35	Design Curves for the Correction Factor $C_R$	276
36	Critical Compressive Loading Coefficient for the General Instability of Stiffened Circular Cylinders	287-311
37	In-plane Running Shear Load $N_{xy}$	317
38	Design Interaction Curve	319
39	Design Correlation (Knock-down) Factor for Pure Axial Load	321
40	Design Correlation (Knock-down) Factor for Pure Bending	322
41	$\frac{\sigma_{cr}}{E}$ vs. $\frac{R}{t}$ for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Axial Load; BEST FIT	324
42	$\frac{\sigma_{cr}}{E}$ vs. $\frac{R}{t}$ for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Axial Load; PROBABILITY = 90%, CONFIDENCE = 95%	325
43	$\frac{\sigma_{cr}}{E}$ vs. $\frac{R}{t}$ for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Axial Load; PROBABILITY = 99%, CONFIDENCE = 95%	326
44	$\frac{\sigma_{cr}}{E}$ vs. $\frac{R}{t}$ for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Bending Moment; BEST FIT	327
45	$\frac{\sigma_{cr}}{E}$ vs. $\frac{R}{t}$ for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Bending Moment; PROBABILITY = 90%, CONFIDENCE = 95%	328

LIST OF FIGURES  
(Continued)

<u>No.</u>	<u>Title</u>	<u>Page</u>
46	$\frac{\sigma_{cr}}{E}$ vs. $\frac{R}{t}$ for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Bending Moment; PROBABILITY = 99%, CONFIDENCE = 95%	329
47	Input Format - Program 3875.	335
48	Sample Input Data - Program 3875.	340
49	Sample Output Listing - Program 3875.	342-344
50	Flow Diagram - Program 3875.	347
51	Input Format - Program 3896.	355
52	Sample Input Data - Program 3896.	358
53	Sample Output Listing - Program 3896.	360
54	Flow Diagram - Program 3896.	362
55	Input Format - Program 3942.	373
56	Sample Input Data - Program 3942.	376
57	Sample Output Listing - Program 3942.	378
58	Flow Diagram - Program 3942.	381
59	Input Format - Program 3962.	393
60	Sample Input Data - Program 3962.	396
61	Sample Output Listing - Program 3962.	398
62	Flow Diagram - Program 3962.	403
63	Input Format - Program 3942I.	410

LIST OF FIGURES  
(Continued)

<u>No.</u>	<u>Title</u>	<u>Page</u>
64	Sample Input Data - Program 3942I.	412
65	Sample Output Listing - Program 3942I.	413
66	Flow Diagram - Program 3942I.	415
67	Buckling of Isotropic Panels (OPTION 2)	A-5 - A-17
68	Buckling of Isotropic Panels (OPTION 3)	A-20 - A-34

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
I	Buckling of Isotropic Skin Panels	36
II	Comparison of Calculations vs. Test Data of Ref. 22.	81
III	Comparison of Calculations vs. Test Data of Ref. 4.	83
IV	Comparison of Calculations vs. Test Data of Ref. 8.	86
V	Comparison of Fixity Factor Analyses With Panel Instability Data of Ref's. 28 and 29.	96
VI	Comparison of First-Iteration Calculations vs. Test Data of Ref. 29.	122
VII	Comparison of Second-Iteration Calculations vs. Test Data of Ref. 29.	123
VIII	Final Comparison of Calculations vs. Test Data of Ref. 29.	125
IX	Calculated Data for Interaction Example Configurations - Axial Compression and External Radial Pressure	137
X	Calculated Data for Interaction Example Configurations - Axial Compression and Internal Radial Pressure	141
XI	Buckling Coefficients for Isotropic Skin Panels Subjected to Edge Compression	167
XII	Table of Contents for the Design Curves "Buckling of Isotropic Panels"	169
XIII	Table of Contents for the Design Curves " $\frac{b}{R}$ vs. $\frac{R}{t}$ for $\sigma_R = 2\sigma_p$ "	170
XIV	Recommended Values for the Fixity Factor $C_F$	191
XV	Recommended Values for the Minimization Factor $\tilde{N}$	191

LIST OF TABLES  
(Continued)

<u>No.</u>	<u>Title</u>	<u>Page</u>
XVI	Recommended Formulas for the $A_{ij}$ 's, $D_{ij}$ 's, and $\bar{t}$ of Longitudinally Stiffened Circular Cylinders	199-205
XVII	Table of Contents for the Design Curves "Compressive Buckling Stress for Longitudinally Stiffened 7075-T6 Al Alloy Circular Cylinders"	207-209
XVIII	Recommended Formulas for the $A_{ij}$ 's and $D_{ij}$ 's of Circular Cylinders Having Both Longitudinal and Circumferential Stiffeners	281-284
XIX	Table of Contents for the Design Curves "Critical Compressive Loading Coefficient for the General Instability of Stiffened Circular Cylinders"	285-286
XX	Program 3875 Notation	345-346
XXI	Fortran Listing - Program 3875	348-353
XXII	Program 3896 Notation	361
XXIII	Fortran Listing - Program 3896	365-369
XXIV	Program 3942 Notation	379-380
XXV	Fortran Listing - Program 3942	382-389
XXVI	Program 3962 Notation	399-402
XXVII	Fortran Listing - Program 3962	404-408
XXVIII	Program 3942I Notation	414
XXIX	Fortran Listing - Program 3942I	416
XXX	Table of Contents for the Supplementary Curves "Buckling of Isotropic Panels" (OPTION 2).	A-2
XXXI	Table of Contents for the Supplementary Curves "Buckling of Isotropic Panels" (OPTION 3).	A-19

LIST OF SYMBOLS

THIS LIST OF SYMBOLS APPLIES TO ALL SECTIONS OF THE REPORT EXCEPT SECTIONS (7.2.2), (8.3), (8.4), AND (18.3.2), ALL OF WHICH RETAIN THE NOTATION OF REFERENCE 5. IN THE INTEREST OF CLARITY, THE SYMBOLS FOR THESE FOUR SECTIONS ARE DEFINED WHERE REQUIRED IN THE REPORT.

$A_r$	Cross-sectional area of ring (no basic cylindrical skin included).
$A_r'$	Cross-sectional area of ring (including effective width of basic cylindrical skin).
$A_s$	Cross-sectional area of stringer (no basic cylindrical skin included).
$A_{11}, A_{22}, A_{12}, A_{21}, A_{33}$	Elastic constants defined by equations (6-4); Computational formulas applicable to conventional configurations are tabulated in Sections 12.1 and 13.1.
$a$	Spacing between rings; Coefficient in quadratic equation (7-7); Postbuckling variable defined by Figures 24 and 25.
$a_e$	Effective width of basic cylindrical skin.
$b$	Spacing between stringers; Coefficient in quadratic equation (7-7); Postbuckling variable defined by Figures 24 and 25.
$b_e$	Effective width of basic cylindrical skin.
$b_s$	Thickness of integral longitudinal stiffener (see Table XVI).
$C$	Coefficient defined by equation (5-7); Symbol to identify clamped boundary condition; Deflectional spring constant (force per unit deflection).

LIST OF SYMBOLS  
(Continued)

$C_F$	Fixity factor.
$C_f$	Experimentally determined constant [see equation (7-1)].
$C_R$	Correction factor defined by equation (7-16).
$C_{11}$	Distance between middle surface of basic cylindrical skin and centroid of skin-stringer combination (positive for internal stringers).
$c$	Constant term in quadratic equation (7-7).
$D$	Diameter of middle surface of basic cylindrical skin.
$D_{11}, D_{22}, D_{12}, D_{21}, D_{33}$	Elastic constants defined by equations (6-4); Computational formulas applicable to conventional configurations are tabulated in Sections 12.1 and 13.1.
$E$	Young's modulus in compression.
$E_{tan}$	Tangent modulus in compression.
$F$	Eccentricity parameter defined by equation (6-17).
$G$	Shear modulus.
$G_{tan}$	Tangent modulus in shear.
$h$	Distance between middle surfaces of sandwich facings; Corrugation pitch $\div 4$ .
$h_s$	Depth of integral longitudinal stiffener (see Table XVI).

LIST OF SYMBOLS  
(Continued)

$I$	Centroidal moment of inertia.
$I_r$	Centroidal moment of inertia of ring cross section (no basic cylindrical skin included).
$I_r'$	Centroidal moment of inertia of ring cross section (including effective width of basic cylindrical skin).
$I_x$	Shell wall local moment of inertia per unit length of circumference, taken about the centroidal axis of skin-stringer combination (including all of the skin and stringer material).
$I_x'$	Same as $I_x$ except for effective width considerations [see note (p) of Table XVI].
$K$	Buckling coefficient for a flat plate; Rotational spring constant (torque per unit rotation); Parameter defined by equation (7-18).
$K_c$	Buckling coefficient for flat plate having loaded edges simply supported and longitudinal edges clamped.
$K_s$	Buckling coefficient for flat plate having all edges simply supported.
$L$	Overall length of entire cylinder.
$l_x$	Axial half-wavelength of buckle pattern.
$l_y$	Circumferential half-wavelength of buckle pattern.

LIST OF SYMBOLS  
(Continued)

M	Overall bending moment.
$M_x, M_y, M_{xy}, M_{yx}$	Stress resultants (see Glossary).
m	Number of longitudinal half-waves in buckle pattern.
$m_i$	Any particular selected value of m.
$\bar{N}$	Thielemann parameter defined in equations (6-5).
$\tilde{N}$	Minimization factor defined by equation (6-30).
$N_s$	Number of stringers.
$N_x, N_y, N_{xy}, N_{yx}$	Stress resultants (see Glossary).
$(N_x)_{CL}$	Classical theoretical value for the critical longitudinal compressive membrane running load.
$(N_x)_{cr}$	Critical longitudinal compressive membrane running load.
$\left[ (N_x)_{cr} \right]_0$	Critical longitudinal compressive membrane running load for case of pure axial load (or pure bending moment) acting alone.
$(N_x)_{MIN}$	Minimum longitudinal compressive membrane running load for the postbuckling equilibrium path (see Figures 24 and 25).
$(N_x)_{wc}$	Wide-column critical longitudinal compressive membrane running load.
$\left[ (N_{xy})_{cr} \right]_0$	Critical in-plane running shear load for case of shear load acting alone.

LIST OF SYMBOLS  
(Continued)

$(N_y)_{cr}$	Critical circumferential compressive membrane running load.
$\left[ (N_y)_{cr} \right]_0$	Critical circumferential compressive membrane running load for case of external pressure acting alone.
$n$	Ramberg-Osgood parameter; Number of circumferential full-waves in buckle pattern.
$P$	Longitudinal force.
$Q_x, Q_y$	Stress resultants (see Glossary).
$R$	Radius to middle surface of basic cylindrical skin; Radius to centroid of ring (only when computing $C_R$ ).
$R_b$	Ratio of peak longitudinal compressive membrane running load from applied bending moment to the critical value under bending moment acting alone.
$R_c$	Ratio of longitudinal compressive membrane running load from applied axial load to the critical value under axial load acting alone.
$R_1$ or $R_2$	Ratio of applied load (or stress) to the critical value for that type of load (or stress) acting alone.
$SS$	Symbol to identify boundary conditions of simple support.
$T$	Torque.

LIST OF SYMBOLS  
(Continued)

$t$	Thickness of isotropic skin panel or isotropic cylinder.
$t'$	Effective skin thickness defined in note (o) of Table XVI.
$\bar{t}$	Effective thickness defined by equation (6-35).
$t_c$	Corrugation skin thickness.
$t_{eff}$	Equivalent thickness defined by equations (9-7) and (9-9).
$t_f$	Sandwich facing thickness.
$t_x$	Wall thickness for a monocoque circular cylinder of same total cross-sectional area as actual composite stiffened wall (including all of the skin and stringer material).
$t_x'$	Same as $t_x$ except for effective width considerations [see note (m) of Table XVI].
$t_y'$	Effective skin thickness defined in note (n) of Table XVI].
$U_b$	Flexural strain energy.
$U_m$	Membrane strain energy.
$\bar{u}$	Reference surface displacement in the x coordinate direction.
$V$	Total potential energy.
$\bar{v}$	Reference surface displacement in the y coordinate direction.

LIST OF SYMBOLS  
(Continued)

W	Discrete radial force depicted in Figure 15(a).
$\bar{w}$	Reference surface displacement in the z coordinate direction.
$w_c$	Uniformly distributed running radial load depicted in Figure 15(b).
X	Variable defined by equations (7-5).
x	Longitudinal coordinate.
Y	Variable defined by equations (7-5).
y	Circumferential coordinate.
z	Radial coordinate.
$\alpha$	Parameter defined by equation (6-10).
$\beta$	Parameter defined by equation (6-7).
$\Gamma$	Correlation (knock-down) factor.
$\Upsilon$	Thielemann parameter defined in equations (6-5).
$\Upsilon_{xy}$	In-plane shear strain.
$\Delta_R$	Radial deflection for the points of load application shown in Figure 15(a).
$\Delta_x$	Deflection defined in note (j) of Table XVI.
$\Delta\theta$	Rotation defined in note (k) of Table XVI.
$\delta_R$	Radial deflection due to uniformly distributed running load shown in Figure 15(b).
$\delta_x$	Deflection defined in note (j) of Table XVI.
$\delta\theta$	Rotation defined in note (k) of Table XVI.

LIST OF SYMBOLS  
(Continued)

$\epsilon_x$	Strain in x direction.
$\epsilon_y$	Strain in y direction.
$\eta_p$	Thielemann parameter defined in equations (6-5).
$\eta_s$	Thielemann parameter defined in equations (6-5).
$\theta$	Half-angle between discrete load points shown in Figure 15(a).
$\nu$	Poisson's ratio.
$\rho$	Radius of gyration.
$\rho_{11}$	Effective local longitudinal radius of gyration of shell wall [see equations (6-33) and (6-34)].
$\rho_{22}$	Effective local circumferential radius of gyration of shell wall.
$(Ed_i)$	Total peripheral length of corrugation center-line.
$\sigma$	Normal stress.
$\sigma_{cc}$	Crippling stress.
$\sigma_{cr}$	Critical buckling stress.
$(\sigma_{cr})_{CL}$	Classical critical stress.
$\sigma_{cy}$	Compressive yield stress.
$\sigma_o$	Critical value of uniformly distributed compressive stress, if acting alone.

LIST OF SYMBOLS  
(Continued)

$\sigma_{PL}$	Assumed proportional limit stress.
$\sigma_p$	Critical stress for buckling of a flat isotropic skin panel.
$\sigma_R$	Critical stress for buckling of an isotropic cylindrical shell.
$\sigma_{wc}$	Wide-column critical stress.
$\sigma_{.7}$	Ramberg-Osgood parameter.
$\tau$	Shear stress.
$\tau_o$	Critical value of torsional shear stress, if acting alone.
$\phi$	Parameter defined by equation (9-3).
$\Omega$	Potential energy of external loading.

NOTE: Subscripts which are preceded by commas denote partial differentiation with respect to the subscript variable. For example, the quantity

$$\bar{w}_{,xy} \text{ is identical to } \frac{\partial^2 \bar{w}}{\partial x \partial y}$$

GDC-DDG66-008

PART II  
APPLICATION

GENERAL DYNAMICS  
Convair Division

10.0 GENERAL

The objective in this part of the report is to present concise working procedures for direct application by structural analysts and designers. The theoretical and empirical foundations for these procedures have been presented in Part I. The sections to follow are intended to serve as an application manual and very little background material will be furnished.

The emphasis here is on approximate methods which are so categorized primarily in view of their

- (1) neglect of certain of the usually less crucial stiffnesses

$$(D_{12} = D_{33} = 0)$$

and

- (2) neglect of the influences from stiffener eccentricities.

The former is a conservative approximation. However, the latter can lead to either conservative or unconservative predictions if the given approximate methods are applied to configurations which incorporate stringer and/or ring eccentricities. The trends in this regard are strongly dependent upon whether the stiffeners are external or internal with respect to the basic cylindrical skin. Additional studies are needed to extend these approximate methods to properly account for such eccentricities.

In order to satisfy the requirements for a final analysis, the approximate methods are supplemented here by an alternative tool which can accommodate all of the inherent stiffness values along with

eccentricities of the longitudinal and/or circumferential stiffeners. This capability is provided in the form of a digital computer program in Section 18.3.2.

11.0 BUCKLING OF ISOTROPIC SKIN PANELS  
SUBJECTED TO EDGE COMPRESSION

11.1 Procedures

FOR  $b/R$  RATIOS LESS THAN THOSE OBTAINED FROM FIGURE 28, THIS PROCEDURE CAN GIVE UNCONSERVATIVE PREDICTIONS [See test data comparisons of Section 5.3, Part I]. TO OBTAIN A RELIABLE DESIGN TOOL, FURTHER DEVELOPMENT OF THE METHOD IS REQUIRED.

Design elastic buckling stress values may be determined for isotropic cylindrical skin panels bounded by conventional stiffening elements using the following procedures. Conventional plasticity reduction factors should be employed for inelastic stresses.

Step 1 - Calculate the ratios  $\frac{R}{t}$ ,  $\frac{b}{R}$ , and  $\frac{a}{b}$ , where

$a$  = Longitudinal dimension of skin panel  
(ring spacing).

$b$  = Circumferential dimension of skin panel  
(stringer spacing).

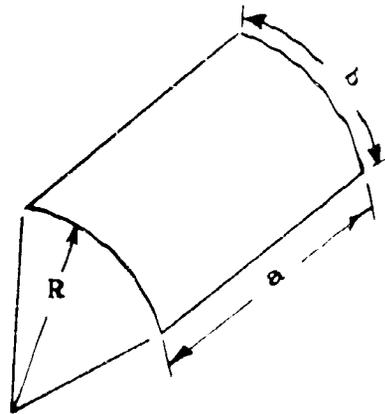


Figure 26 - Geometry of Isotropic Skin Panel

$t$  = Skin thickness.

$R$  = Radius to middle surface of skin.

Step 2 - Based on the computed  $\frac{a}{b}$  value and the boundary conditions, select an appropriate buckling coefficient  $K$  from the following table:

TABLE XI - Buckling Coefficients For Isotropic Skin Panels Subjected to Edge Compression

Boundary Conditions		$\frac{a}{b}$	K
Unloaded Edges	Loaded Edges		
C	SS	0.4	7.0
C	SS	0.6 $\rightarrow \infty$	5.7
SS	SS	0.4	7.0
SS	SS	0.6	4.0
SS	SS	0.8 $\rightarrow \infty$	5.29

Note that two combinations of boundary conditions are considered here. However, because of the scatter in the test data reported in Section 5.3 (Part I), it is recommended that, for design purposes, all edges be considered simply supported.

Step 3 - Find the appropriate  $\frac{\sigma_{cr}}{E}$  or  $\sigma_{cr}$  value from the design curves of Section 11.2. These curves were drawn by the SC-4020 plotting machine which does not provide any capability for the print-out of lower case letters. Hence the ratio  $\frac{a}{b}$  appears on these curves as  $\frac{A}{B}$ .

As discussed in Part I, the design curves of Section 11.2 are based on the Schapitz criterion [6] which considers two regimes of response dependent upon the two values  $\sigma_p$  and  $\sigma_R$  which are defined as follows:

$\sigma_p$  = Critical stress for buckling of a flat isotropic skin panel.

$\sigma_R$  = Critical stress for buckling of an isotropic cylindrical shell (OPTION 1 employing the lower bound criterion of Seide, et al. [13] is used here).

When  $\sigma_R \leq 2\sigma_p$ , the critical stress for the curved skin panel is established by a relationship which provides a smooth transition between flat plate theory and the full-cylinder behavior. For  $\sigma_R > 2\sigma_p$  the critical stress is simply taken as  $\sigma_{cr} = \sigma_R$ . To establish the point of separation between these two regimes, Section 11.2 includes curves which show  $\frac{b}{R}$  vs.  $\frac{R}{t}$  for the condition  $\sigma_R = 2\sigma_p$ . Here again, the plotting machine converted lower-case notation into upper case print-outs.

Section 18.1 gives the digital computer program which was used to obtain the design curves of Section 11.2. This program may be used to obtain additional plots or single-point solutions as desired. The program also includes capability to obtain critical stresses for the special case of flat plates.

## 11.2 Design Curves

All of the design curves in this section are based on a Poisson's ratio of 0.30. The curves for aluminum employ  $E = 10 \times 10^6$  psi while the curves for steel use  $E = 30 \times 10^6$  psi.

Tables XII and XIII list the families provided here.

TABLE XII - Table of Contents for the  
Design Curves "Buckling of  
Isotropic Panels"

<u>Figure Number</u>	<u>Ordinate</u>	<u>Abscissa</u>	<u>a/b</u>	<u>K</u>	<u>Page</u>
27(a)	$\left( \frac{\text{Buckling Stress}}{\text{Elastic Modulus}} \right)$	$\frac{R}{t}$	0.4	7.0	171
27(b)	"	"	0.6 $\rightarrow$ $\infty$	5.7	172
27(c)	"	"	0.6	4.0	173
27(d)	"	"	0.8 $\rightarrow$ $\infty$	3.29	174
27(e)	Buckling Stress-psi For Aluminum	$\frac{R}{t}$	0.4	7.0	175
27(f)	"	"	0.6 $\rightarrow$ $\infty$	5.7	176
27(g)	"	"	0.6	4.0	177
27(h)	"	"	0.8 $\rightarrow$ $\infty$	3.29	178
27(i)	Buckling Stress-psi For Steel	$\frac{R}{t}$	0.4	7.0	179
27(j)	"	"	0.6 $\rightarrow$ $\infty$	5.7	180
27(k)	"	"	0.6	4.0	181
27(l)	"	"	0.8 $\rightarrow$ $\infty$	3.29	182

TABLE XIII - Table of Contents  
for the Design Curves  
"b/R vs. R/t for  $\sigma_R = 2\sigma_p$ "

<u>Figure Number</u>	<u>Ordinate</u>	<u>Abscissa</u>	<u>a/b</u>	<u>K</u>	<u>Page</u>
28(a)	$\frac{b}{R}$ ; (For $\sigma_R = 2\sigma_p$ )	$\frac{R}{t}$	0.4	7.0	183
28(b)	"	"	0.6 $\rightarrow \infty$	5.7	184
28(c)	"	"	0.6	4.0	185
28(d)	"	"	0.8 $\rightarrow \infty$	3.29	186

For informational purposes, additional families of  $\sigma_{cr}/E$  vs.  $\frac{R}{t}$  are presented in Appendix A. These supplementary plots were obtained using OPTIONS 2 and 3 which are based on 50% and 90% probability values respectively for  $\sigma_R$  as discussed in Part I, Section 5.

### BUCKLING OF ISOTROPIC PANELS

A/B = 0.400

K = 7.000

OPTION 1

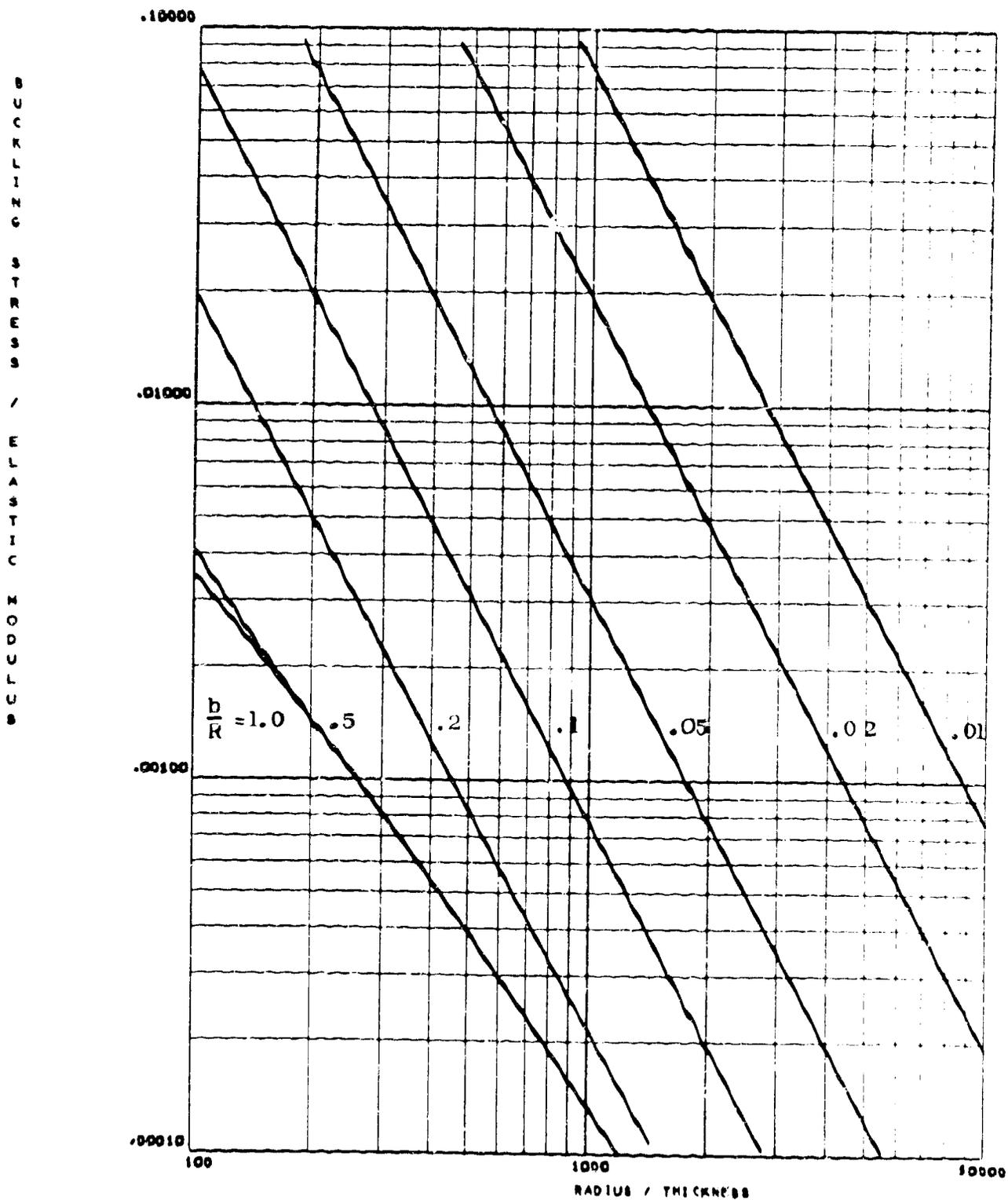


Figure 27(a) - (See Table XII)

BUCKLING OF ISOTROPIC PANELS

$\lambda/E \cdot \mu$  0.600  $\rightarrow \infty$

$K = 1.700$

OPTION 1

BUCKLING STRESS / ELASTIC MODULUS

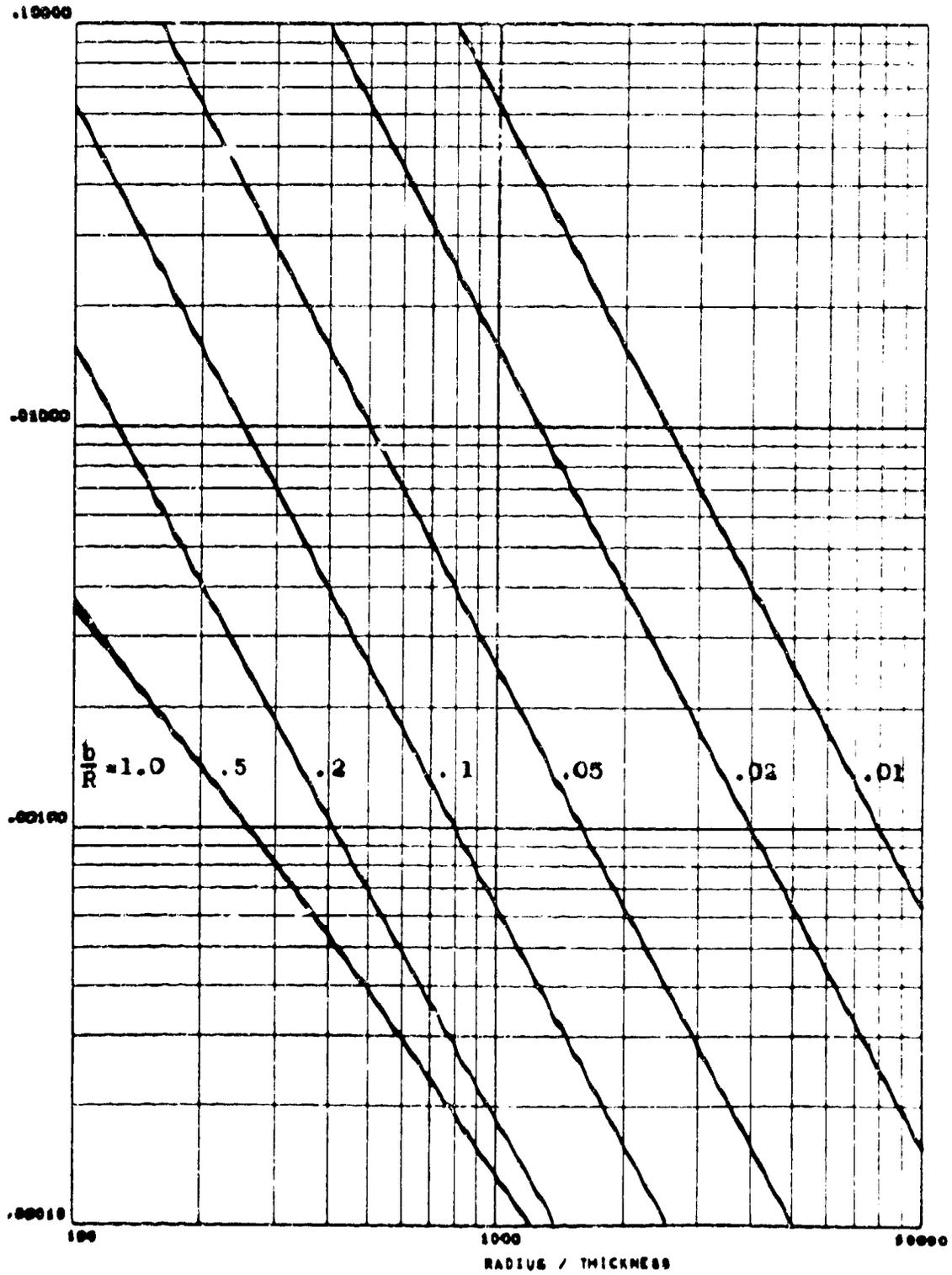


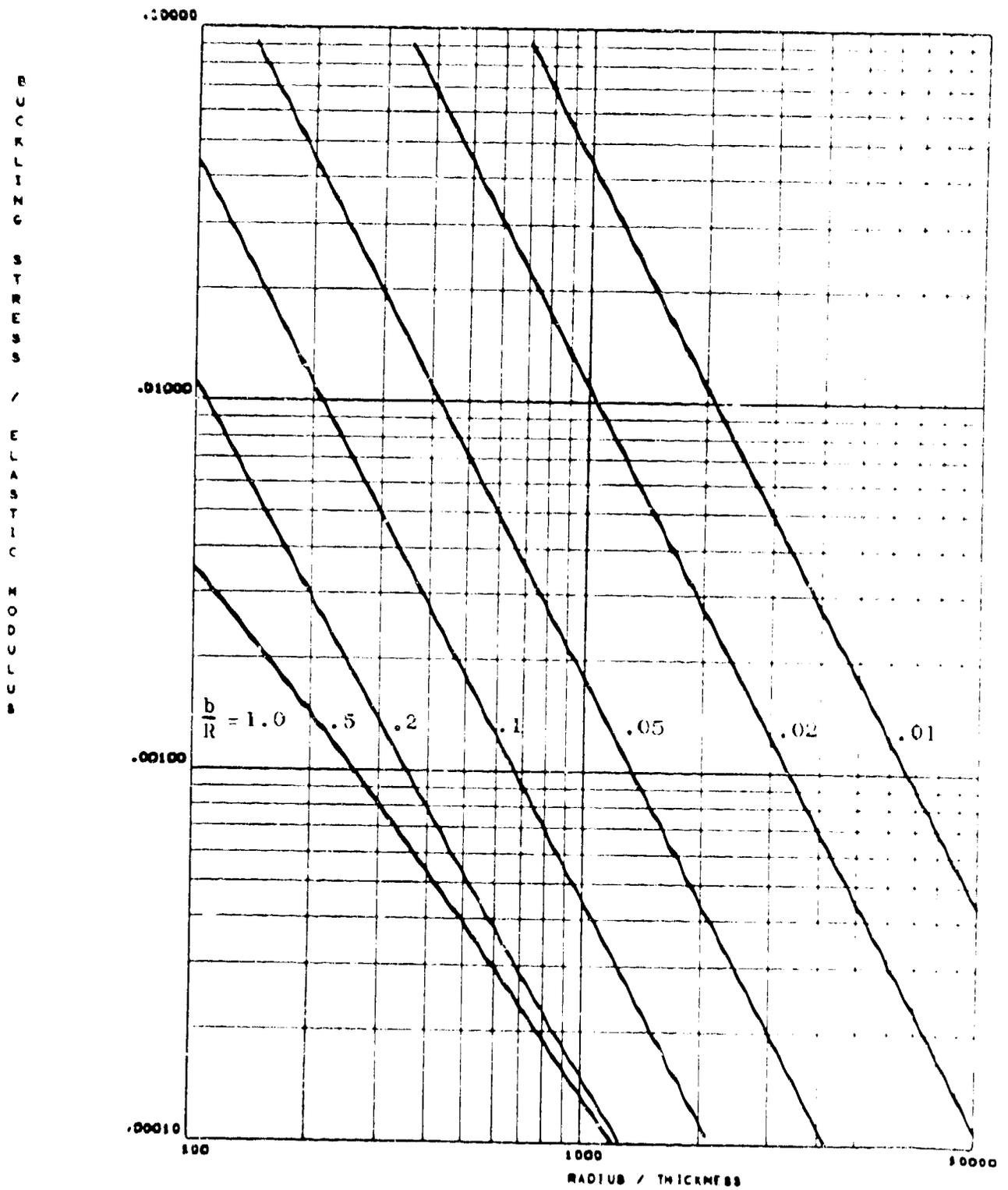
Figure 27 (b) - (See Table XII)

# BUCKLING OF ISOTROPIC PANELS

$\mu/B = 0.400$

$\kappa = 4.000$

OPTION 1



GENERAL DYNAMICS  
Convair Division

Figure 27 (c) - (See Table XII)

### BUCKLING OF ISOTROPIC PANELS

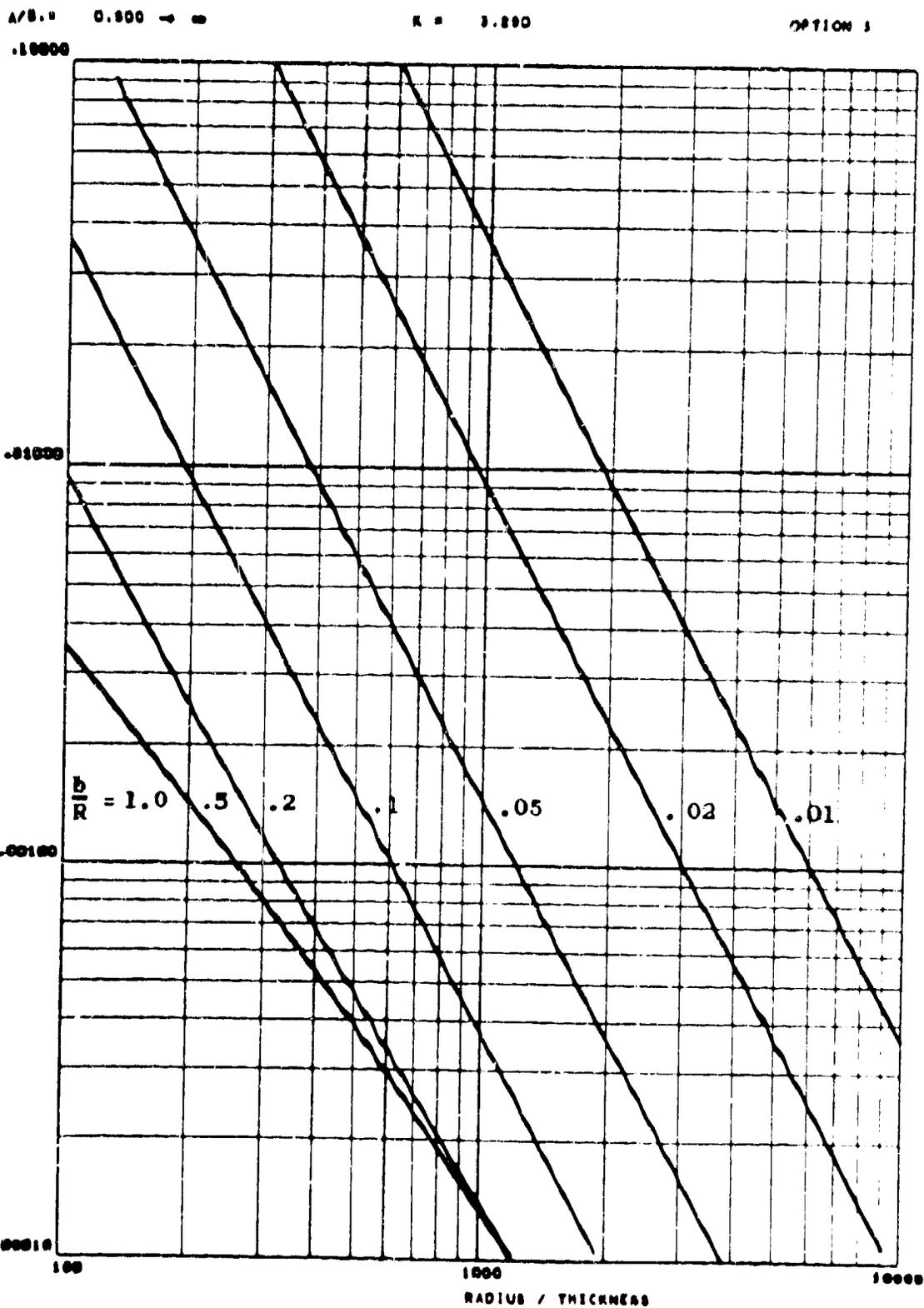


Figure 27 (d) - (See Table XII)

GENERAL DYNAMICS  
Convair Division

### BUCKLING OF ISOTROPIC PANELS

A/B = 0.400

K = 7.000

OPTION 1

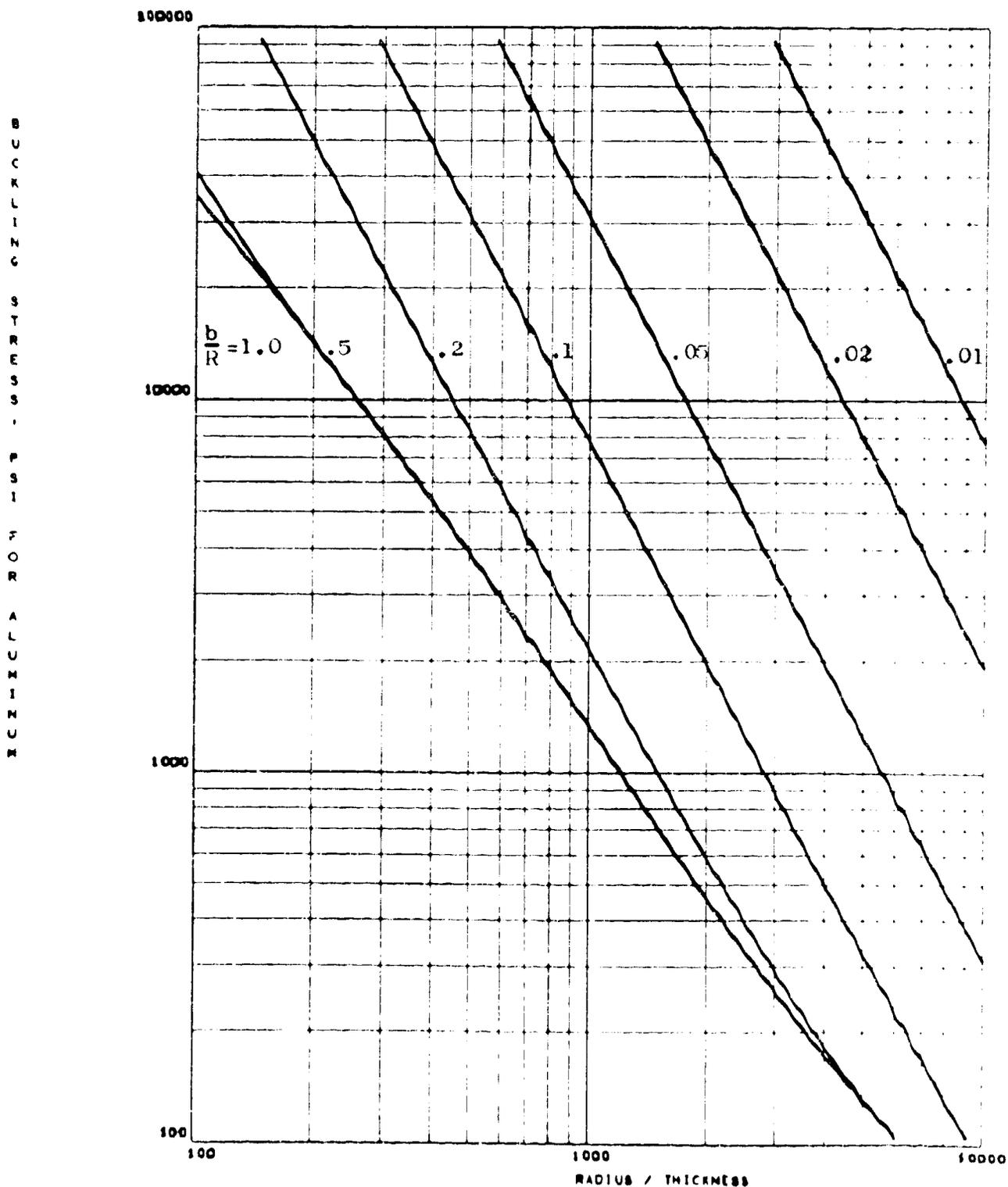


Figure 27 (c) - (See Table III)

# BUCKLING OF ISOTROPIC PANELS

$A/R = 0.000 \rightarrow \infty$

$\nu = 0.300$

OPTION 1

BUCKLING STRESS - PSI

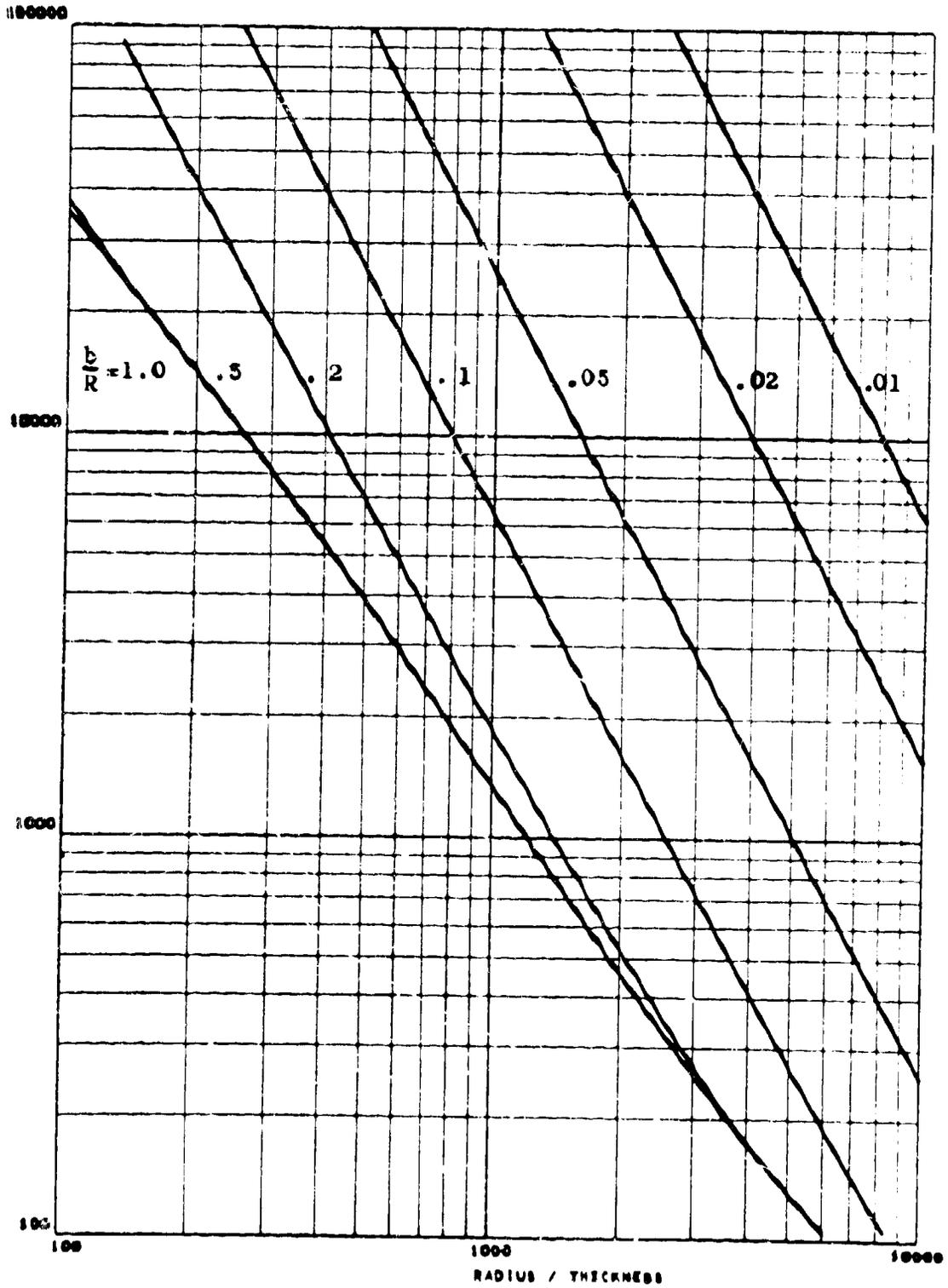


Figure 27 (f) - (See Table XII)

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.800

K = 4.000

OPTION 1

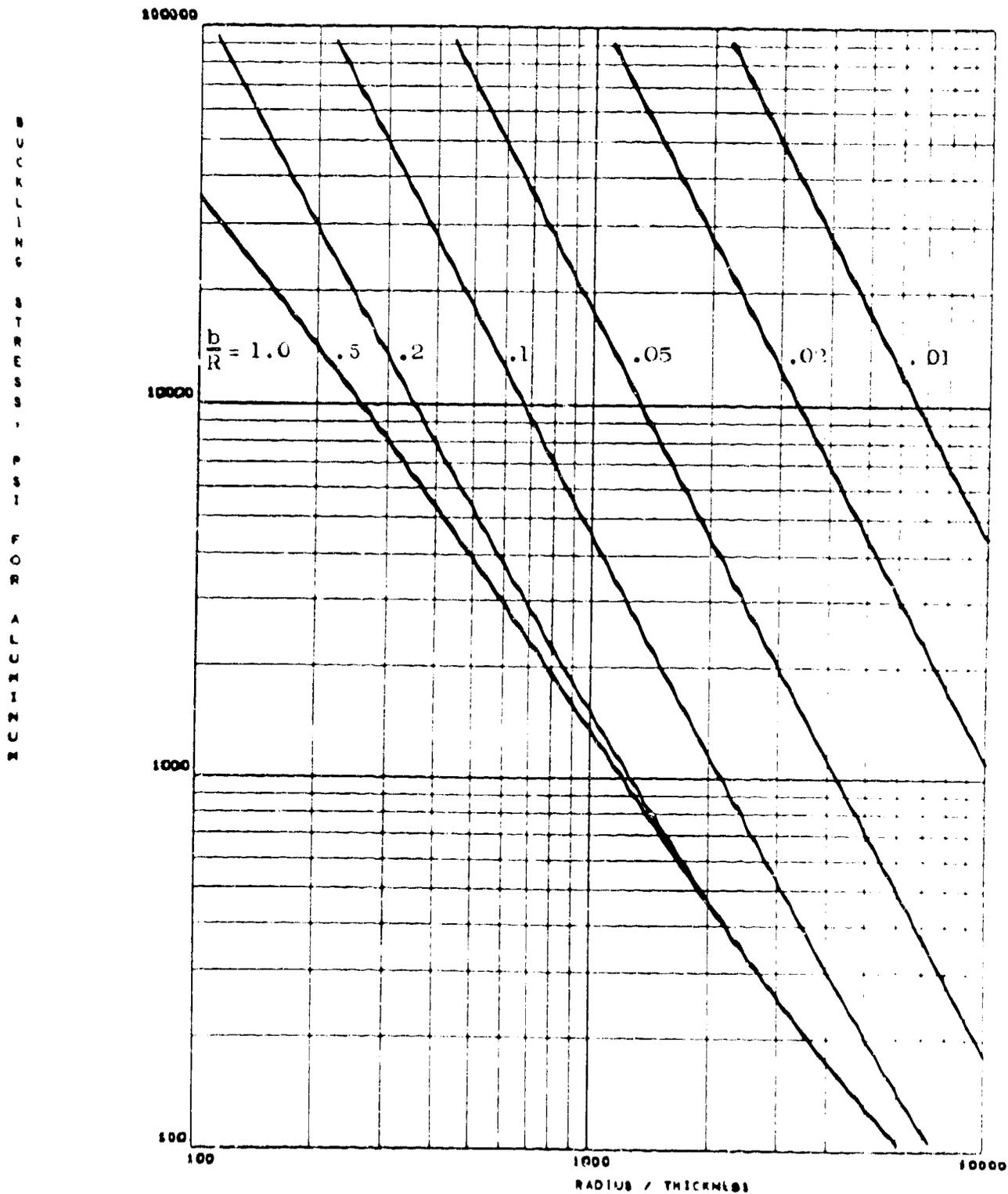


Figure 17 (g) - (See Table XII)

# BUCKLING OF ISOTROPIC PANELS

$\mu = 0.600 \rightarrow \infty$

$K = 1.230$

OPTION 1

SCHEMATIC OF PANEL BUCKLING

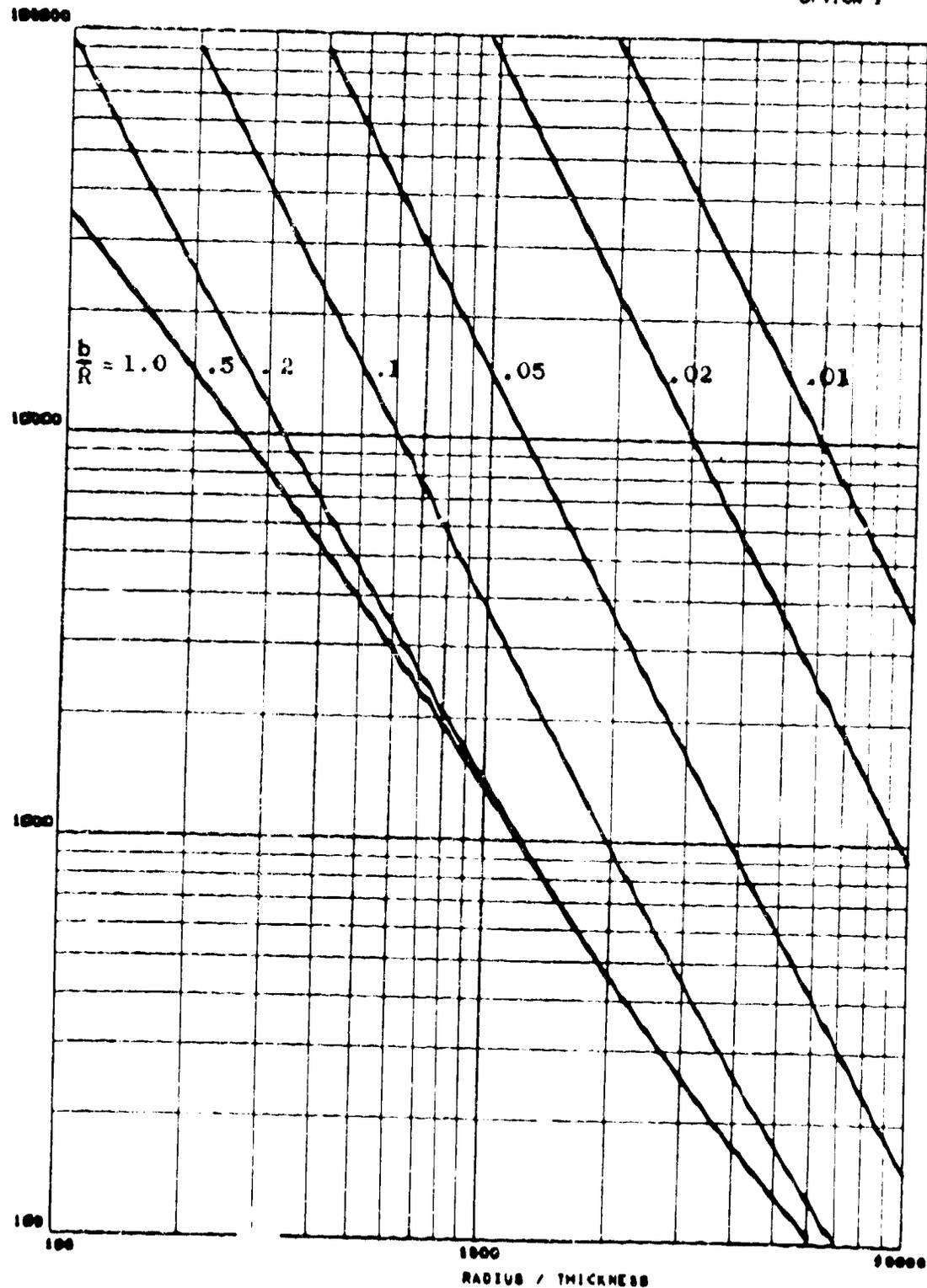


Figure 27 (h) - (See Table XII)

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.400

K = 7.000

OPTION 1

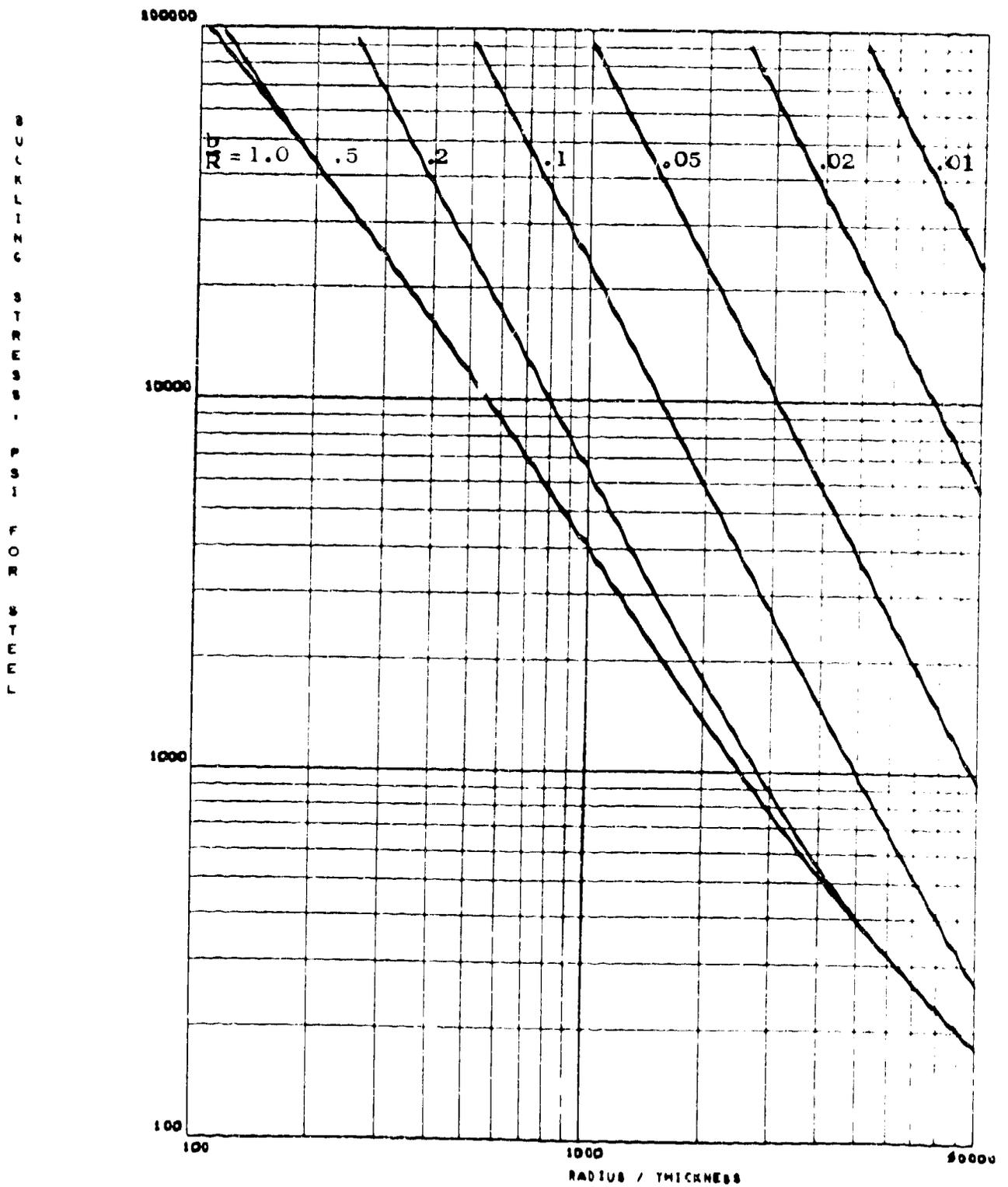


Figure 27 (i) - (See Table XII)

# BUCKLING OF RECTANGULAR PANELS

A/B = 0.800 →

ν = 0.30

OPTION 1

BUCKLING STRESS FOR STEEL

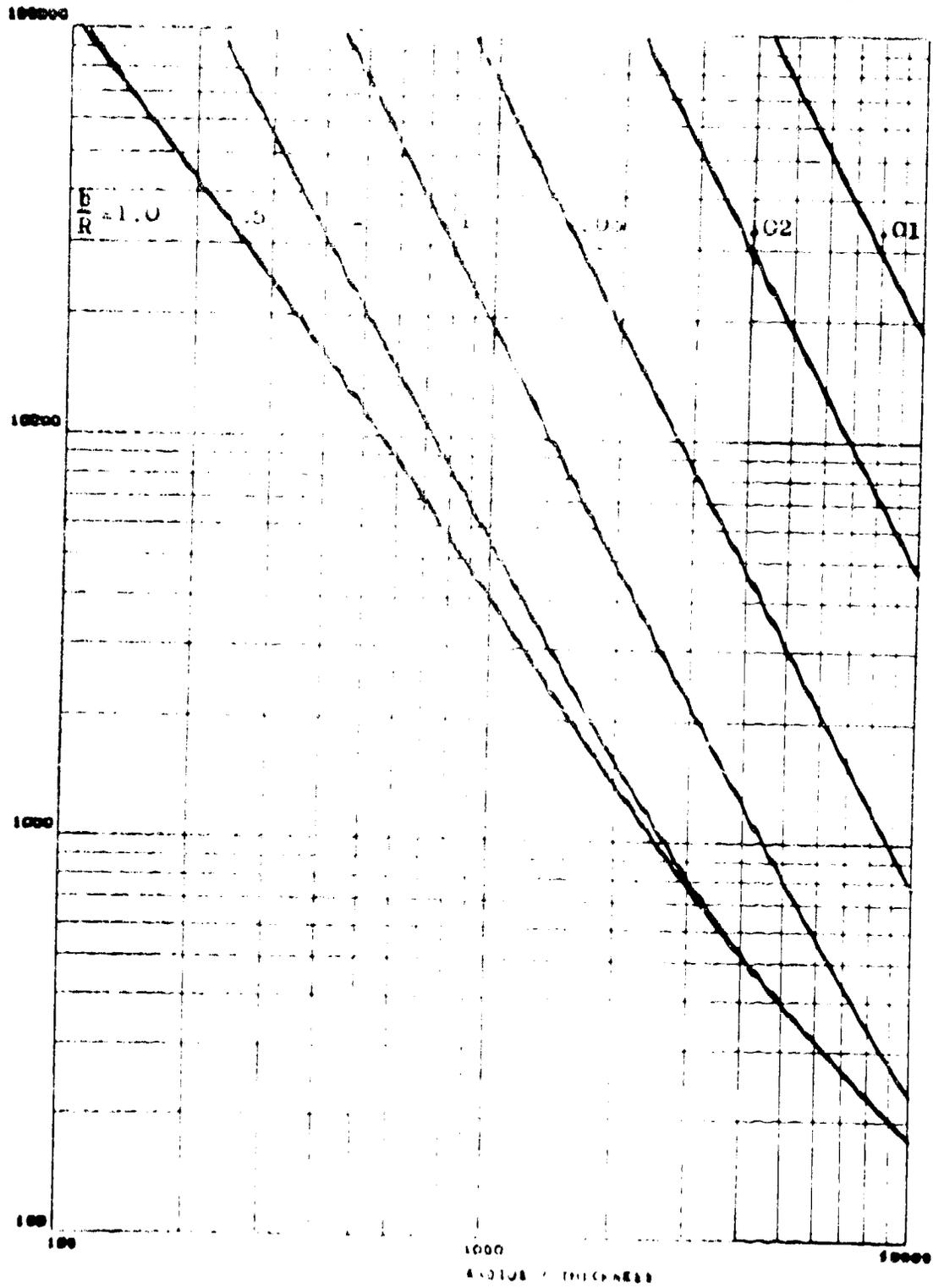


FIGURE 11 - BUCKLING STRESS FOR STEEL

BUCKLING OF ISOTROPIC PANELS

$\lambda/B = 0.600$

$K = 4.000$

OPTION 1

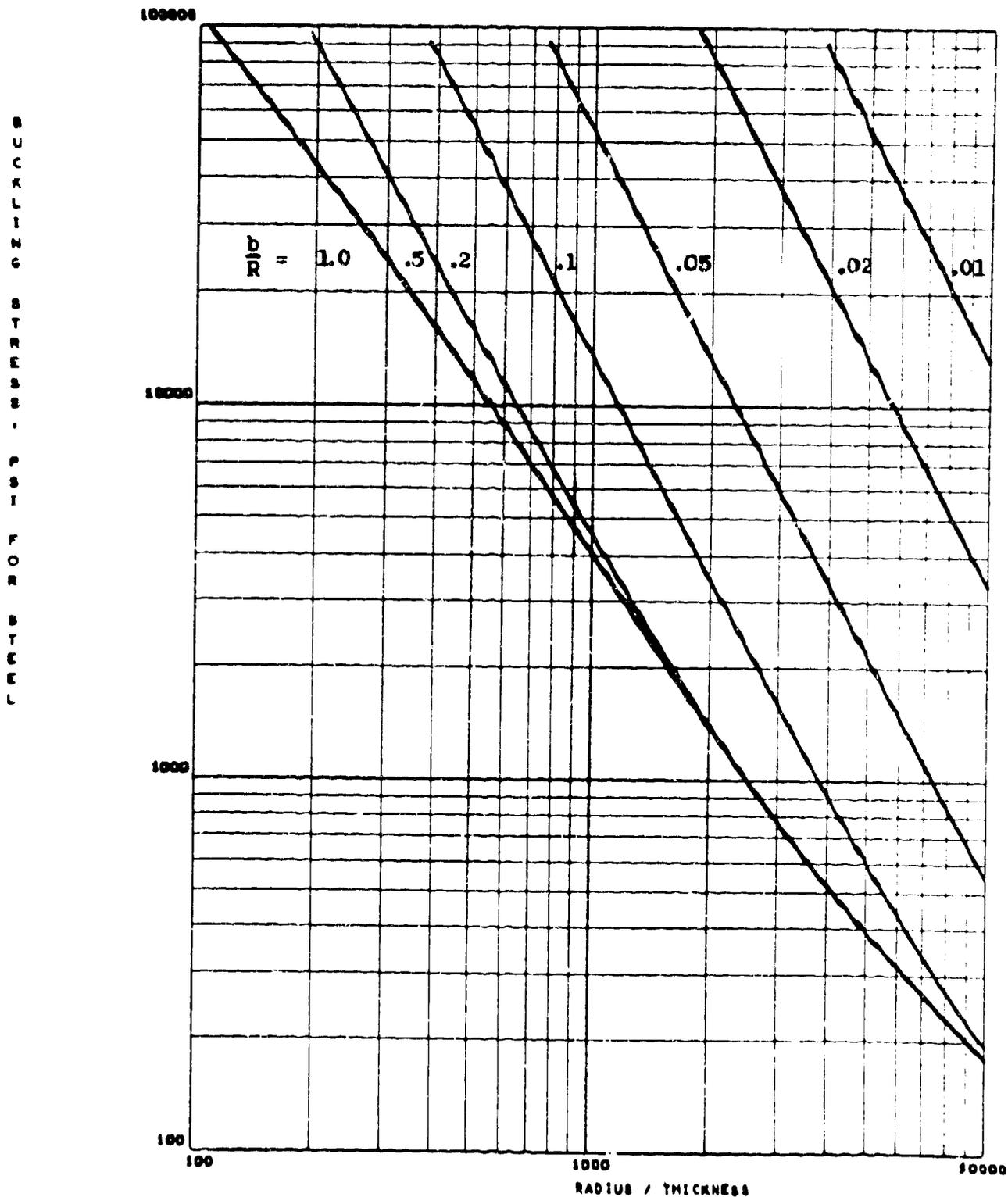


Figure 27 (k) - (See Table XII)

# BUCKLING OF ISOTROPIC PANELS

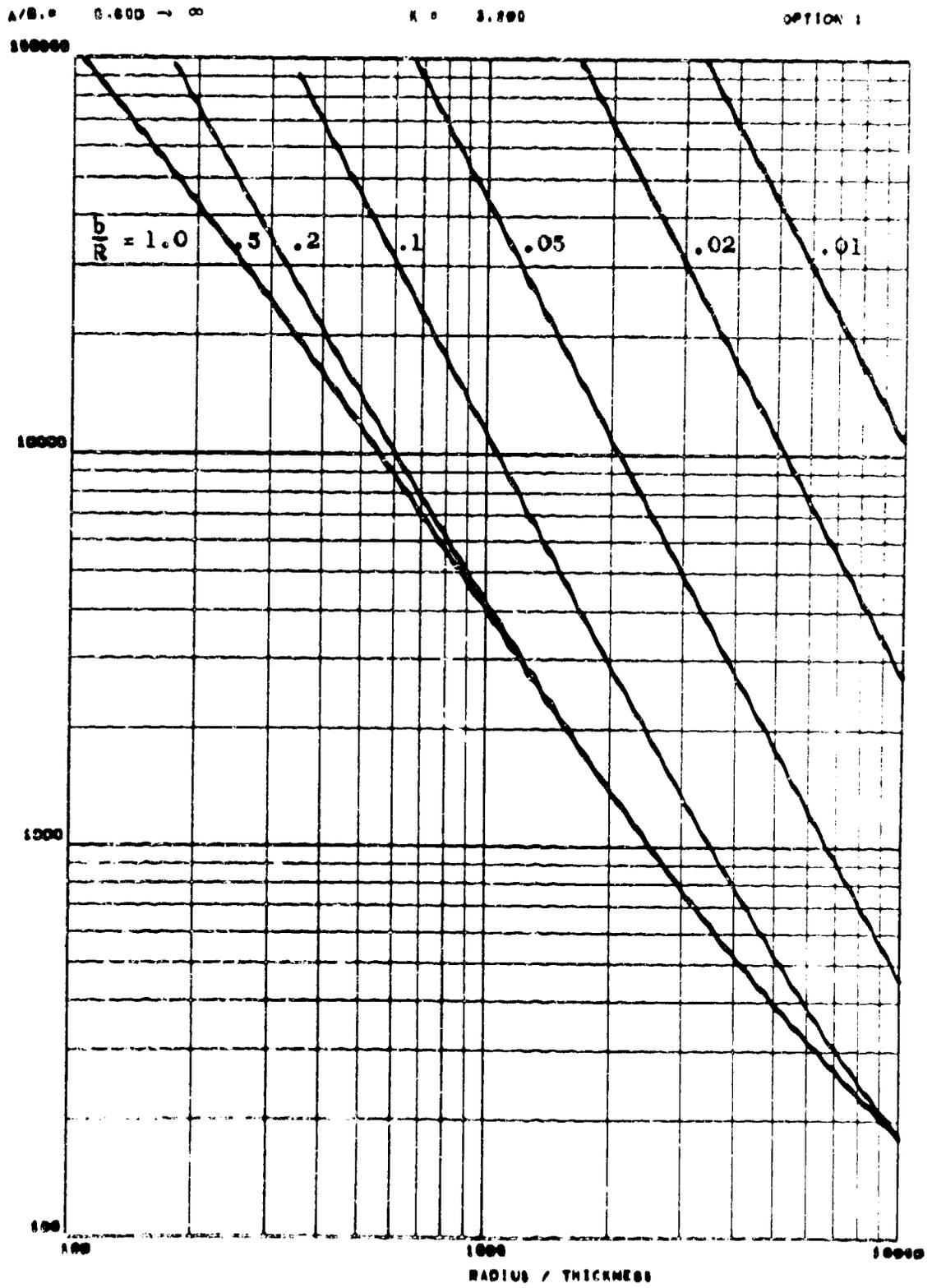


Figure 27 (1) - (See Table XII)

B/R VS R/T FOR SIGR=2\*SIGP

A/B = 0.400

K = 7.000

OPTION 1

P  
A  
N  
E  
L  
W  
I  
D  
T  
H  
/  
R  
A  
D  
I  
U  
S

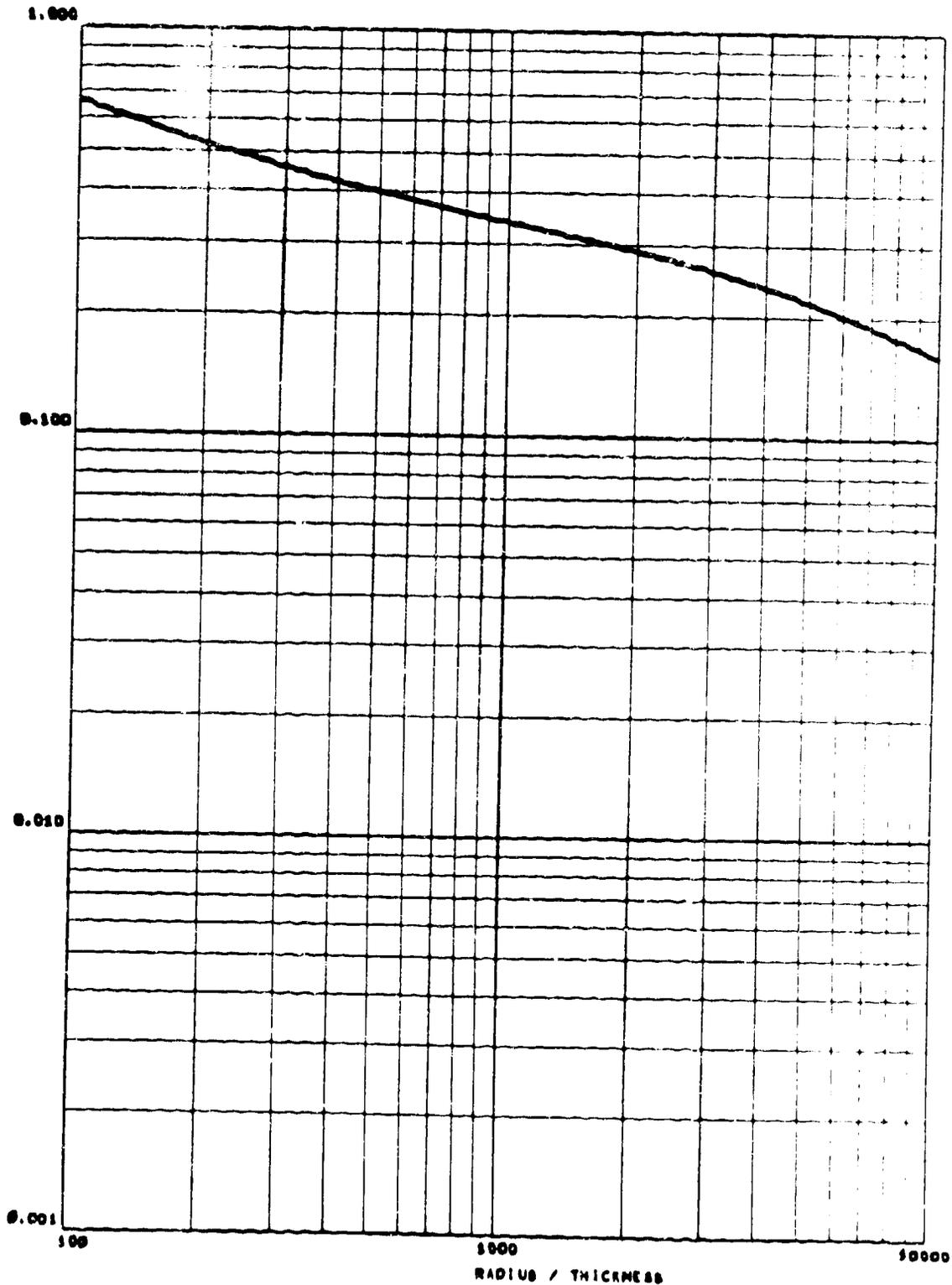


Figure 28 (a) - (See Table XIII)

### B/R VS R/T FOR SIGR=2#SIGP

$\lambda/S = 0.000$

$K = 1.700$

OPTION 1

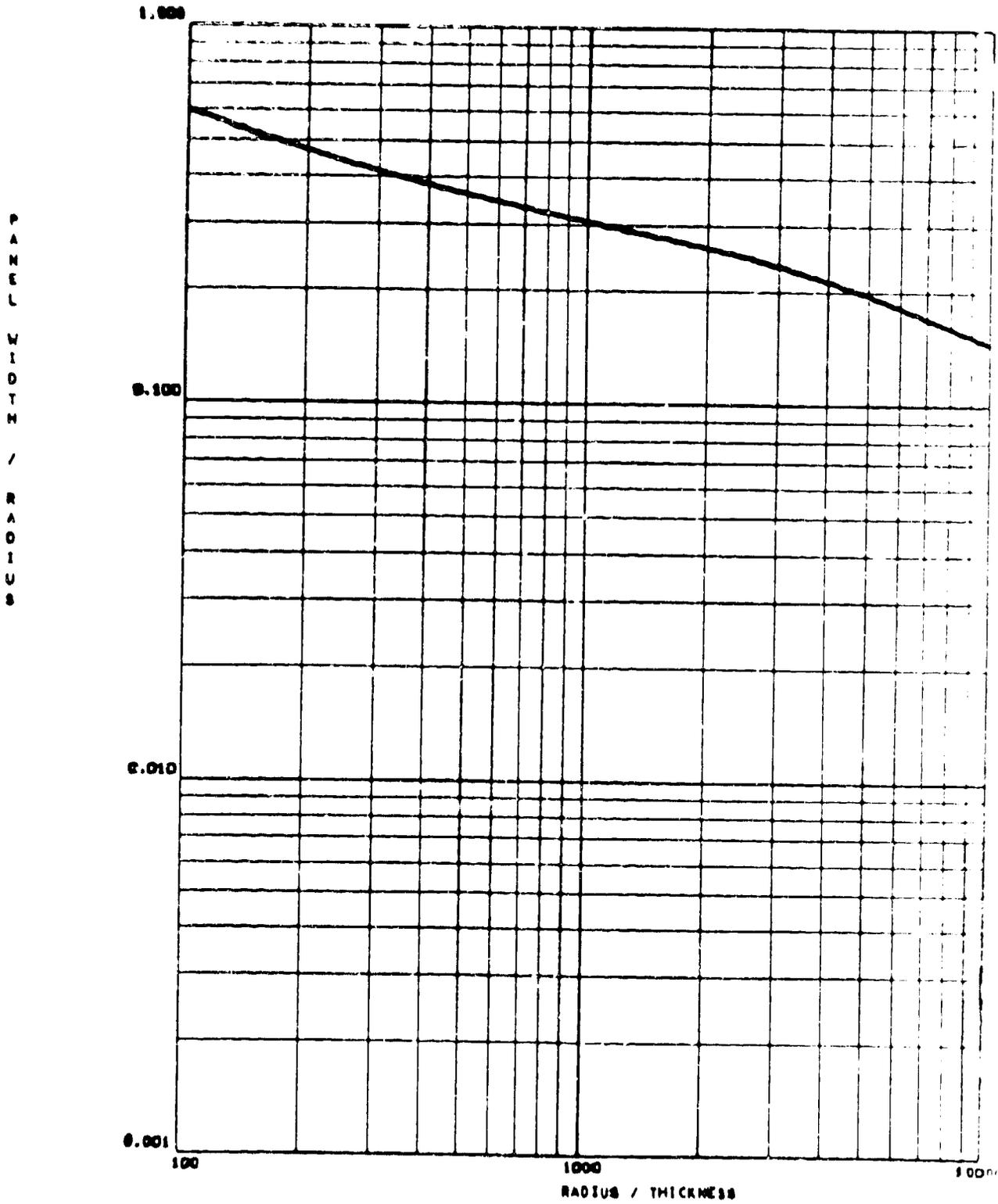


Figure 28 (b) - (See Table XIII)

B/R VS R/T FOR SIGR=2\*SIGP

A/B = 0.600

K = 4.000

OPTION 1

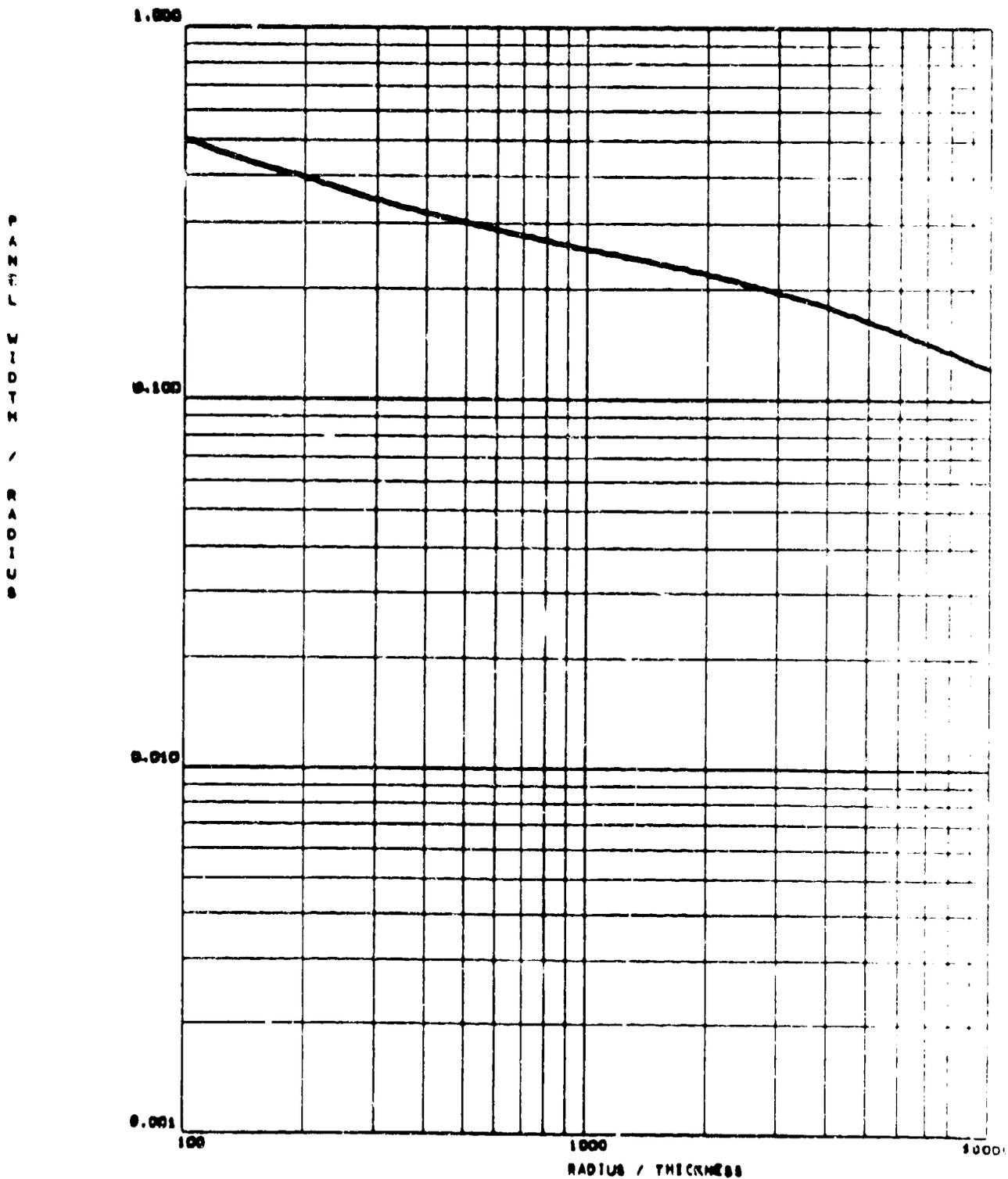


Figure 28 (c) - (See Table XIII)

### B/R VS R/T FOR SIGR=2\*SIGP

A/B = 0.600 → ∞

K = 3.290

OPTION 1

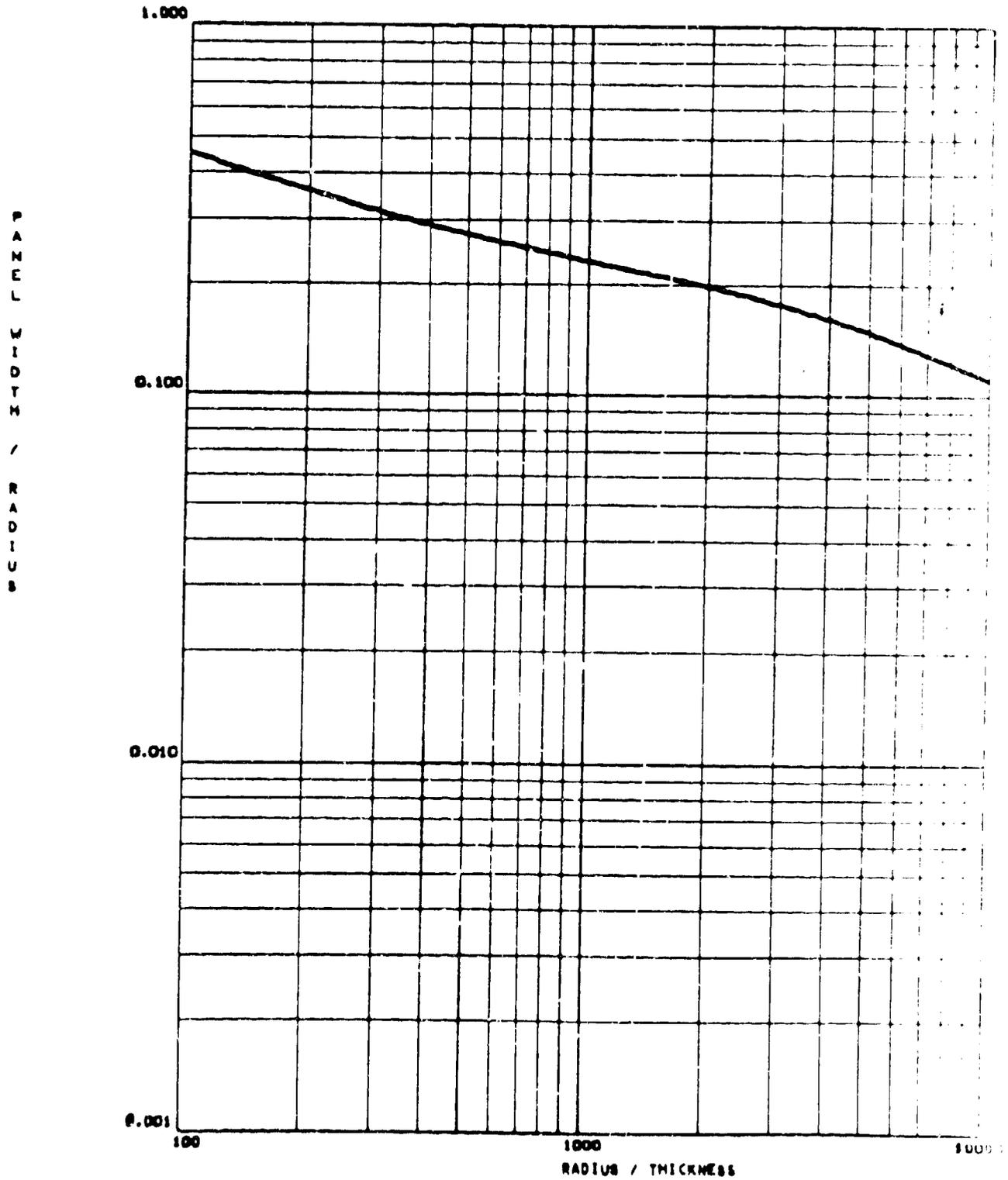


Figure 28 (d) - (See Table XIII)

12.0 COMPRESSIVE BUCKLING OF LONGITUDINALLY  
STIFFENED CIRCULAR CYLINDRICAL SHELLS

12.1            Procedures

The procedures given in this section apply equally well to cylinders which incorporate only longitudinal stiffening and to sections which lie between rings in cylinders incorporating both axial and hoop stiffening. Application to the latter case is only valid where general instability (see Glossary) does not precede the panel instability mode (see Glossary).

The given procedures employ the smearing-out technique whereby discrete stiffness values are averaged over the entire surface of the cylinder. One must therefore exercise engineering judgement to prevent misapplication to configurations having excessively large stringer spacings.

Simple formulas for the  $A_{ij}$ ,  $D_{ij}$ , and  $\bar{t}$  values are given in Table XVI. This table only considers the following two cases:

- (a) No pre-buckling occurs and all of the stringer and skin material is fully effective.
- (b) Buckling of the isotropic skin panels and/or local buckling of the stringers occurs, requiring the use of effective width concepts.

In some practical applications, one might encounter stringer spacings sufficiently large to justify the use of effective width concepts even in the absence of any pre-buckling. Aside from the effective width criterion, the approach would then be quite similar to that for case (b) above.

GENERAL DYNAMICS  
Convair Division

The following procedure can be used for the approximate analysis of the compressive buckling of longitudinally stiffened circular cylinders:

Step 1 - Using Table XVI, compute the values  $A_{11}$ ,  $A_{22}$ ,  $A_{12}$ ,  $A_{33}$ ,  $D_{11}$ , and  $D_{22}$ .

Step 2 - Compute the ratio  $\frac{R}{\bar{t}}$  where

$\bar{t}$  = Effective thickness from Table XVI [Note equation (6-35), Part I].

$R$  = Radius to middle surface of basic cylindrical skin.

Step 3 - Compute the effective local longitudinal radius of gyration as follows:

$$\rho_{11} = \sqrt{D_{11} A_{11}} \quad (12-1)$$

Step 4 - For cylinders having only longitudinal stiffening, compute the ratio  $\frac{L}{\rho_{11}}$  where

$L$  = Overall length of cylinder.

Step 5 - For sections which lie between rings in cylinders having both axial and hoop stiffening, compute the ratio  $\frac{a}{\rho_{11}}$  where

$a$  = Spacing between rings.

Step 6 - Using conventional methods, compute the crippling stress  $\sigma_{cc}$  for the local cross section of the shell wall (including both stringer and basic cylindrical skin).

Step 7 - Using Table XIV, select a value for the fixity factor  $C_F$ .

Step 8 - Compute the Thieleman parameter  $\eta_s$  from the following:

$$\eta_s = \frac{A_{12} + \frac{A_{33}}{2}}{\sqrt{A_{11}A_{22}}} \quad (12-2)$$

Step 9 - Use the procedure of Section 15 to establish the correlation (knock-down) factor  $\Gamma$ .

Step 10 - Compute the length parameter  $a$  as follows:

$$a = \frac{L^2}{2Rm^2 \pi^2 A_{22} \sqrt{D_{22}/A_{11}}} \quad (12-3)$$

where,

$m$  = Number of longitudinal half-waves (Use  $m^2 = C_F$ ).

For sections which lie between rings on cylinders having both axial and hoop stiffening, make the substitution  $L = a$ .

Step 11 - Using Table XV, establish the minimization factor  $\tilde{N}$ .

Step 12 - Compute the quantity

$$\text{Correction Factor} = \Gamma \tilde{N} \quad (12-4)$$

This is the correction factor denoted on the buckling curves of Section 12.2

Step 13 - For configurations made of 7075-T6 aluminum alloy, enter the buckling curves of Section 12.2 to obtain the critical longitudinal compressive buckling stress  $\sigma_{cr}$ . Section 18.2 gives the digital computer program which was used to obtain these design curves. This program may be used to obtain additional plots or single-point solutions for any material.

Step 14 - Check to insure that the configuration falls into the short-cylinder classification.

THIS IS THE FINAL STEP FOR THE METHOD.

To clarify the means by which Step 14 is to be executed, note that both the design curves of Section 12.2 and the digital computer program of Section 18.2 are applicable only to the relatively short cylinders for which the critical loading condition corresponds to  $m^2 = C_F$ . The check cited in Step 14 is to establish that these conditions are satisfied. Since most practical configurations likely to be encountered will fall into this short cylinder category, in actual

TABLE XIV - Recommended Values for the Fixity Factor  $C_F$ 

CASE DESCRIPTION	Sections Between Intermediate Rings	"Apparent Full Fixity" (See data in section 6.4.1)	Whenever Reliable K Available
RECOMMENDED $C_F$	1.0	3.75	From Figure 29

Note: When using Figure 29, one must carefully coordinate the criteria used for computing the local moment of inertia  $I$  and the rotational spring constant  $K$ .

TABLE XV - Recommended Values for the Minimization Factor  $\tilde{N}$ 

* CASE DESCRIPTION	$\alpha = 0.1$ to 10,000 Together With $\eta_s = 0.05$ to 50.0	$\alpha < 0.1$ Together With $\eta_s < 0.05$	$\alpha < 0.1$ Together With $\eta_s \geq 0.05$
RECOMMENDED $\tilde{N}$	From Figure 30	$\tilde{N} \approx \alpha$	** 0.05

\*For any cases not covered by these ranges,  $\tilde{N}$  may be found by plotting

$$\left[ \frac{1}{4\alpha\beta^4} + \frac{\alpha\beta^4}{1 + 2\eta_s^2 + \beta^4} \right] \text{ vs. } \beta^2$$

in order to find a relative minimum for the bracketed expression. This minimum is  $\tilde{N}$ . It can be shown that not more than one such relative minimum can exist.

\*\* This is an approximation which can introduce a maximum error of

$$\pm (0.05) \frac{E_{\tan}}{\sqrt{3(1-\nu^2)}} \left( \frac{R}{t} \right) \text{ psi}$$

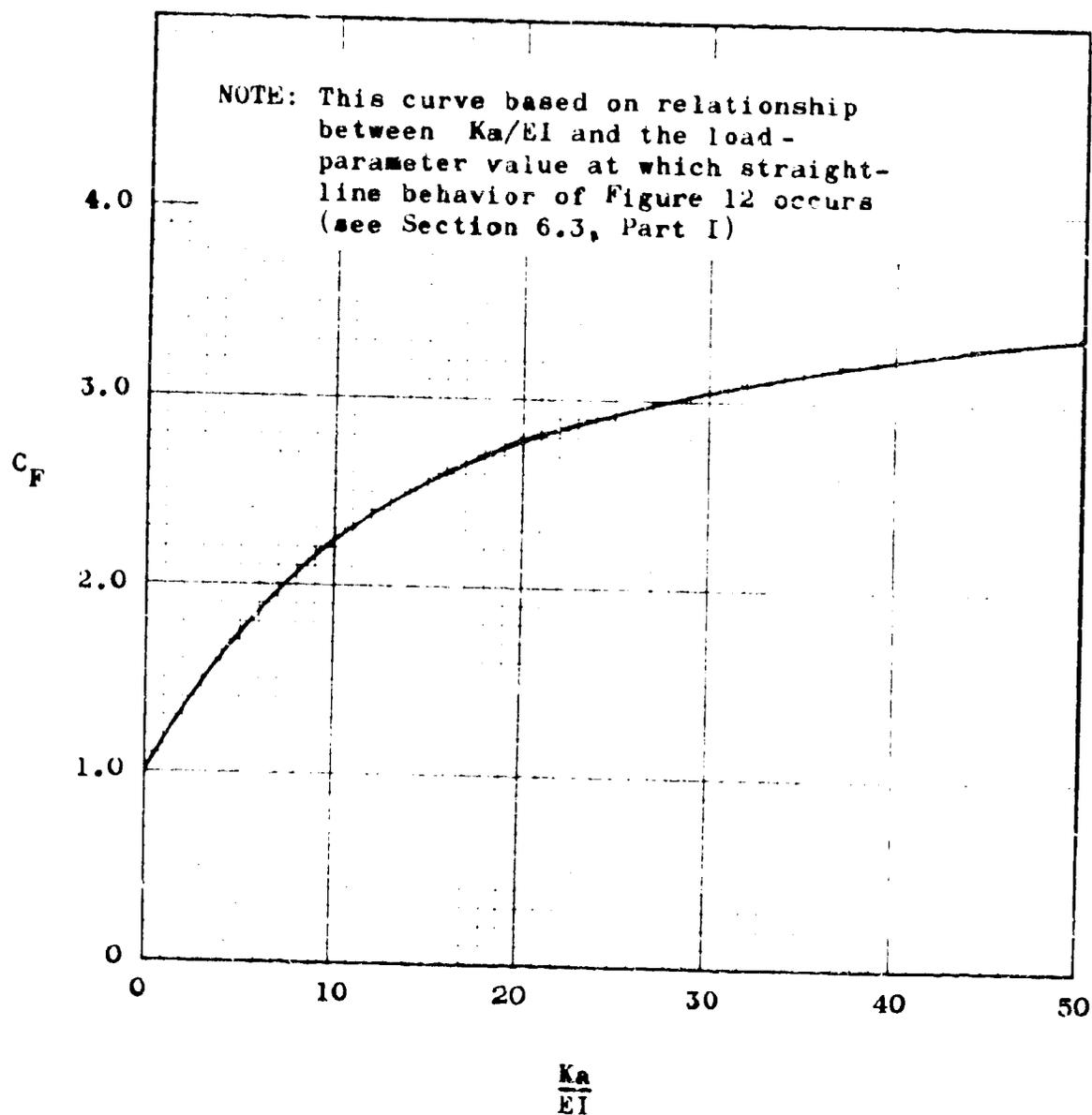


Figure 29 - Fixity Factor vs. Rotational Stiffness Parameter

GENERAL DYNAMICS  
Convair Division

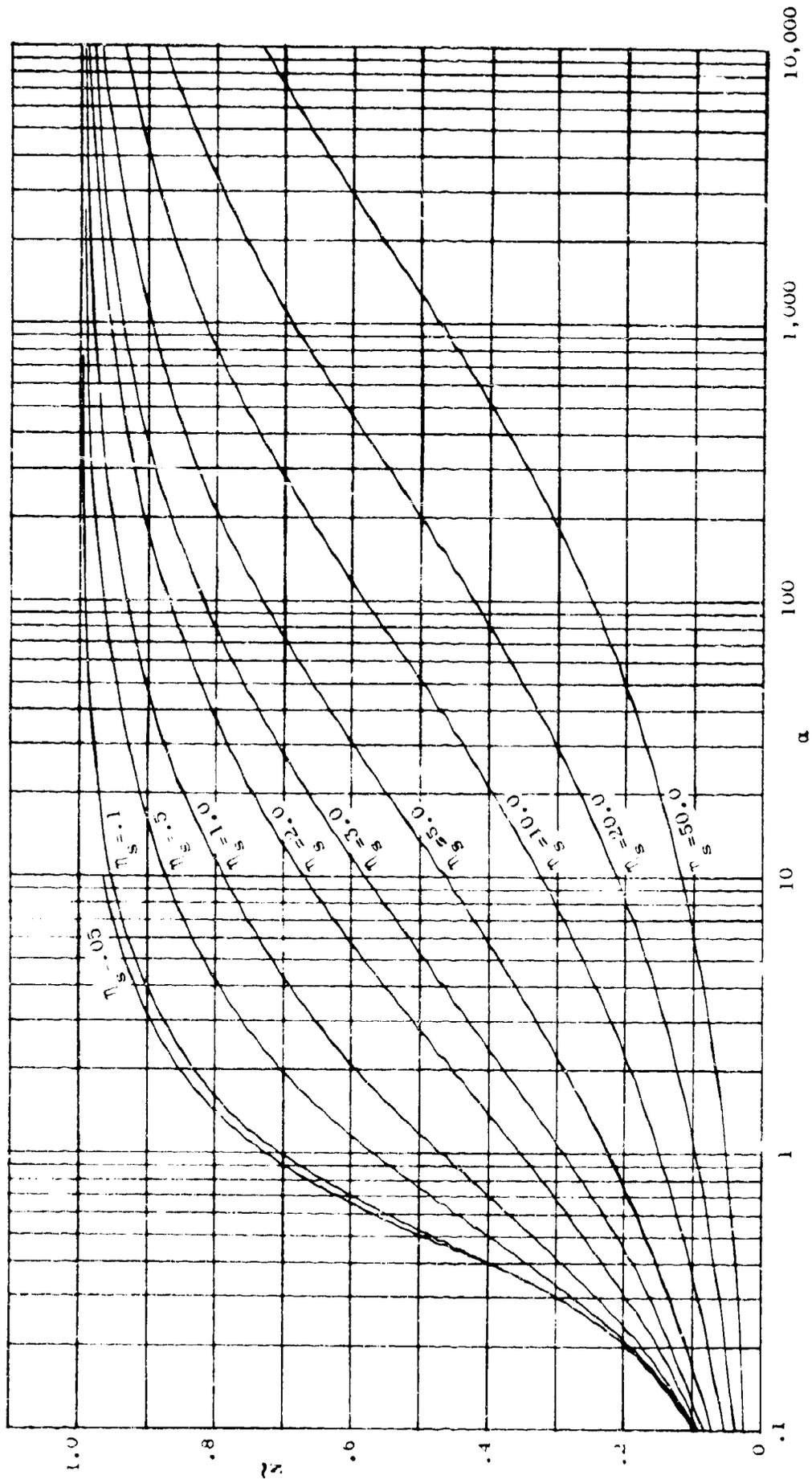


Figure 30 - Minimization Factor  $\tilde{N}$  vs.  $\alpha$   
and the Parameter  $s$  ( $F = 0$ )

practice it might not be necessary to make this check for every configuration investigated. When studying a large number of candidate designs, it will often be reasonable to start by assuming short-cylinder behavior and only make the check as a final operation for a few selected cases.

To accomplish the check for shortness in cases where

$$C_F = 1.0 \text{ or } 4.0 \tag{12-5}$$

one must investigate those situations for which

$$\Gamma = 1$$

$$m^2 = \left[ \sqrt{C_F} + C_i \right]^2 \tag{12-6}$$

and

$$C_i = 0, 1, 2, 3, \dots \tag{12-7}$$

That is, ideally one would determine the classical  $(\sigma_{cr})_{CL}$  values which respectively correspond to the following individual cases:

$$\Gamma = 1 \quad m^2 = C_F \quad = \text{Fixity Factor} \tag{12-8}$$

$$\Gamma = 1 \quad m^2 = \left[ \sqrt{C_F} + 1 \right]^2 \quad = \text{"Artificial Fixity Factor"} \tag{12-9}$$

$$\Gamma = 1 \quad m^2 = \left[ \sqrt{C_F} + 2 \right]^2 \quad = \text{"Artificial Fixity Factor"} \tag{12-10}$$

$$\Gamma = 1 \quad m^2 = \left[ \sqrt{C_F} + 3 \right]^2 \quad = \text{"Artificial Fixity Factor"} \tag{12-11}$$

↓  
etc.

↓  
etc.

For each of these situations, one would assume  $\Gamma = 1$  and repeat Steps 10 through 13, using the appropriate  $m^2$  value. Condition (12-8) must give the lowest  $(\sigma_{cr})_{CL}$  value for the short-cylinder criterion to apply. In actual practice, multiple iterations will usually prove unnecessary. Normally, one will be able to conclude that the cylinder is "short" by inspection of the results from only (12-8) and (12-9).

For cases where

$$1.0 < C_F < 4.0 \quad (12-12)$$

the procedure to check cylinder shortness is exactly the same except that non-integer  $C_i$  values must also be considered. However, as a practical expediency, it is recommended that whenever

$$\begin{aligned} C_F &= 1.00 \text{ to } 1.10 \\ \text{or} \\ C_F &= 3.75 \text{ to } 4.00 \end{aligned} \quad (12-13)$$

consideration of only the integral  $C_i$  be regarded as adequate.

The "artificial fixity factor" denoted in equations (12-9) through (12-11) should be used in exactly the same manner as the conventional fixity factor. That is, the "artificial" value corresponds to the FIXITY FACTOR denoted on the curves of Section 12.2. Likewise, the "artificial" value is inserted into Section 18.2 coding form columns that are reserved for the fixity factor. Note that the

"artificial" factor can exceed the value 4.0 whereas

$$1.0 \leq C_F \leq 4.0 \quad (12-14)$$

For those situations where equation (12-8) does not give the lowest  $(\sigma_{cr})_{CL}$  value, the short-cylinder criterion is not satisfied and neither the buckling curves of Section 12.2 nor the digital computer program of Section 18.2 is directly applicable. In such cases one may find the critical stress  $\sigma_{cr}$  by the following method:

- (a) Employing the procedure indicated by equations (12-8), (12-9), (12-10), (12-11), ---- etc., find the minimum classical stress  $(\sigma_{cr})_{CL}$ . The program of Section 18.2 may be used for this purpose.
- (b) Compute the wide-column stress  $\sigma_{wc}$  as follows:

Whenever the applicable slenderness ratio satisfies

$$\left[ \frac{L \text{ (or } a)}{\rho_{11}} \right] \geq (\sqrt{2C_F})(\pi) \left( \sqrt{\frac{E}{\sigma_{cc}}} \right) \quad (12-15)$$

use

$$\sigma_{wc} = \frac{C_F \pi^2 E_{tan}}{\left[ \frac{L \text{ (or } a)}{\rho_{11}} \right]^2} \quad (12-16)$$

Whenever the applicable slenderness ratio satisfies

$$\left[ \frac{L \text{ (or } a)}{\rho_{11}} \right] < (\sqrt{2C_F})(\pi) \left( \sqrt{\frac{E}{\sigma_{cc}}} \right) \quad (12-17)$$

use the lower of the values from equations (12-16) and (12-18)

$$\sigma_{wc} = \sigma_{cc} - \frac{\sigma_{cc}^2 \left[ \frac{I}{\rho_{11}} \text{ (or } a) \right]^2}{4 C_{FN}^2 E} \quad (12-18)$$

The digital computer program of Section 18.2 can be employed to find the applicable  $\sigma_{wc}$  by using

$$\text{Correction Factor} = (\tilde{\Gamma}) = 0 \quad (12-19)$$

(c) Find the predicted critical stress  $\sigma_{cr}$  as follows:

$$\sigma_{cr} = \sigma_{wc} + \Gamma \left[ (\sigma_{cr})_{CL} - \sigma_{wc} \right] \quad (12-20)$$

where  $\Gamma$  is found from the procedures of Section 15.

For any situations where condition (12-15) is satisfied and one might want to

(a) Include the effects of stringer eccentricities

and/or

(b) Include the effects of non-zero  $D_{12}$  and/or  $D_{33}$  values

( $\eta_p \neq 0$ )

and/or

(c) Be unrestricted insofar as maximum cylinder length is concerned, the digital computer program of Section 18.3.2 can be used. Although this program was developed primarily for application to configurations which include both longitudinal and circumferential stiffeners, it can be

specialized to longitudinally stiffened cylinders through the input values. The restriction here to cases which satisfy (12-15) is due to the fact that this program does not consider the crippling stress influence. However, further work could be readily accomplished to eliminate this limitation. In addition, note that the Section 18.3.2 program only considers boundary conditions of simple support. As an engineering approximation, it is recommended that one account for fixity influences by investigating only those buckle configurations for which  $m^2 \geq C_F$ . Taking the program result to represent the classical critical running load  $(N_x)_{CL}$ , the predicted critical running load is found as follows:

$$(N_x)_{cr} = (N_x)_{wc} + \Gamma \left[ (N_x)_{CL} - (N_x)_{wc} \right] \quad (12-21)$$

where

$(N_x)_{wc}$  = Running load corresponding to  $\sigma_{wc}$  [see Step (b) in foregoing procedure].

$\Gamma$  = From procedure of Section 15.

TABLE XVI - Recommended Formulas for the  $A_{ij}$ 's,  $D_{ij}$ 's, and  $\bar{t}$  of Longitudinally Stiffened Circular Cylinders.

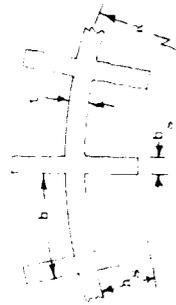
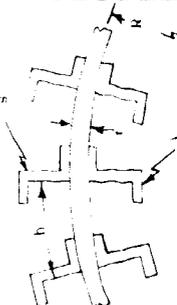
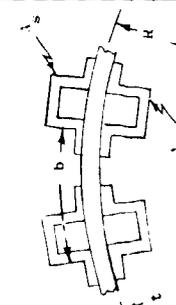
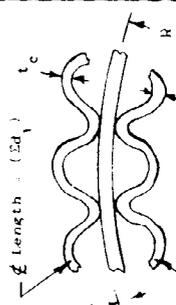
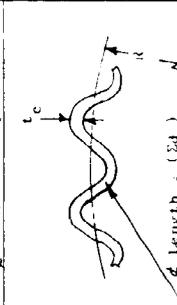
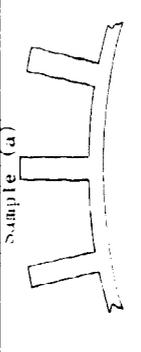
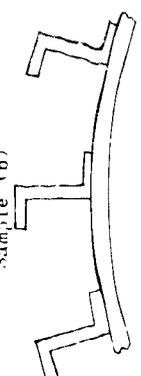
Case	Configuration	$t_x$	$A_{11}$	$A_{12}$	$A_{22}$	$A_{33}$	$D_{11}$	$D_{12}$	$D_{22}$	$D_{33}$	$D_{31}$	$D_{32}$	$\bar{t}$
A	 <p>No eccentricities. Stiffeners symmetric with respect to skin. A buckling of isotropic skin panels. No local buckling of stiffeners.</p>	$\left(\frac{2b^2h}{3s} + t\right)$	$\frac{1}{E t_x}$	$\frac{-\nu}{E t_x}$	$\frac{1}{E t_x}$	$\frac{1}{E t_x}$	$\frac{1}{E t_x}$	$\frac{-\nu}{E t_x}$	$\frac{1}{12(1-\nu^2)}$	0	0	0	$t_x \left(\frac{t}{t_x}\right)^{1/2}$
B	 <p>"</p>	$\left(\frac{2A_s}{b} + t\right)$	"	"	"	"	"	"	"	"	"	"	"
C	 <p>"</p>	"	"	"	"	"	"	"	"	"	"	"	"
D	 <p>Corrugations attached to continuous cylindrical skin. No eccentricities (corrugations symmetric with respect to skin). No local buckling.</p>	$\left[\frac{(E d_1)}{4R} t_c + t\right]$	"	$\frac{1}{E} \left[ \frac{2t_c}{\left(\frac{\Delta x}{b_x}\right)} + t \right]$	$\frac{1}{G} \left[ \frac{4\pi R}{(E d_1)} t_c + t \right]$	"	"	$\frac{E}{12(1-\nu^2)} \left[ 2t_c^3 \left(\frac{\Delta R}{\Delta C}\right) + t^3 \right]$	"	"	"	"	$\left[ \frac{2t_c^3 \left(\frac{\Delta R}{\Delta C}\right) + t^3}{t_x} \right]^{1/2}$
E	 <p>Simple corrugation. No eccentricities. No local buckling.</p>	$\left[ \frac{(E d_1)}{2\pi R} t_c \right]$	"	$\left(\frac{1}{E t_c}\right) \left(\frac{\Delta x}{b_x}\right)$	0	$\frac{1}{G} \left[ \frac{2\pi R}{(E d_1)} t_c \right]$	"	$\left[ \frac{E t_c^3}{12(1-\nu^2)} \left(\frac{\Delta R}{\Delta C}\right) \right]$	"	"	"	"	$\left[ \frac{3 \left(\frac{\Delta R}{\Delta C}\right)}{t_x} \right]^{1/2}$

TABLE XVI - Recommended Formulas for the  $A_{ij}$ 's,  $D_{ij}$ 's, and  $\bar{t}$  of Longitudinally Stiffened Circular Cylinders (Continued)

CASE	CONFIGURATION	$t_x'$	$A_{11}$	$A_{22}$	$A_{12}$	$A_{33}$	$D_{11}$	$D_{22}$	$D_{12}$	$D_{33}$	$\bar{t}$
F	Same as cases A, B, and/or C, except that buckling of isotropic skin panels occurs.	See Note (m) Below	$\frac{1}{Et_x'}$	$\frac{1}{Et_y'}$	0	$\frac{1}{Gt'}$	$Et_x'$	$\frac{Et^3}{12(1-\nu^2)}$	0	0	$\left(\frac{t}{t_x'}\right)^{1/2}$
G	Same as cases A, B, and/or C, except that local buckling of longitudinal stiffeners occurs.	"	"	$\frac{1}{Et}$	$\frac{-\nu}{Et_x'}$	$\frac{1}{Gt'}$	"	"	"	"	"
H	Same as cases A, B, and/or C, except that both buckling of isotropic skin panels and local buckling of longitudinal stiffeners occur.	"	"	$\frac{1}{Et_y'}$	0	$\frac{1}{Gt'}$	"	"	"	"	"
I	Any of cases A through H, except STIFFENING IS ECCENTRIC WITH RESPECT TO SKIN.										



Sample (a)



Sample (b)

etc.

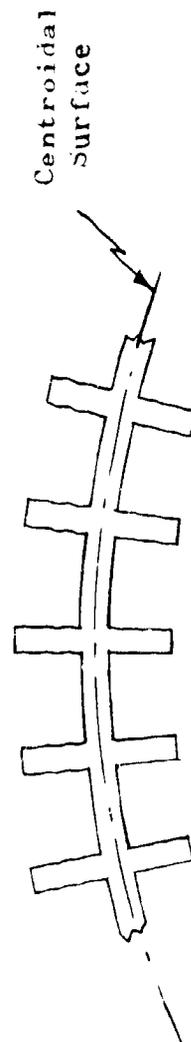
Digital computer program of Section 18.3.2 may be used to obtain classical critical running load.  
 Since this program does not account for crippling stress influences, its current application should be restricted to cases where

$$\left[ \frac{L \text{ (or } a)}{p_{11}} \right] \geq \left( \sqrt{\frac{E}{G}} \right) \left( \sqrt{\frac{E}{G}} \right)$$

This program only considers boundary conditions of simple support. Is an engineering approximation, it is recommended that account for fixity be made by investigating only those buckle patterns for which  $m^2 \geq C_p$ .

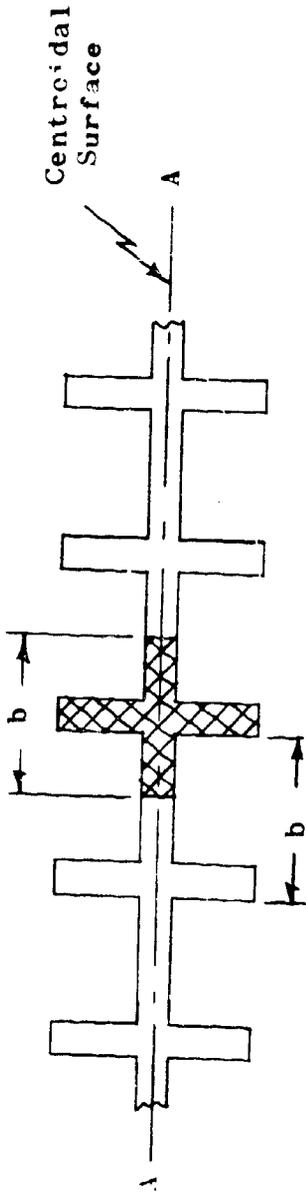
Notes for TABLE XVI:

- (a) The tabulated formulas constitute simplifications suitable for practical engineering purposes. To be rigorous, much more complicated expressions would be required.
- (b) For cases where the buckling stress exceeds the proportional limit of the stress-strain curve, use  $E_{\tan}$  and  $G_{\tan}$  in place of  $E$  and  $G$ , respectively.
- (c) The  $A_{ij}$ 's and  $D_{ij}$ 's arise out of mathematical integrations involving the distribution of the composite wall material about the cylindrical centroidal surface. (When either buckling of the isotropic skin panels or local buckling of the longitudinal stiffeners occurs, only effective widths are considered). Note that the centroidal surface has curvature of its own. Therefore, the related material distribution is equivalent to that which exists about the centroidal plane of the flat plate obtained by unfolding the composite circular shell wall into a flat configuration. All influences of curvature, in this regard, are inherent in the basic shell equations into which the  $A_{ij}$ 's and  $D_{ij}$ 's are substituted.
- (d) The quantity  $t_x$  is the wall thickness for a monocoque circular cylinder of the same radius as the middle surface of the stiffened-cylinder basic skin, and of the same total cross-sectional area as the actual composite stiffened wall, including both skin and stringers. The cross section referred to here is obtained by passing a plane through the entire cylinder, normal to the axis of revolution. Since  $t_x$  is used only where neither buckling of the isotropic skin panels nor local buckling of the longitudinal stiffeners occurs, no effective width considerations are needed here.
- (e) The quantity  $I_x$  is the local longitudinal centroidal running moment of inertia for the flat configuration obtained by unfolding the composite circular wall. Since  $I_x$  is used only where neither buckling of the isotropic skin panels nor local buckling of the longitudinal stiffeners occurs, no effective width considerations are needed here. That is, all of the skin and stringer material is to be included in the  $I_x$  computation. For example, take the case of a cylinder having a local wall cross section of the type:



Notes for TABLE XVI: (Continued)

In such a case, one must consider the unfolded geometry shown below.



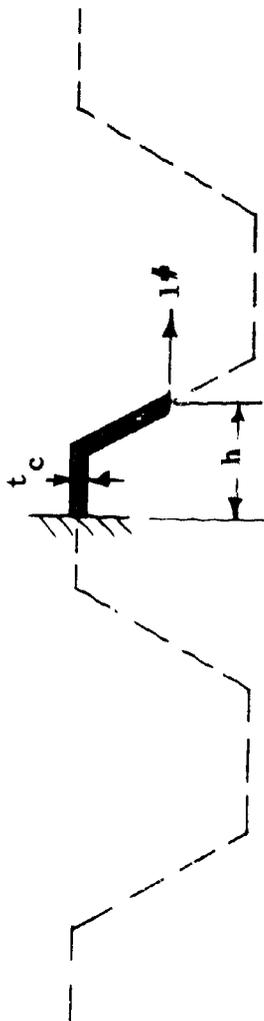
After computing  $I_{A-A}$  for the cross-hatched area shown,  $I_x$  is found as follows:

$$I_x = \frac{I_{A-A}}{b}$$

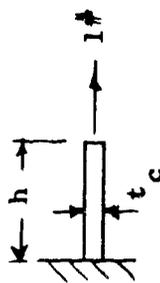
- (f) The term  $(1-\nu^2)$  has been omitted from the formulas for  $D_{11}$  since the specified configurations provide incomplete restraint to anticlastic bending (see Glossary). However, the  $(1-\nu^2)$  factor has been retained in all of the formulas for  $D_{22}$  since the usually broad axial extent of the skin panel affords restraint to anticlastic bending in the same manner as that customarily recognized for flat plates.
- (g) The quantities  $D_{12}$  and  $D_{33}$  are assumed equal to zero in the interest of simplicity. This is a conservative practice.
- (h) The quantity  $A_s$  is the stiffener cross-sectional area denoted in the figures for cases B and C. This value should not include any of the basic cylindrical skin.
- (i) The symbol  $(\Sigma d_i)$  is used to denote the total peripheral length of a single corrugation center-line for the wave-type cross section obtained by passing a plane through the entire cylinder, normal to the axis of revolution. Hence,  $(\Sigma d_i) > 2\pi R$  and the total area for the single corrugation cross section may be taken equal to  $(\Sigma d_i)(t_c)$ .

Notes for TABLE XVI: (Continued)

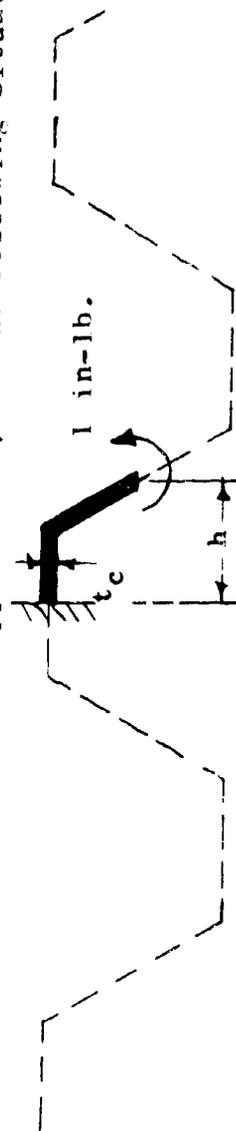
(j) The factor  $(\Delta_x / \delta_x)$  accounts for the accordion-like hoop extensional flexibility of a corrugation. For example, consider a corrugation of the type:  In this case, the quantity  $\Delta_x$  is the linear deflection, in the direction of loading, for the point of load application, in the following situation:



The quantity  $\delta_x$  is the linear deflection, in the direction of loading, for the point of load application, in the following situation:

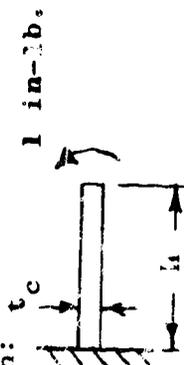


(k) The factor  $(\frac{\delta\theta}{\Delta\theta})$  accounts for the increased length over which circumferential bending occurs in the case of a corrugation. For example, consider a corrugation of the same type as in note (j) above. In this case, the quantity  $\Delta\theta$  is the rotation, in the direction of loading, for the point of load application, in the following situation:



## Notes for TABLE XVI: (Continued)

The quantity  $\delta\theta$  is the rotation, in the direction of loading, for the point of load application, in the following situation:



Hence it follows that  $\left(\frac{\delta\theta}{\Delta\theta}\right) = \frac{2\pi R}{(Ed)^3}$ . It is pointed out that the skin thicknesses  $t$  and  $t_c$  enter into the  $\bar{t}$  expression through the elastic constant  $D_{22}$ . Hence, wherever the factor  $\left(\frac{\delta\theta}{\Delta\theta}\right)$  appears in the  $D_{22}$  formula, the expression for  $\bar{t}$  must likewise include this factor.

- (1) For any cases where buckling of the isotropic skin panels and/or local buckling of the longitudinal stiffeners occurs, the listed formulas assume the same effective widths are used for both stressed area considerations and stiffness computations. More refined effective width concepts recognize differences in this regard.
- (m) The quantity  $t'_x$  differs from  $t_x$  [defined in note (d) above] only in that the former does not include all of the skin and stringer material. Since  $t'_x$  is used only where buckling of the isotropic skin panels and/or local buckling of the longitudinal stiffeners occurs, this value should be based on the total areas of non-prebuckled elements plus only the effective areas of the pre-buckled elements.
- (n) The quantity  $t'_y$  is an effective skin thickness value used to describe the hoop extensional stiffness of isotropic skin panels that have been buckled by axial compression. That is,
- $$t'_y = \left[ \frac{a_e}{L(\text{or } a)} \right] (t)$$
- where  $a_e$  is the related effective skin width. Methods for determination of  $a_e$  have not yet been developed to the same degree as the formulas for the effective width ( $b_e$ ) by which the stringer cross sections are augmented. Hence, engineering judgement must be employed in the selection of appropriate  $a_e$  values.

Notes for TABLE XVI: (Continued)

- (o) The quantity  $t'$  is an effective skin thickness which accounts for the reduced in-plane shear stiffness of isotropic skin panels which have been buckled by longitudinal compressive loading. It is recommended that  $t'$  be found using the approach of reference 37.
- (p) The quantity  $I'_x$  differs from  $I_x$  [defined in note (e) above] only in that the former does not include all of the skin and stringer material. Since  $I'_x$  is used only where buckling of the isotropic skin panels and/or local buckling of the longitudinal stiffeners occurs, this value should be based on the total areas of non-prebuckled elements together with only the effective areas of the prebuckled elements.

12.2 Design Curves for Bare 7075-T6 Aluminum Alloy

All of the design curves in this section are based on the following material properties:

$$E = 10.5 \times 10^6 \text{ psi}$$

$$\nu = 0.33$$

$$\sigma_{cy} = 67,000 \text{ psi}$$

$$\text{Ramberg-Osgood } \sigma_{.7} = 70,000 \text{ psi}$$

$$\text{Ramberg-Osgood } n = 10$$

In cases where

(a) buckling of the isotropic skin panels (see Glossary)

and/or

(b) local buckling of the longitudinal stiffeners

(see Glossary)

occurs, these curves assume the effective widths of stressed material to be the same as the effective widths used in computing the  $A_{ij}$ 's and  $D_{ij}$ 's. More refined effective width concepts recognize differences between stressed-area phenomena and stiffness mechanisms. To include such features, some modification would be required to the digital computer program of Section 18 2.

Table XVII lists the families provided here:

TABLE XVII - Table of Contents for the Design Curves  
"Compressive Buckling Stress for Longi-  
tudinally Stiffened 7075-T6 Al Alloy  
Circular Cylinders"

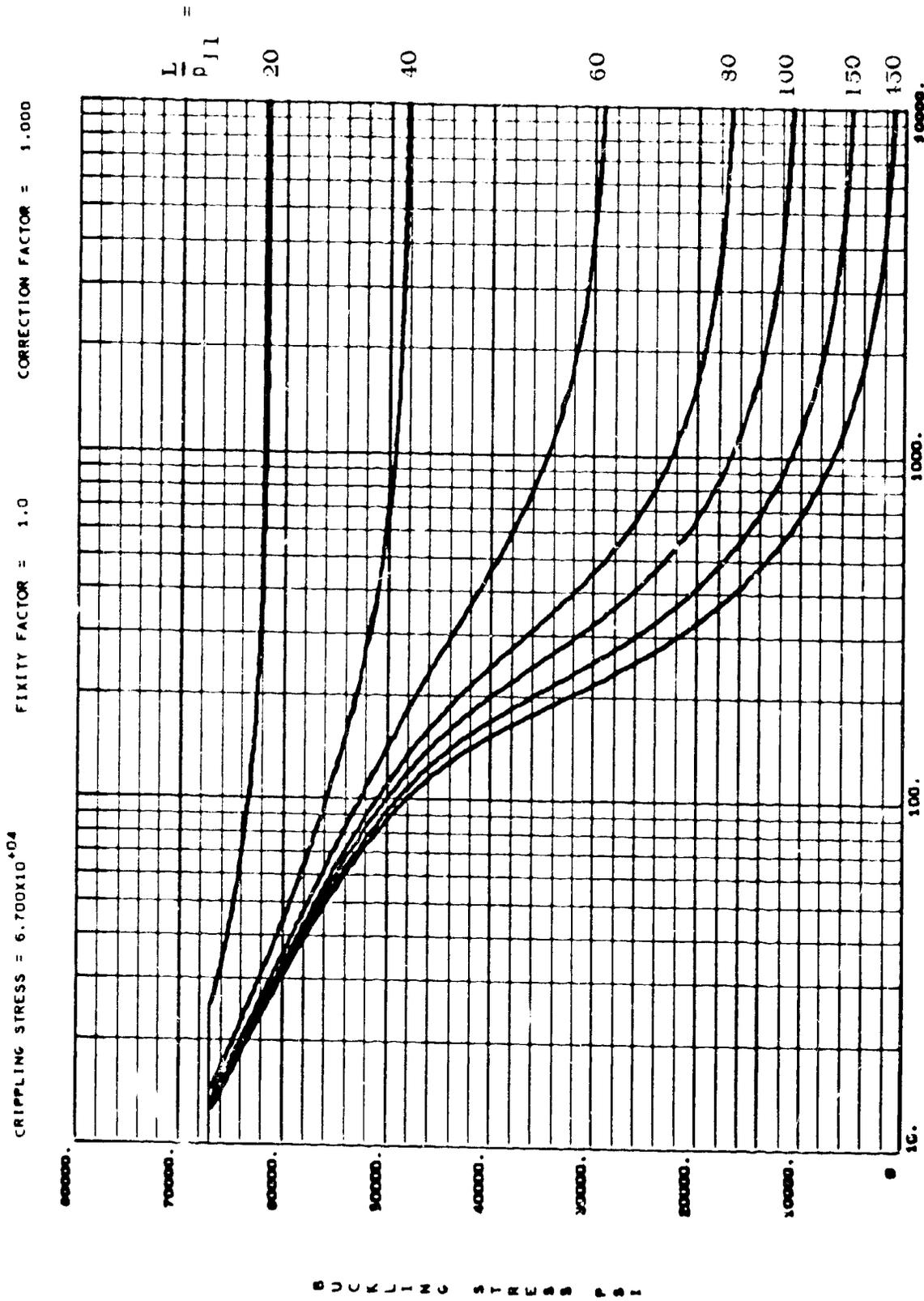
<u>Figure</u> <u>Number</u>	<u>Crippling</u> <u>Stress</u> <u><math>\sigma_{cc}</math></u>	<u>Fixity</u> <u>Factor</u>	<u>Correction</u> <u>Factor</u> <u>(<math>\Gamma_N</math>)</u>	<u>Page</u>
31(a)	67,000	1.0	1.0	210
31(b)	67,000	1.0	0.8	211
31(c)	67,000	1.0	0.6	212
31(d)	67,000	1.0	0.4	213
31(e)	67,000	2.0	1.0	214
31(f)	67,000	2.0	0.8	215
31(g)	67,000	2.0	0.6	216
31(h)	67,000	2.0	0.4	217
31(i)	67,000	3.0	1.0	218
31(j)	67,000	3.0	0.8	219
31(k)	67,000	3.0	0.6	220
31(l)	67,000	3.0	0.4	221
31(m)	67,000	4.0	1.0	222
31(n)	67,000	4.0	0.8	223
31(o)	67,000	4.0	0.6	224
31(p)	67,000	4.0	0.4	225
32(a)	60,000	1.0	1.0	226
32(b)	60,000	1.0	0.8	227
32(c)	60,000	1.0	0.6	228
32(d)	60,000	1.0	0.4	229
32(e)	60,000	2.0	1.0	230
32(f)	60,000	2.0	0.8	231
32(g)	60,000	2.0	0.6	232
32(h)	60,000	2.0	0.4	233

TABLE XVII - Table of Contents for the Design Curves  
"Compressive Buckling Stress for Longi-  
tudinally Stiffened 7075-T6 Al Alloy  
Circular Cylinders" (Continued)

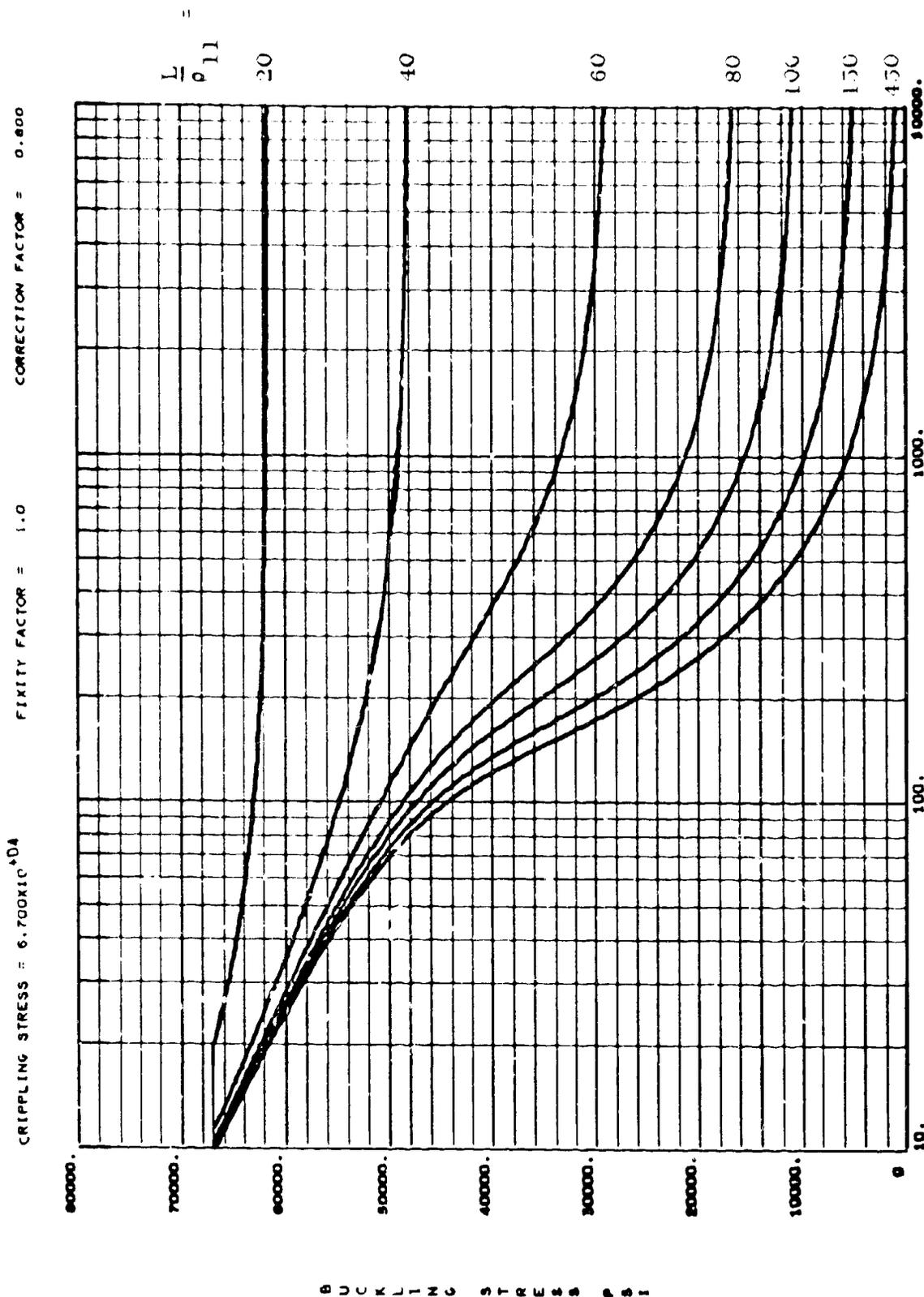
<u>Figure</u> <u>Number</u>	<u>Crippling</u> <u>Stress</u> <u><math>\sigma_{cc}</math></u>	<u>Fixity</u> <u>Factor</u>	<u>Correction</u> <u>Factor</u> <u>(<math>\Gamma_N</math>)</u>	<u>Page</u>
32(i)	60,000	3.0	1.0	234
32(j)	60,000	3.0	0.8	235
32(k)	60,000	3.0	0.6	236
32(l)	60,000	3.0	0.4	237
32(m)	60,000	4.0	1.0	238
32(n)	60,000	4.0	0.8	239
32(o)	60,000	4.0	0.6	240
32(p)	60,000	4.0	0.4	241
33(a)	50,000	1.0	1.0	242
33(b)	50,000	1.0	0.8	243
33(c)	50,000	1.0	0.6	244
33(d)	50,000	1.0	0.4	245
33(e)	50,000	2.0	1.0	246
33(f)	50,000	2.0	0.8	247
33(g)	50,000	2.0	0.6	248
33(h)	50,000	2.0	0.4	249
33(i)	50,000	3.0	1.0	250
33(j)	50,000	3.0	0.8	251
33(k)	50,000	3.0	0.6	252
33(l)	50,000	3.0	0.4	253
33(m)	50,000	4.0	1.0	254
33(n)	50,000	4.0	0.8	255
33(o)	50,000	4.0	0.6	256
33(p)	50,000	4.0	0.4	257

TABLE XVII - Table of Contents for the Design Curves  
"Compressive Buckling Stress for Longi-  
tudinally Stiffened 7075-T6 Al Alloy  
Circular Cylinders" (Continued)

<u>Figure</u> <u>Number</u>	<u>Crippling</u> <u>Stress</u> $\sigma_{cc}$	<u>Fixity</u> <u>Factor</u>	<u>Correction</u> <u>Factor</u> ( $\Gamma_N$ )	<u>Page</u>
34(a)	40,000	1.0	1.0	258
34(b)	40,000	1.0	0.8	259
34(c)	40,000	1.0	0.6	260
34(d)	40,000	1.0	0.4	261
34(e)	40,000	2.0	1.0	262
34(f)	40,000	2.0	0.8	263
34(g)	40,000	2.0	0.6	264
34(h)	40,000	2.0	0.4	265
34(i)	40,000	3.0	1.0	266
34(j)	40,000	3.0	0.8	267
34(k)	40,000	3.0	0.6	268
34(l)	40,000	3.0	0.4	269
34(m)	40,000	4.0	1.0	270
34(n)	40,000	4.0	0.8	271
34(o)	40,000	4.0	0.6	272
34(p)	40,000	4.0	0.4	273

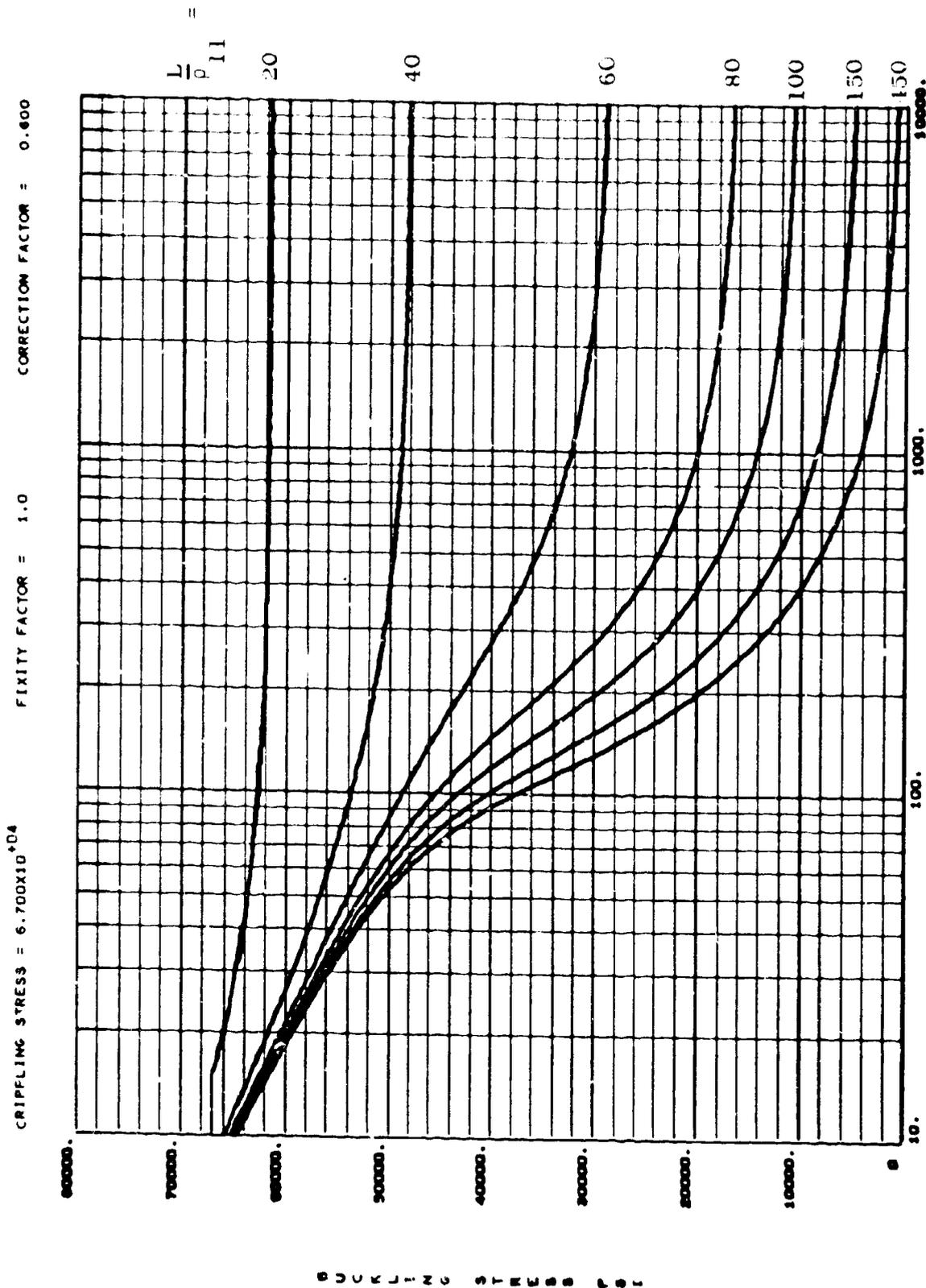


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 51 (a) - (see Table XVII)

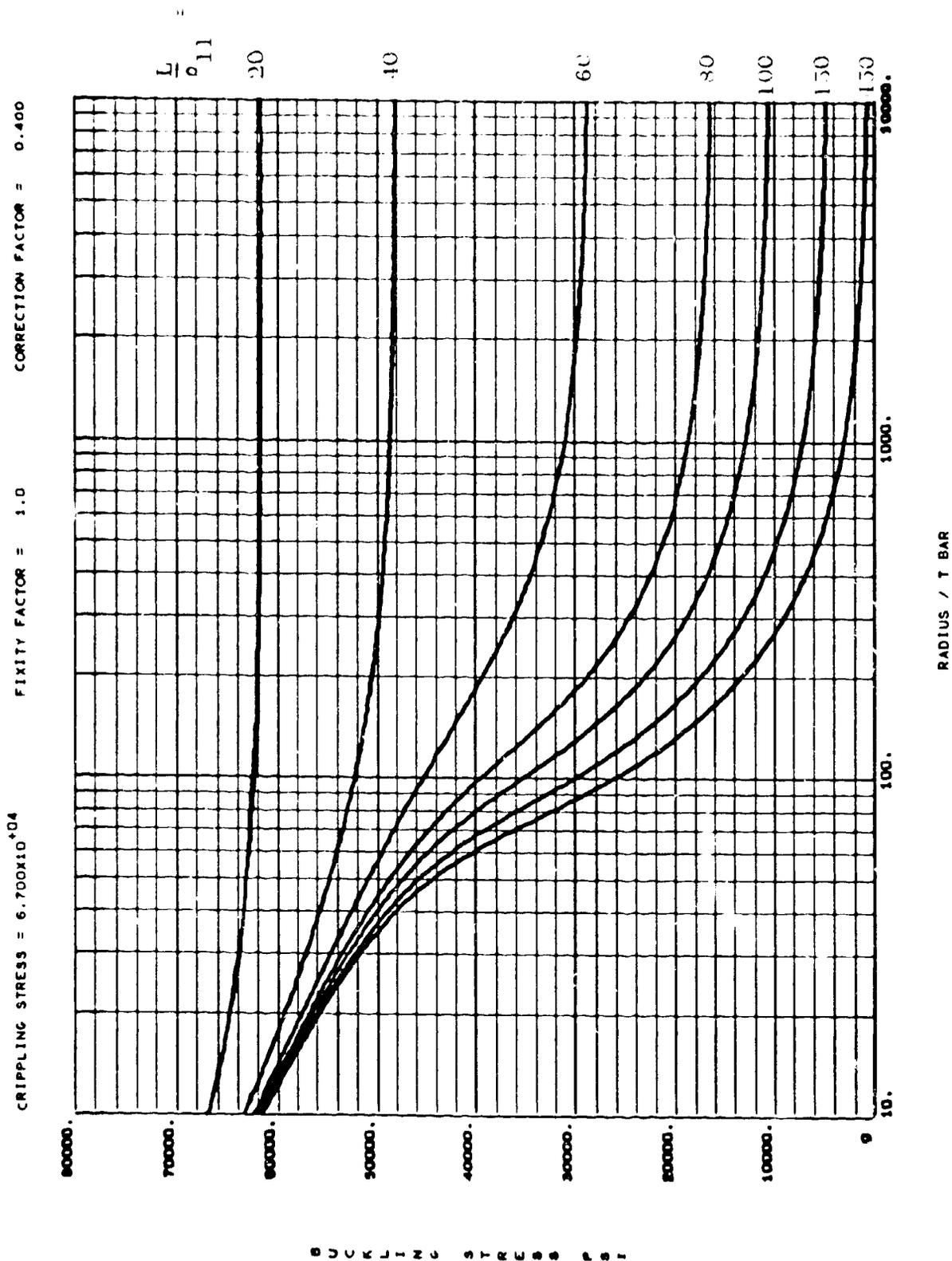


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

(Continued from Page XVII)

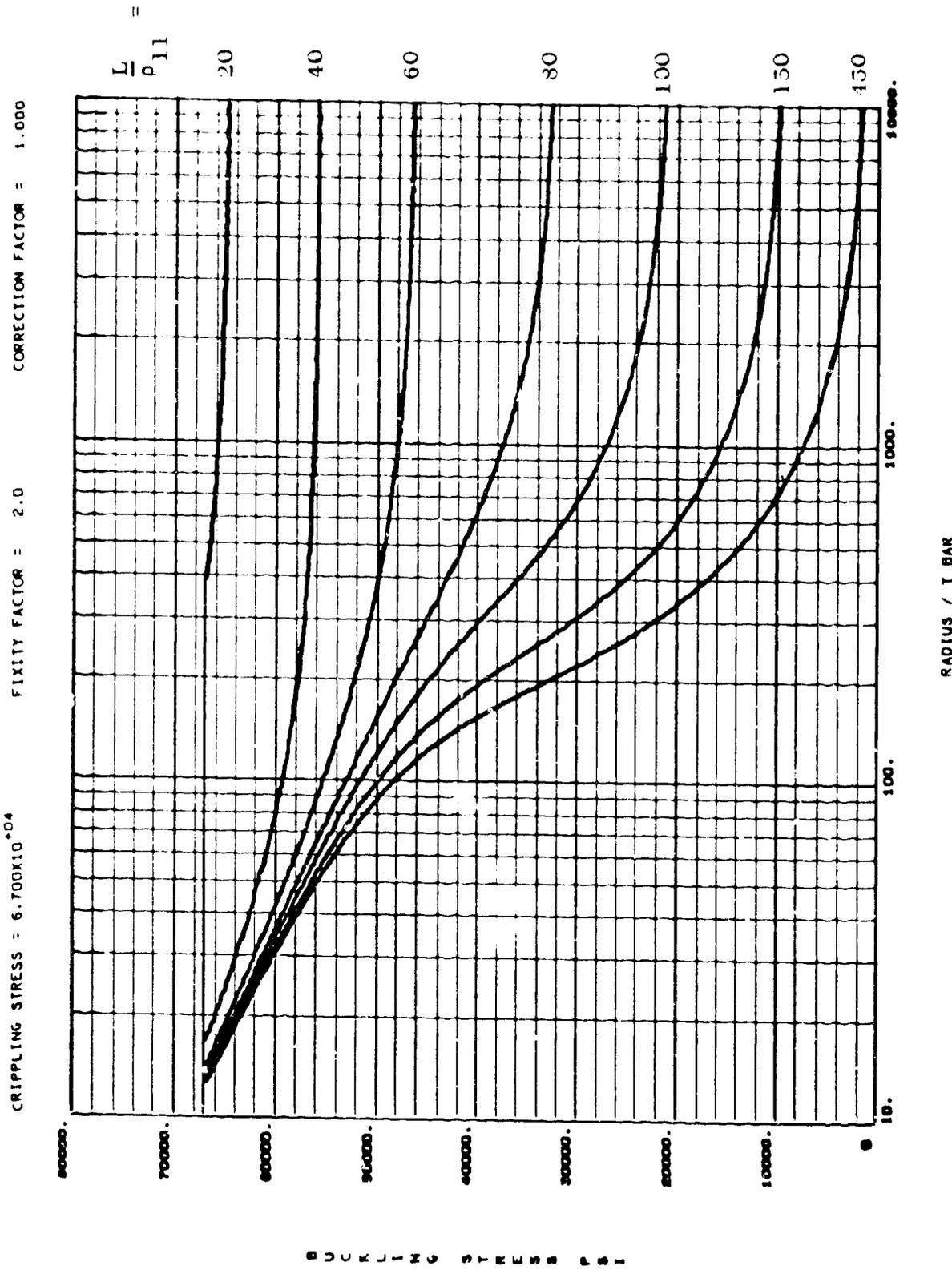


COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS  
 Figure 31 (c) - (See Table XVII)



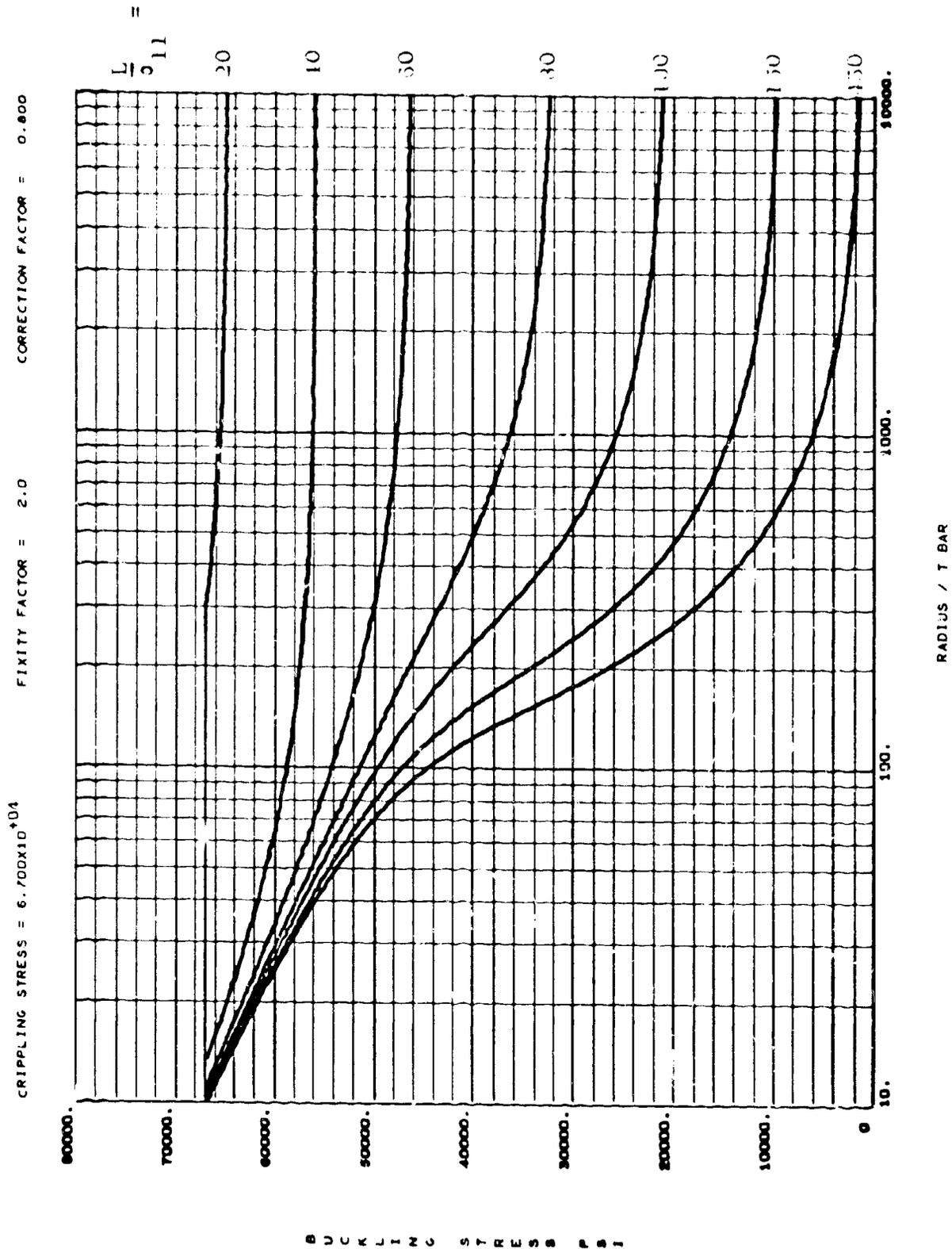
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

GENERAL DYNAMICS CORPORATION (GDC)

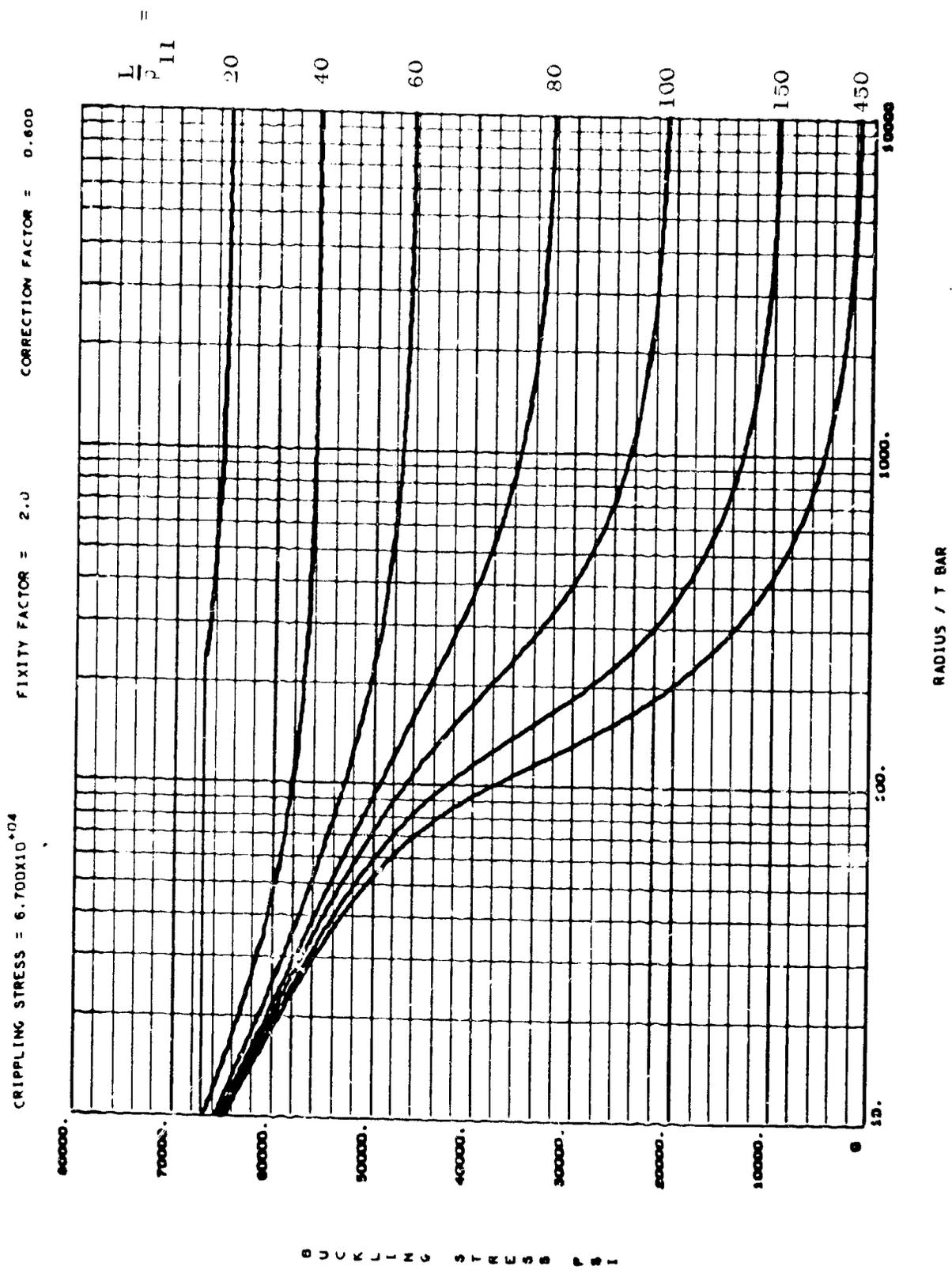


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

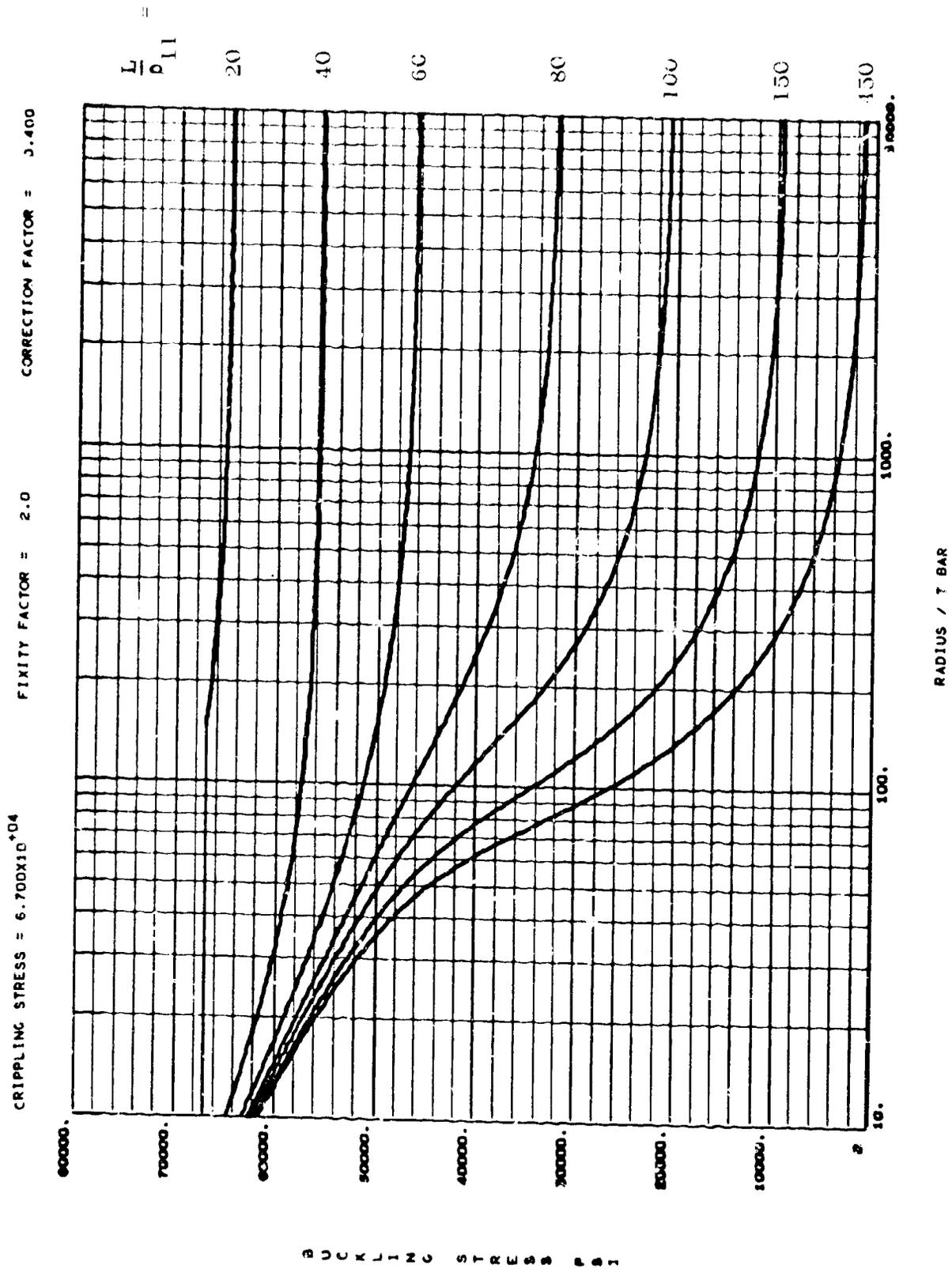
Figure 31 (e) - (See Table XVII)



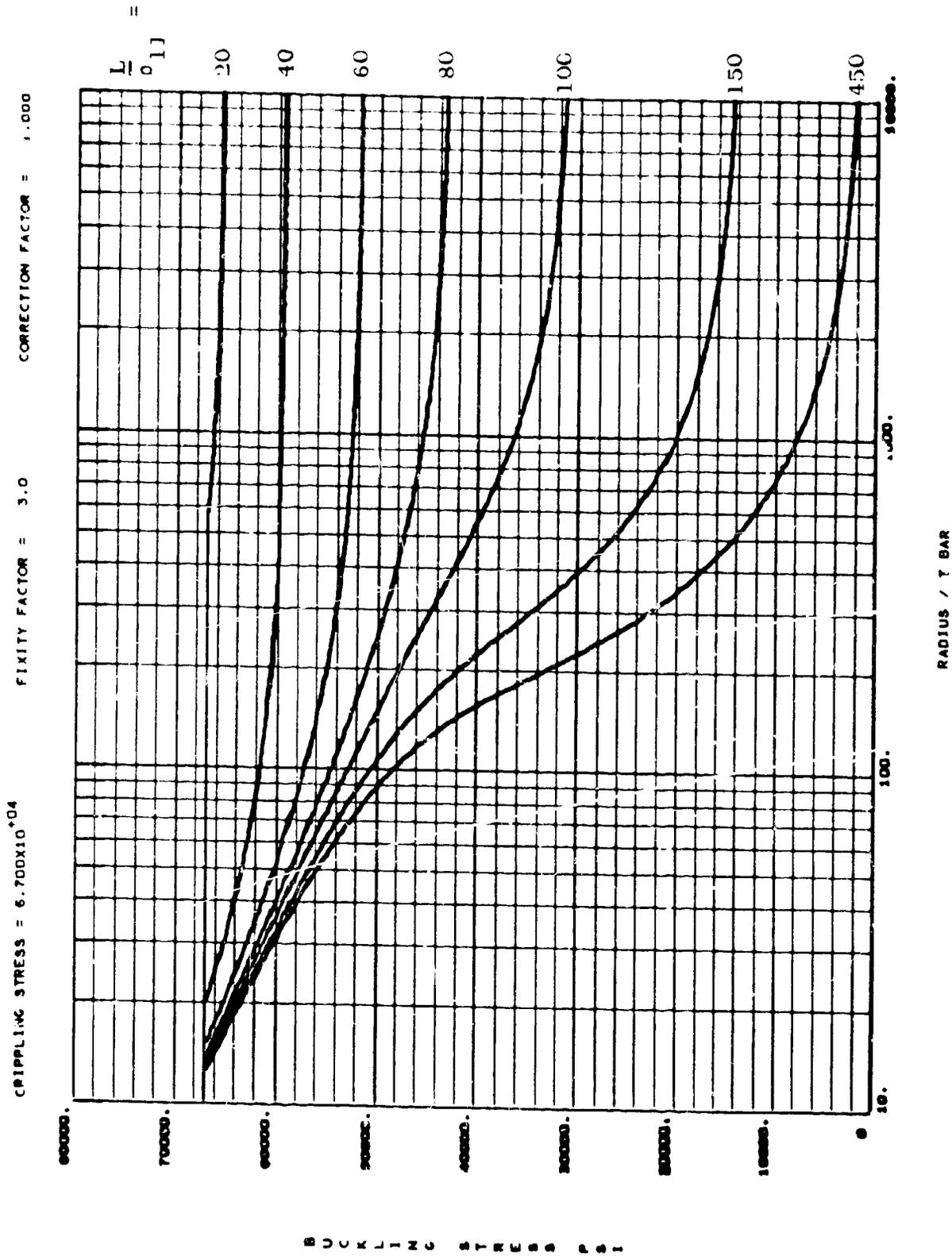
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 31 (F) - (See Table XVII)



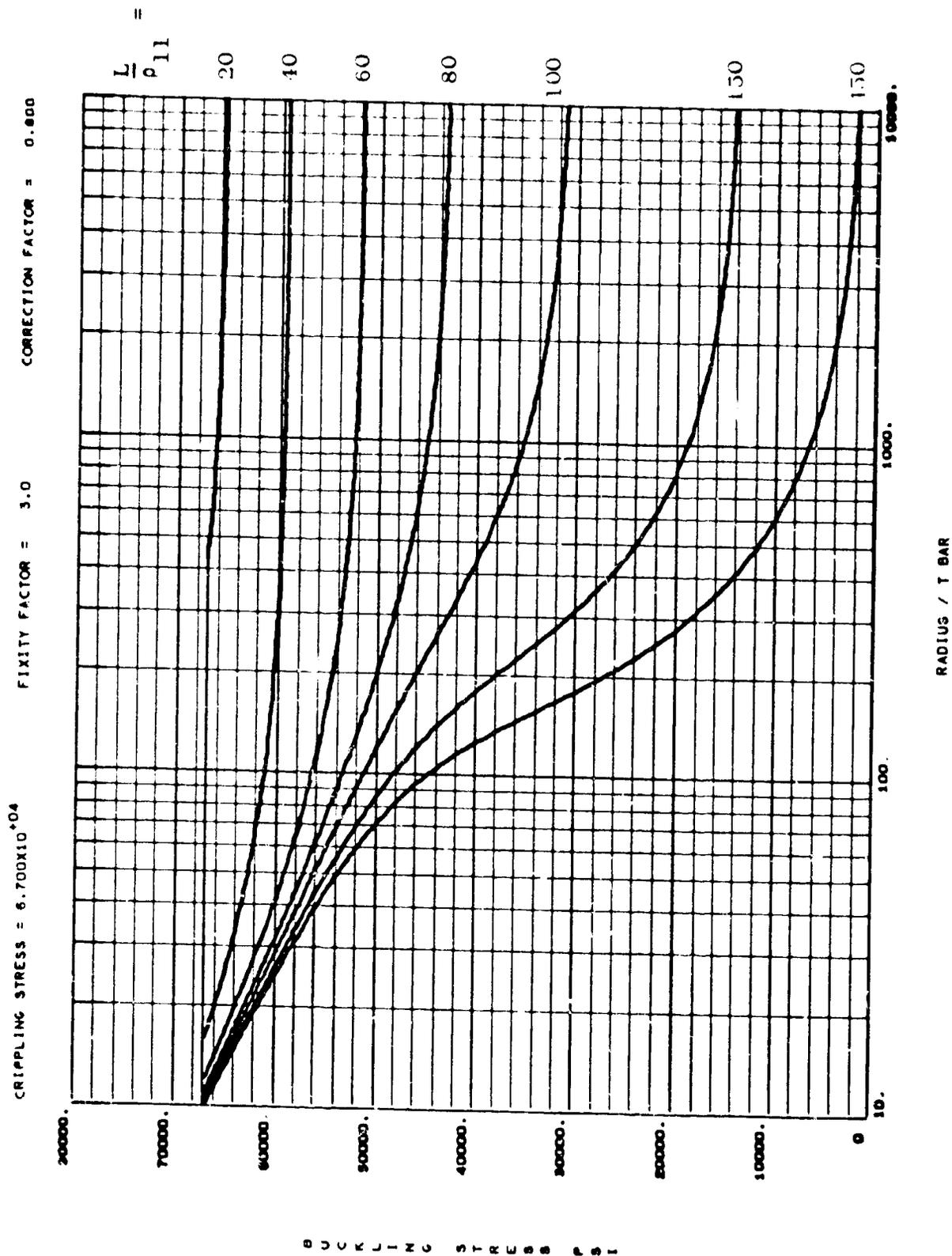
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 31 (g) - (See Table XVII)



COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 51 (h) - (see Table XVII)

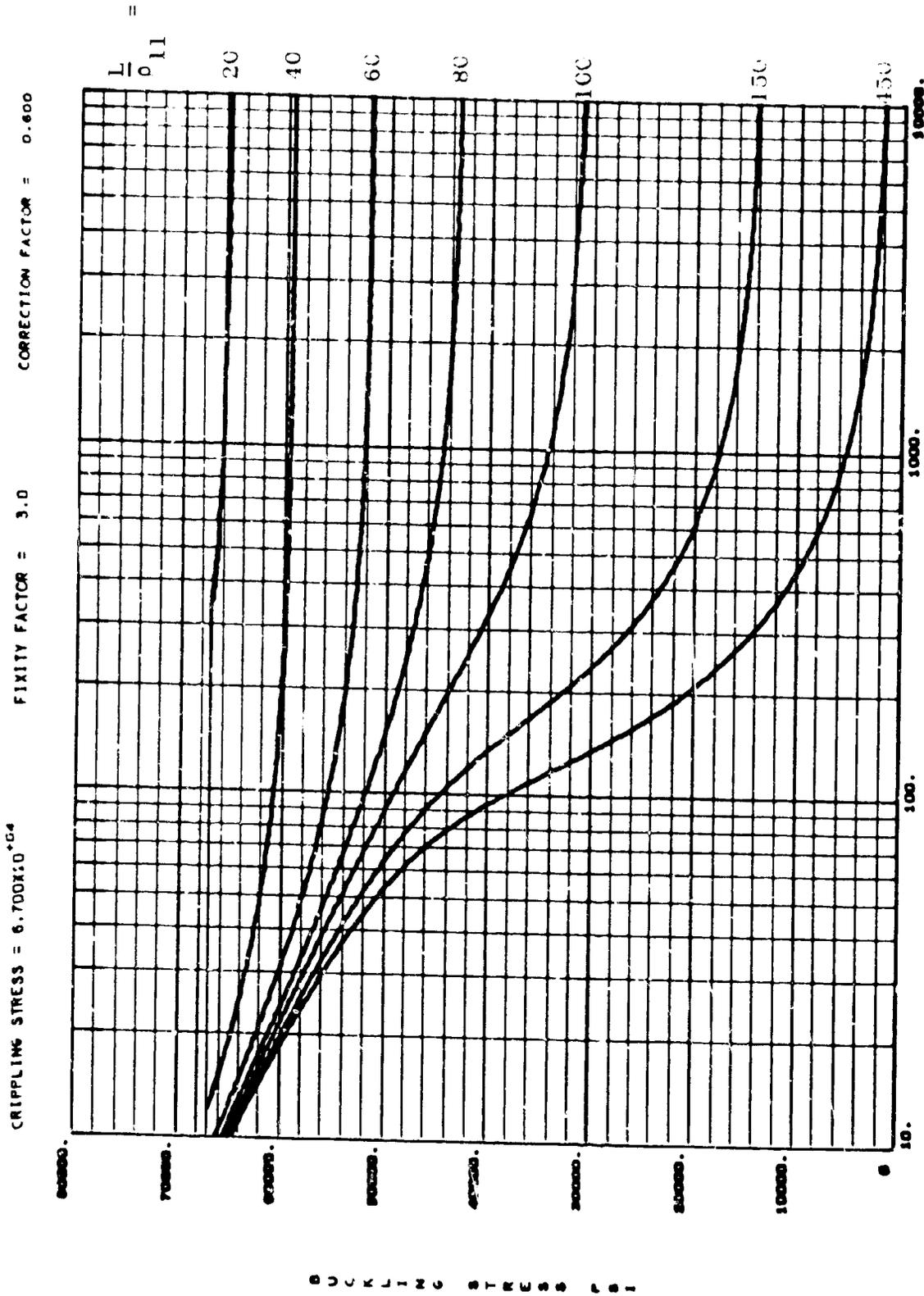


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 31 (i) - (See Table XVII)



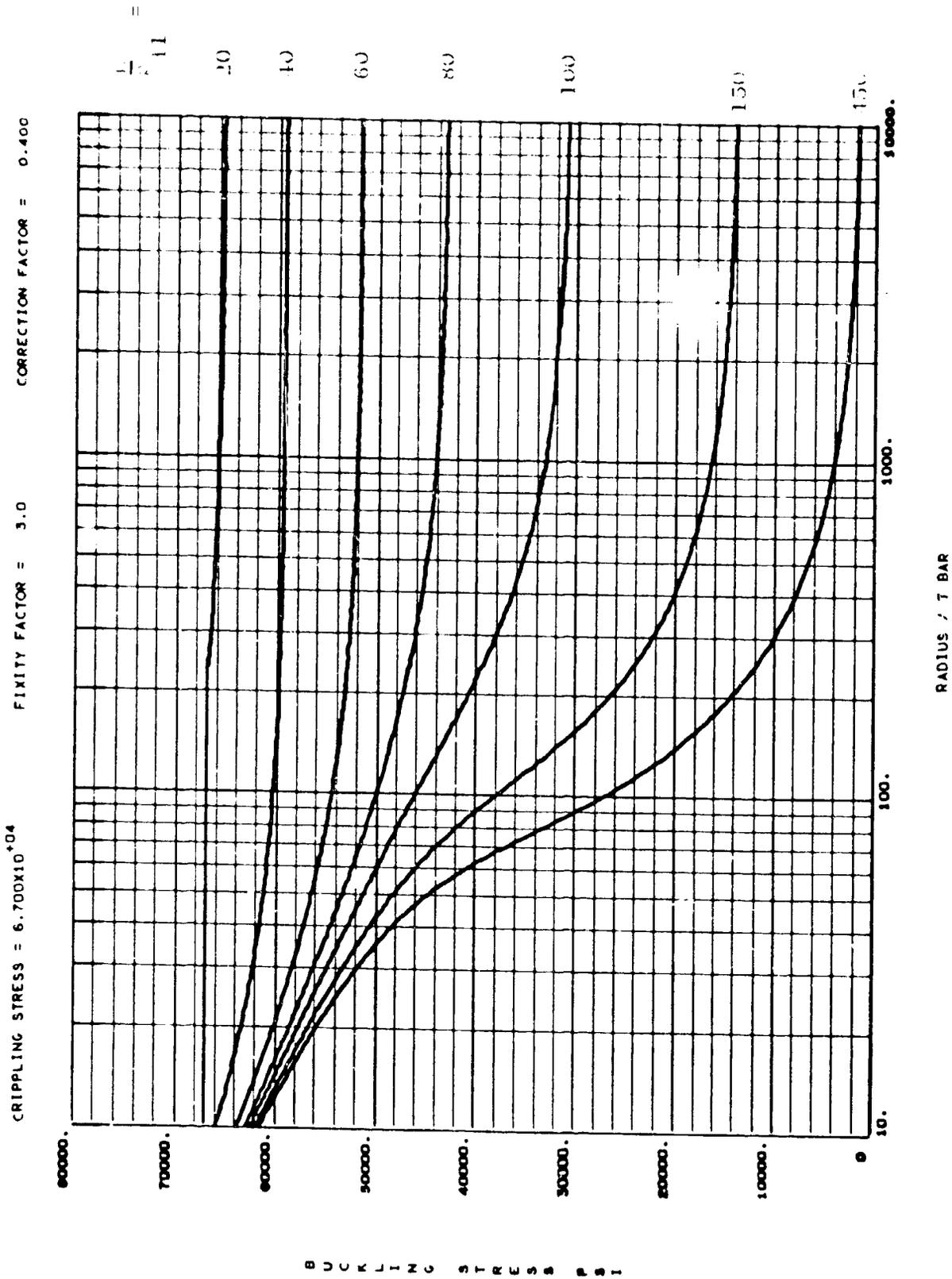
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 31 (j) - (See Table VII)



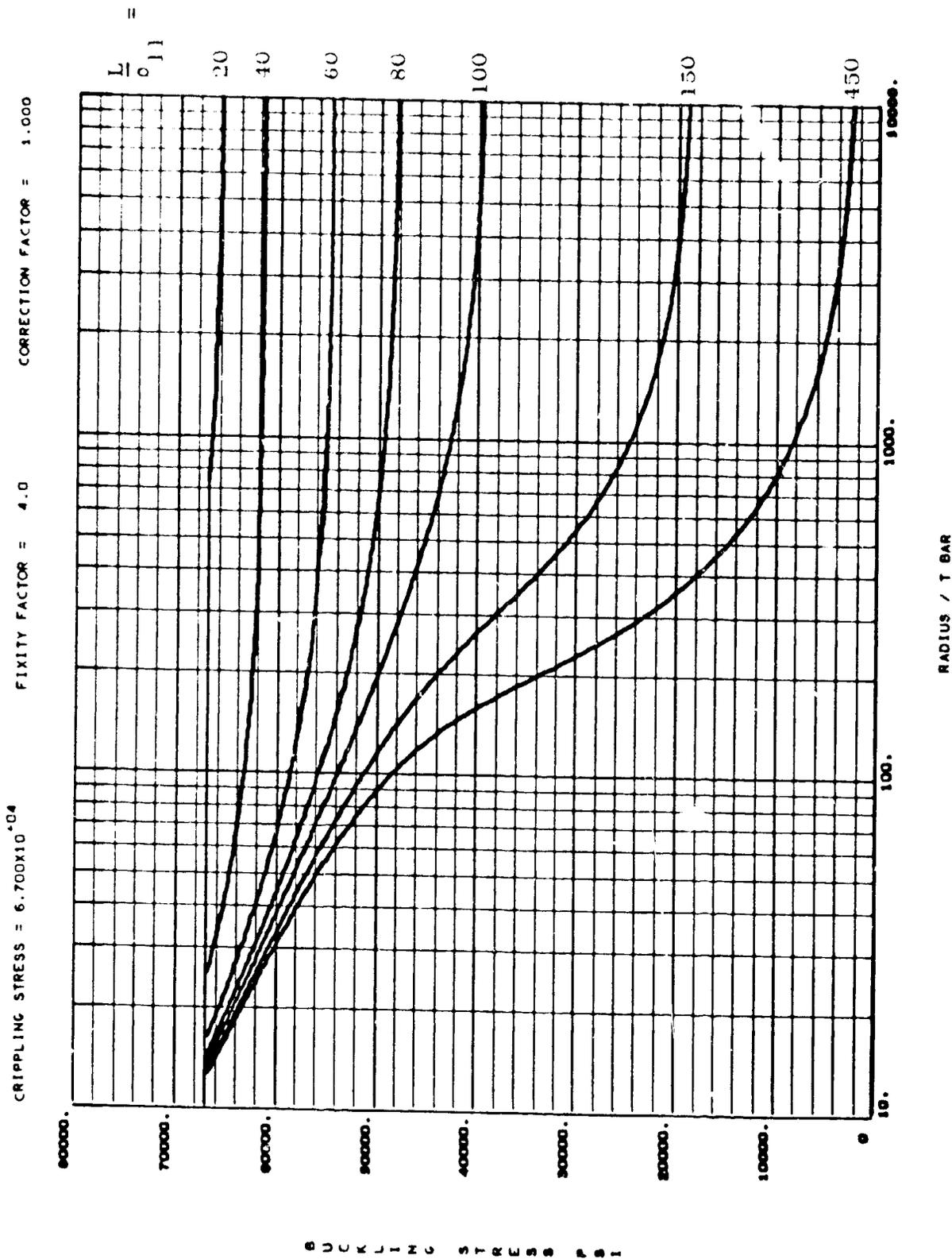
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 51 (k) - (See Table XVII)



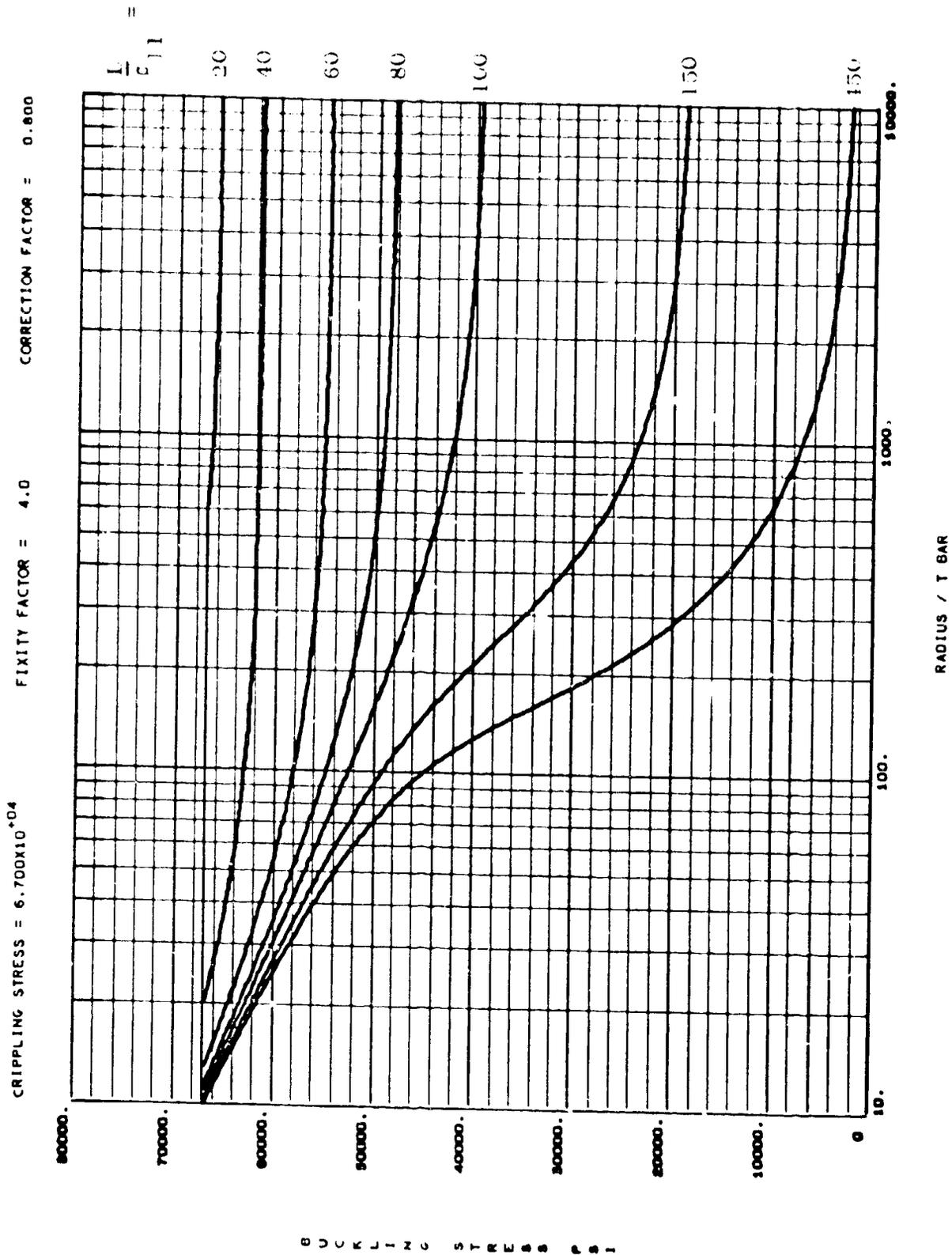
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 51 (1) - (See Table XVII)



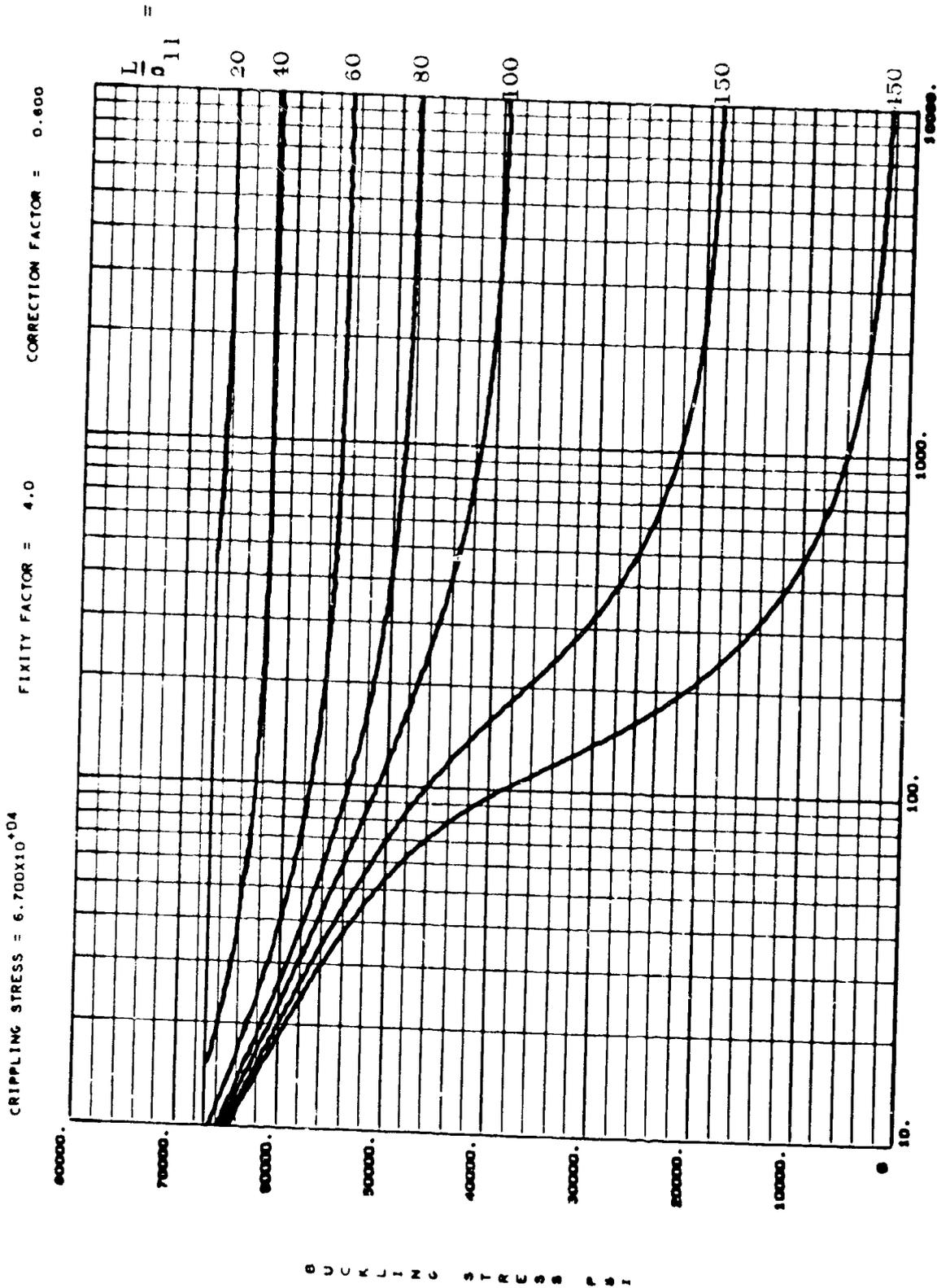
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 31 (m) - (see Table XVII)



COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

(Figure 51 (n) - (see Table XVII))



COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 51 (a) - (see Table XVII)

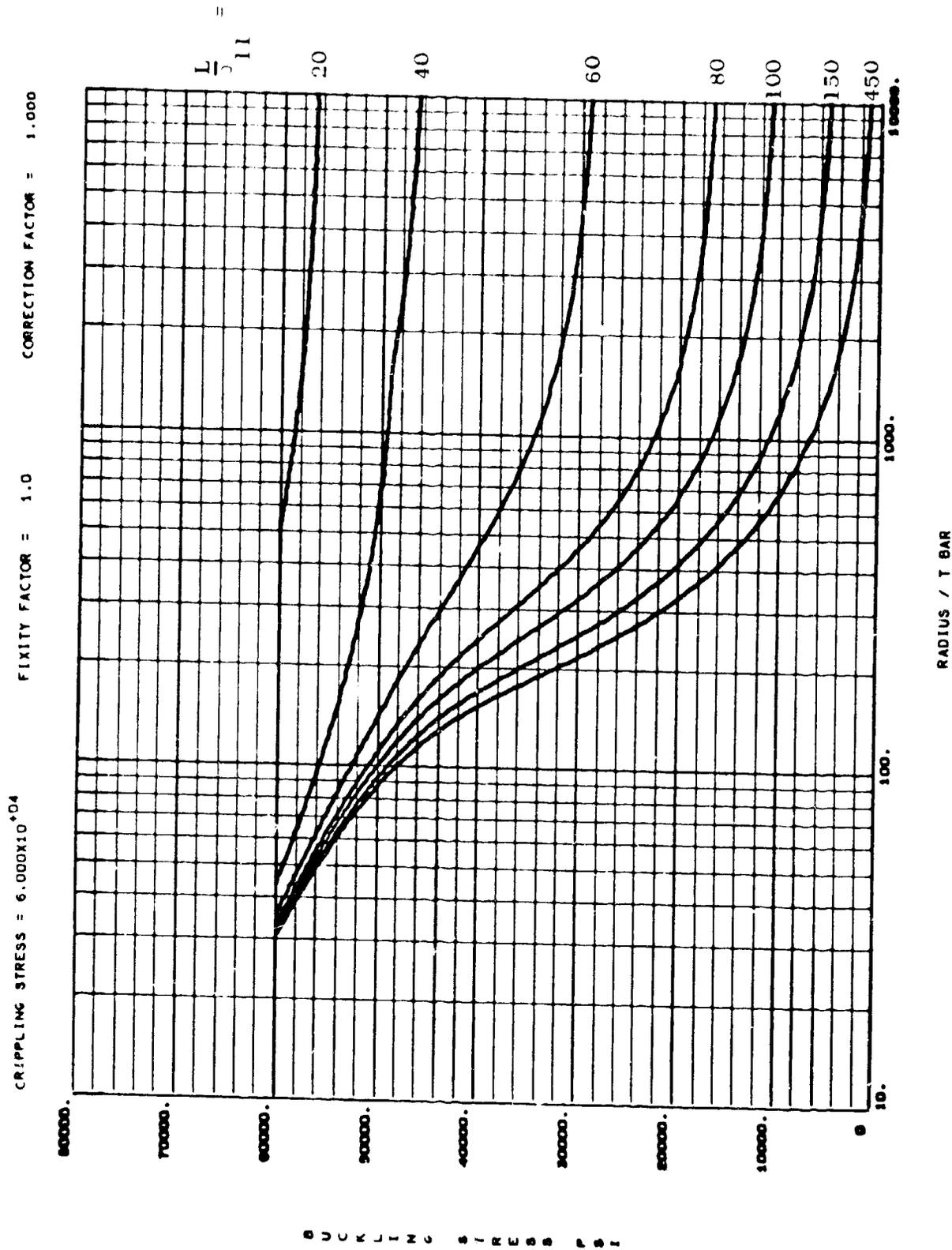


RADIUS / T BAR

COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

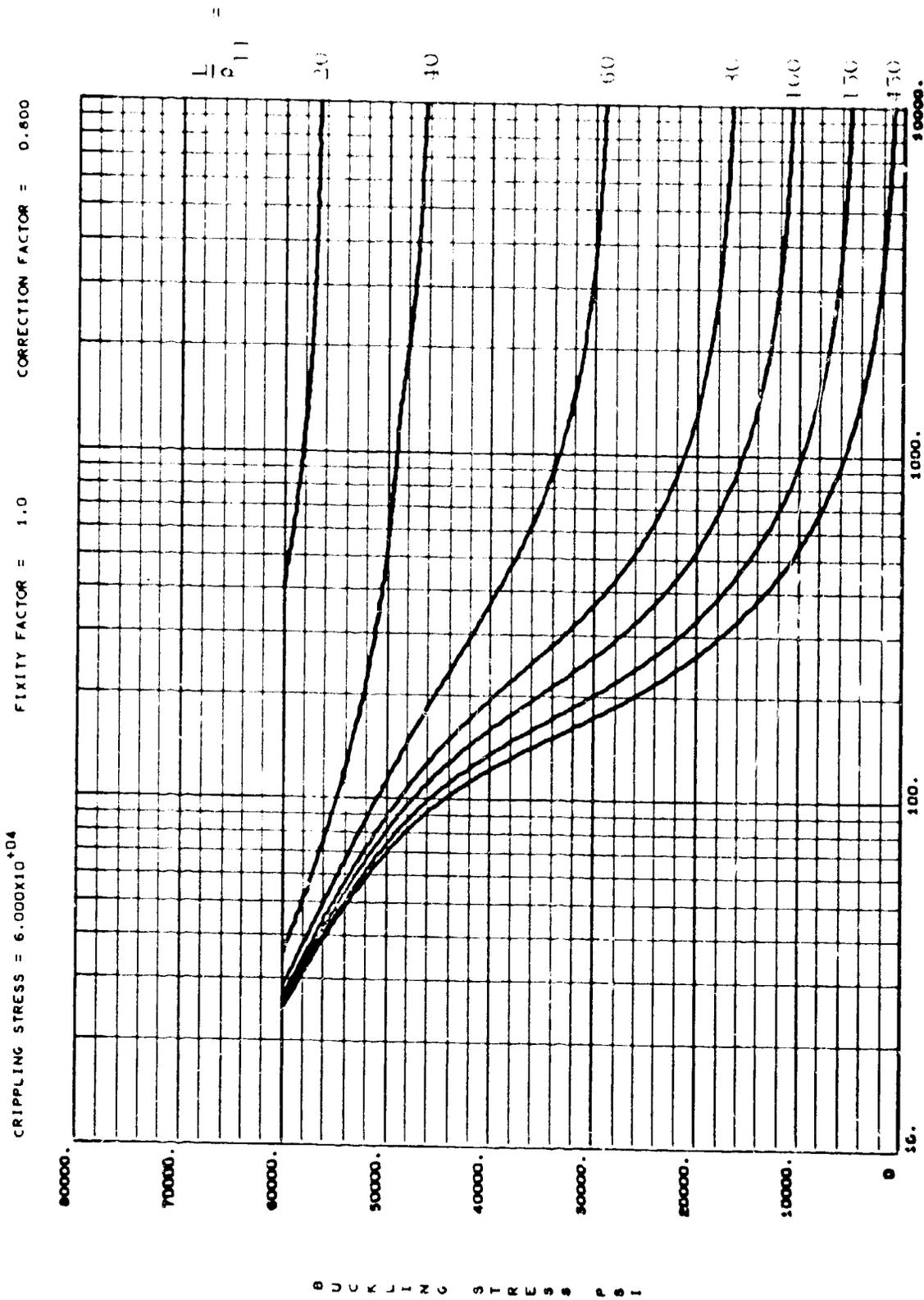
Figure 31 (p) - (see Table XVII)

GENERAL DYNAMICS  
Convair Division



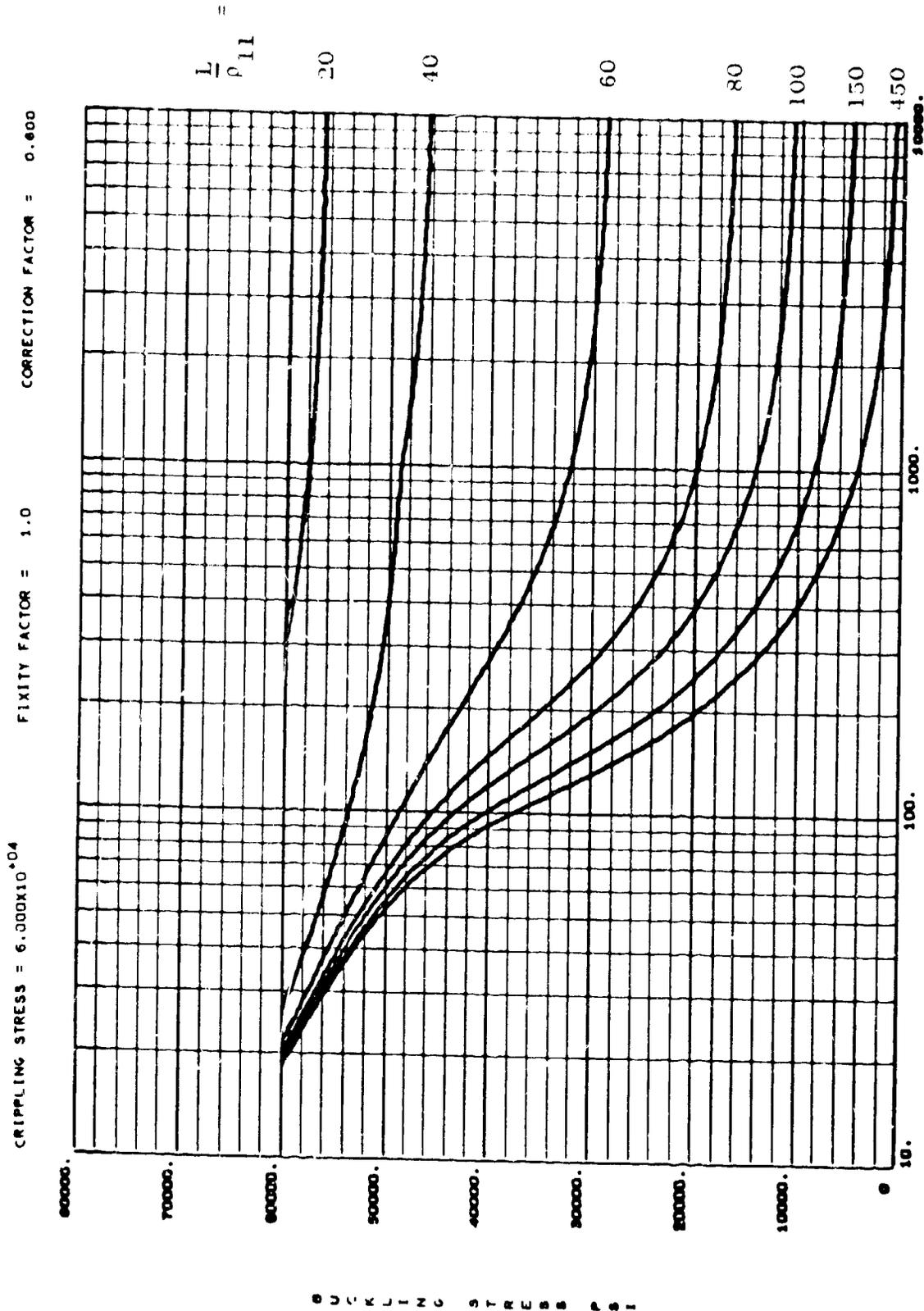
COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS  
 Figure 52 (a) - (see Table XVII)

GENERAL DYNAMICS  
 Convair Division



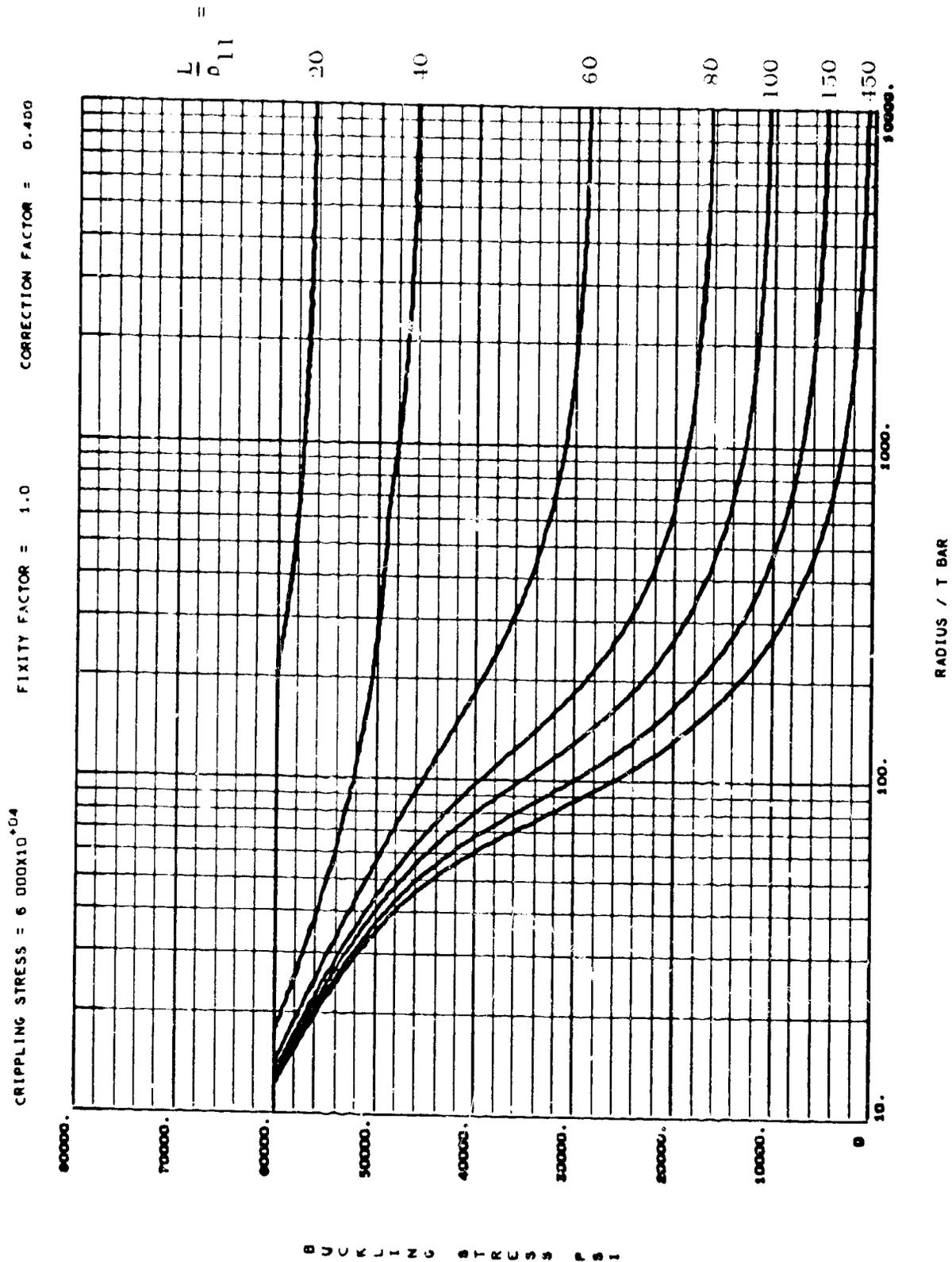
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 32 (b) - (See Table VII)



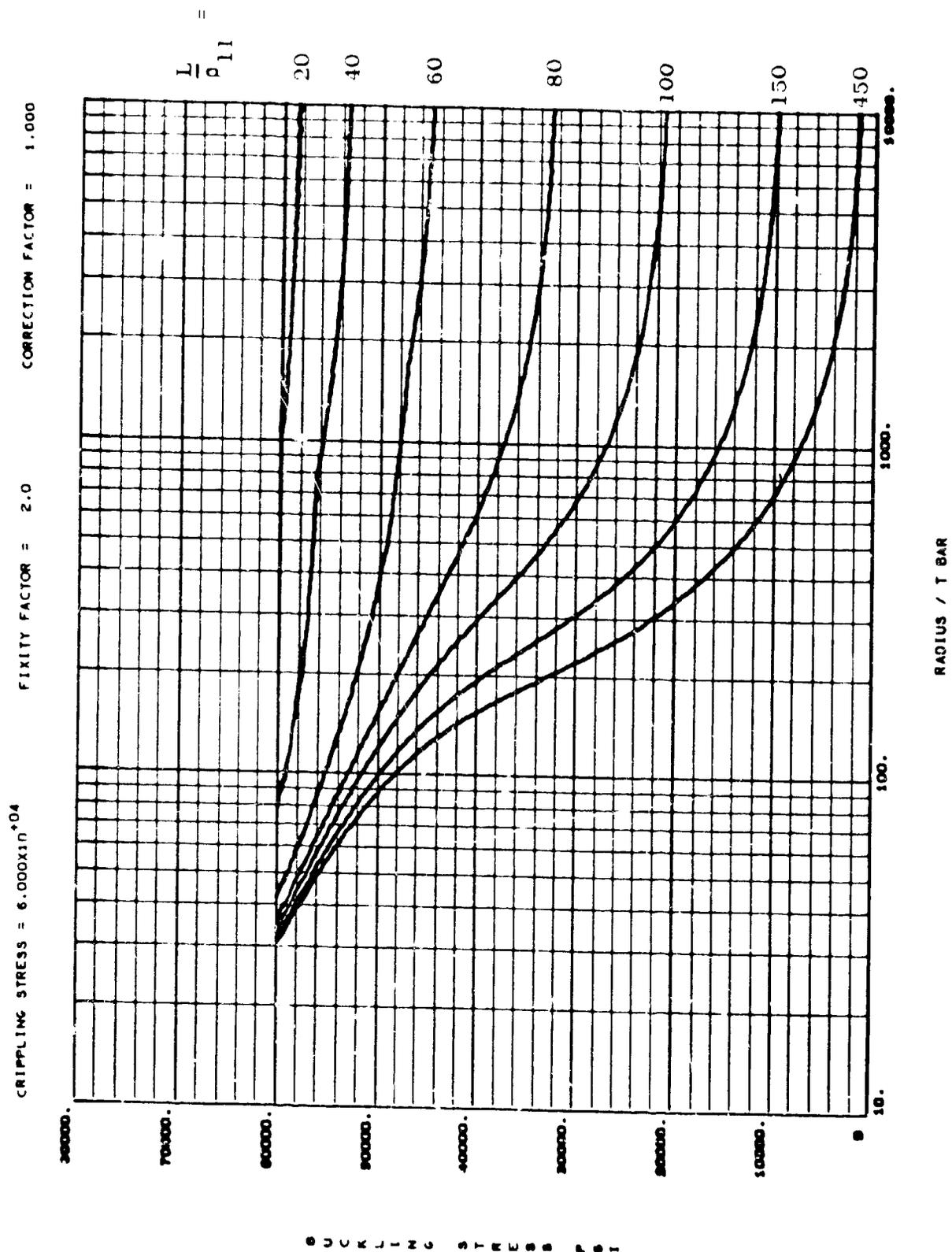
**COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS**

Figure 32 (c) - (See Table XVII)



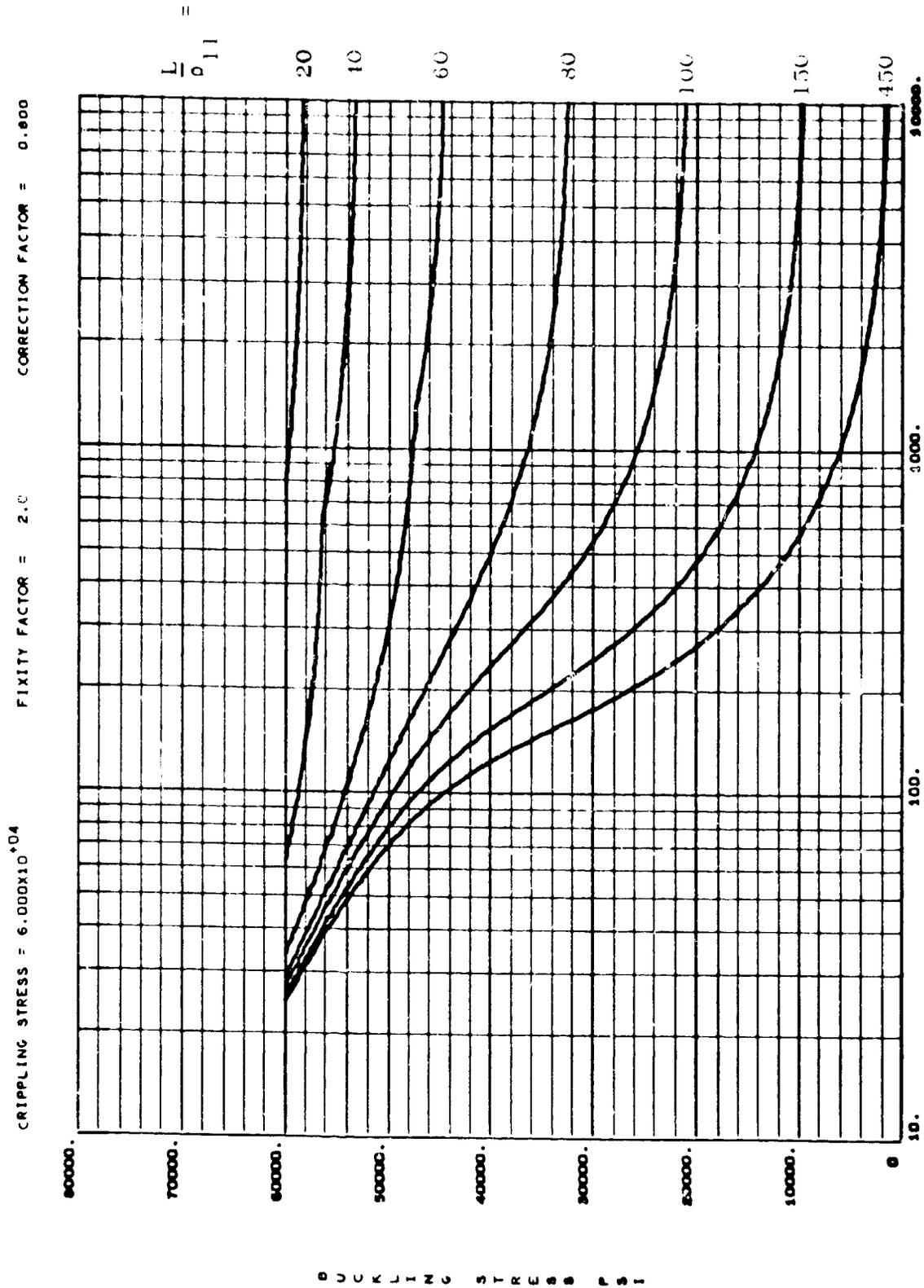
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

(Figure 5213 - (See Table XVI))



COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

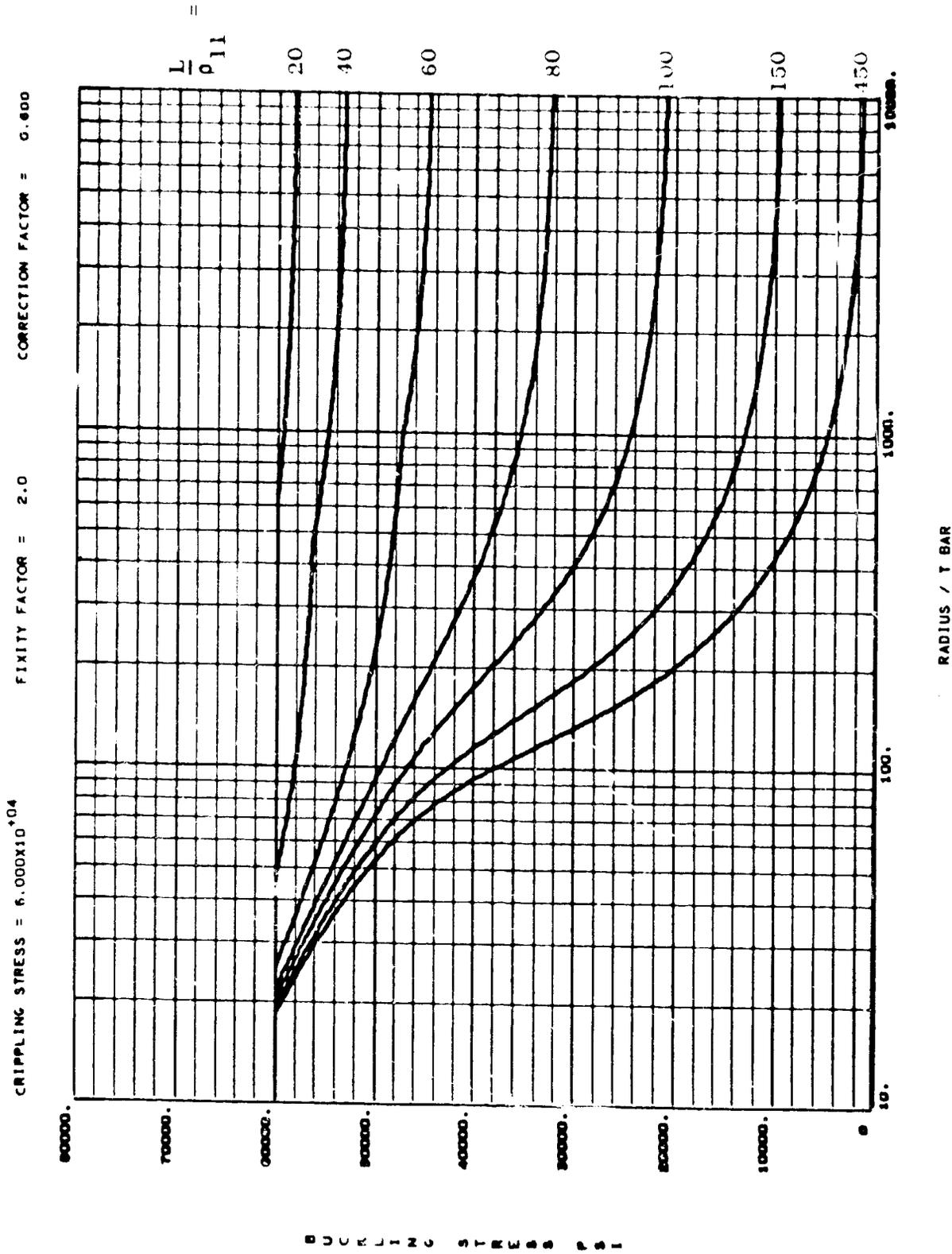
Figure 32(e) - (See Table XVII)



RADIUS / T BAR

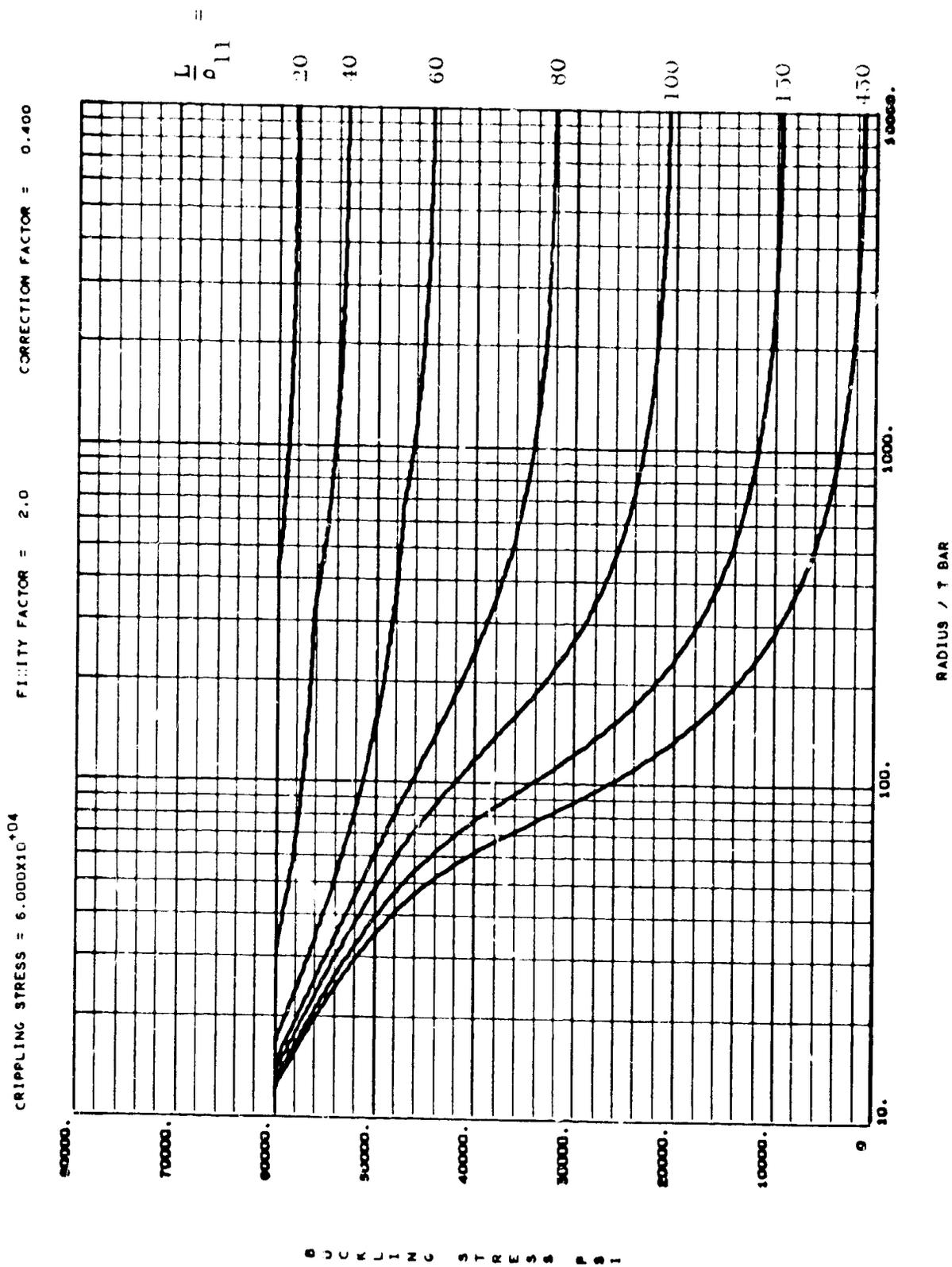
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 2(f) - (See Table XVI)



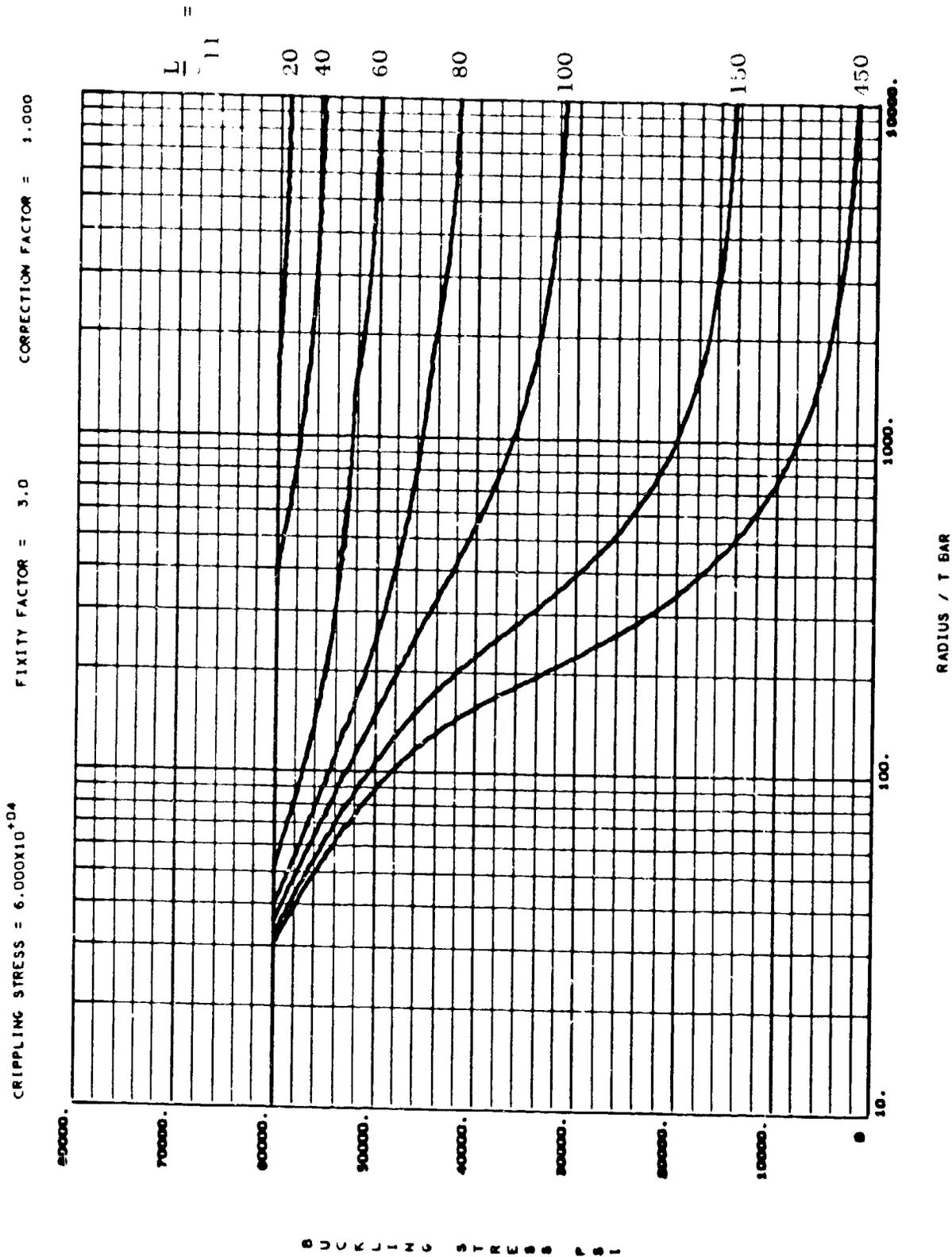
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 52(g) - (See Table XVII)

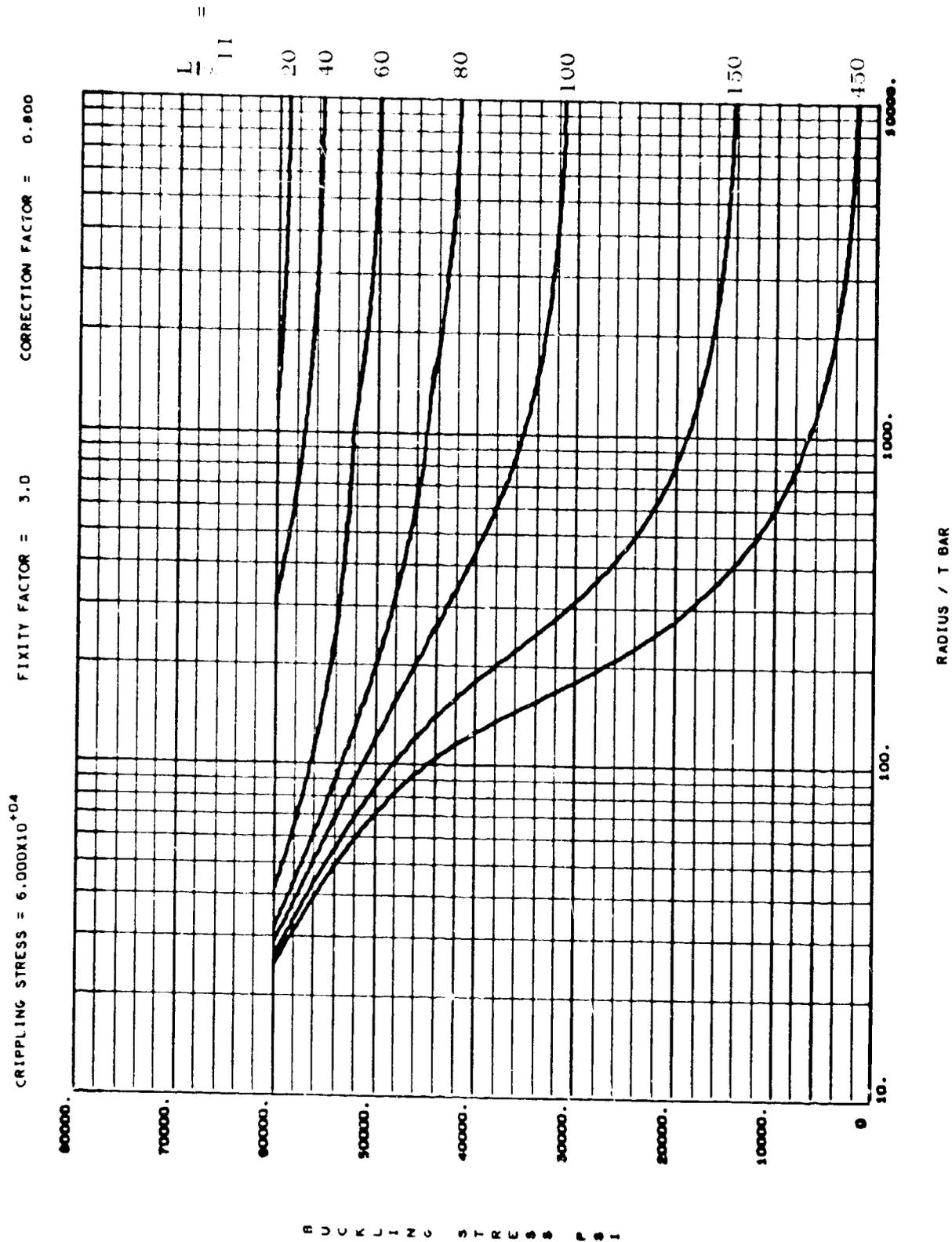


COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

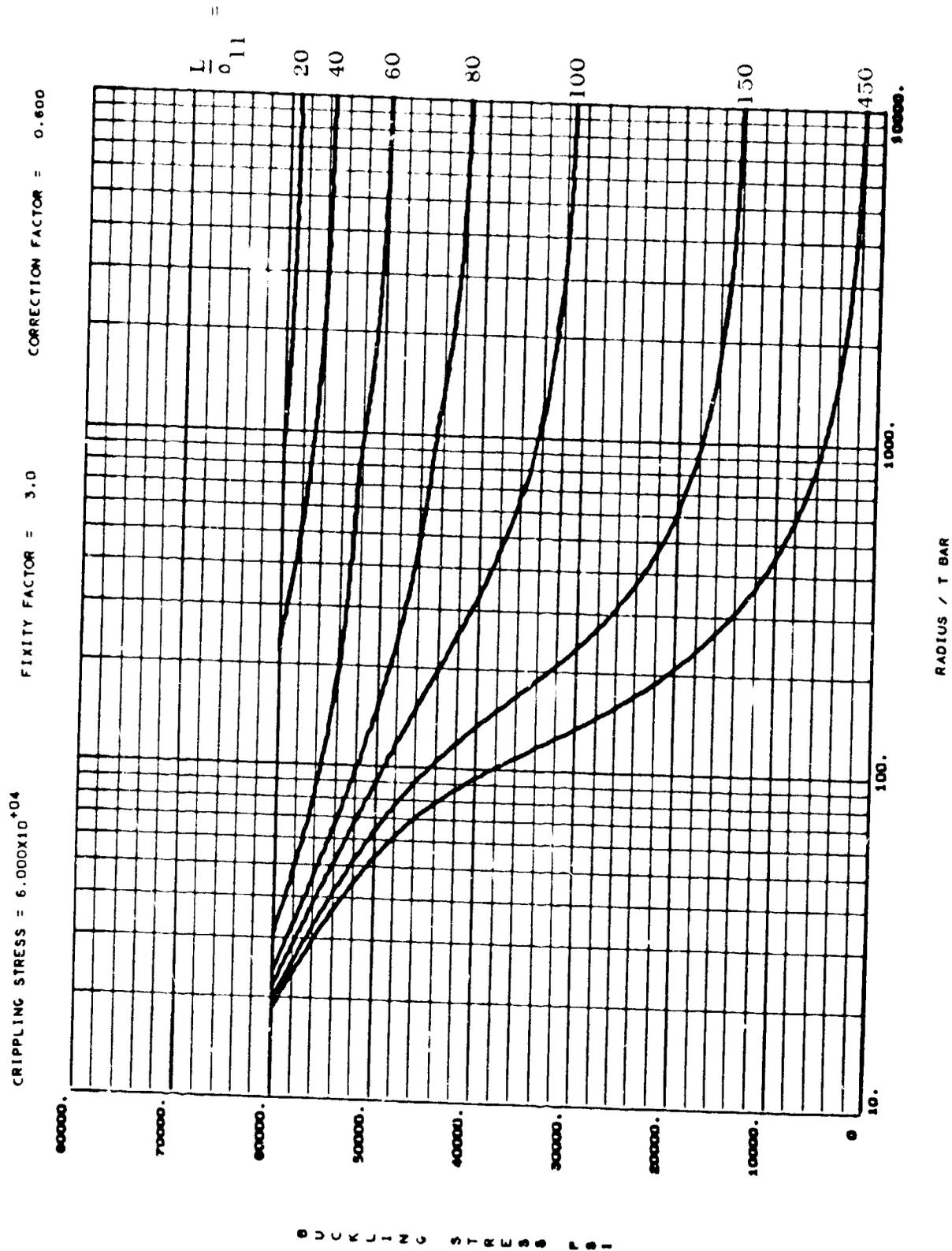
Figure 12(b) - Cont. Table VIII



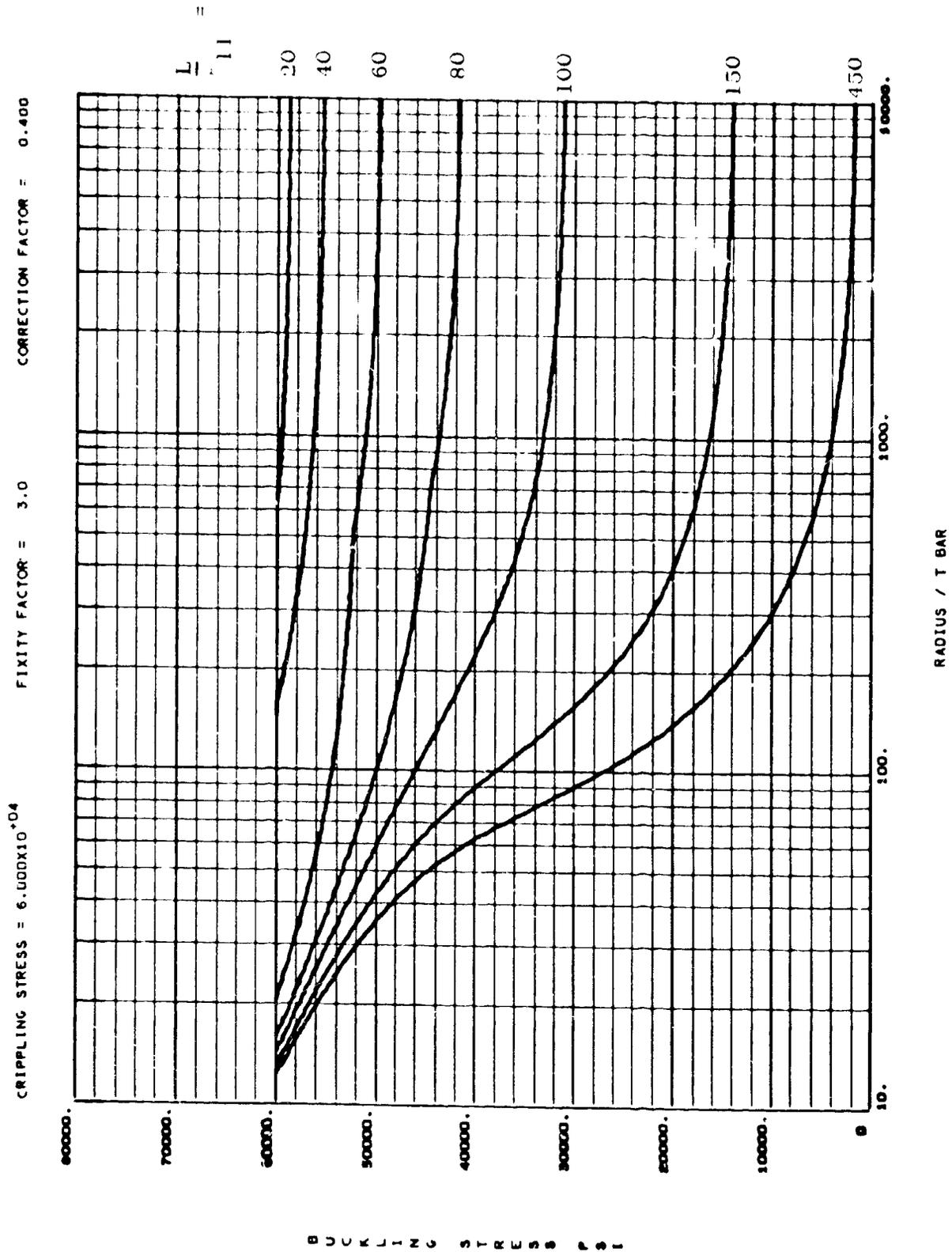
**COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS**  
 Figure 32(i) - (See Table XVII)



COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 24(j) - (See Table XVII)

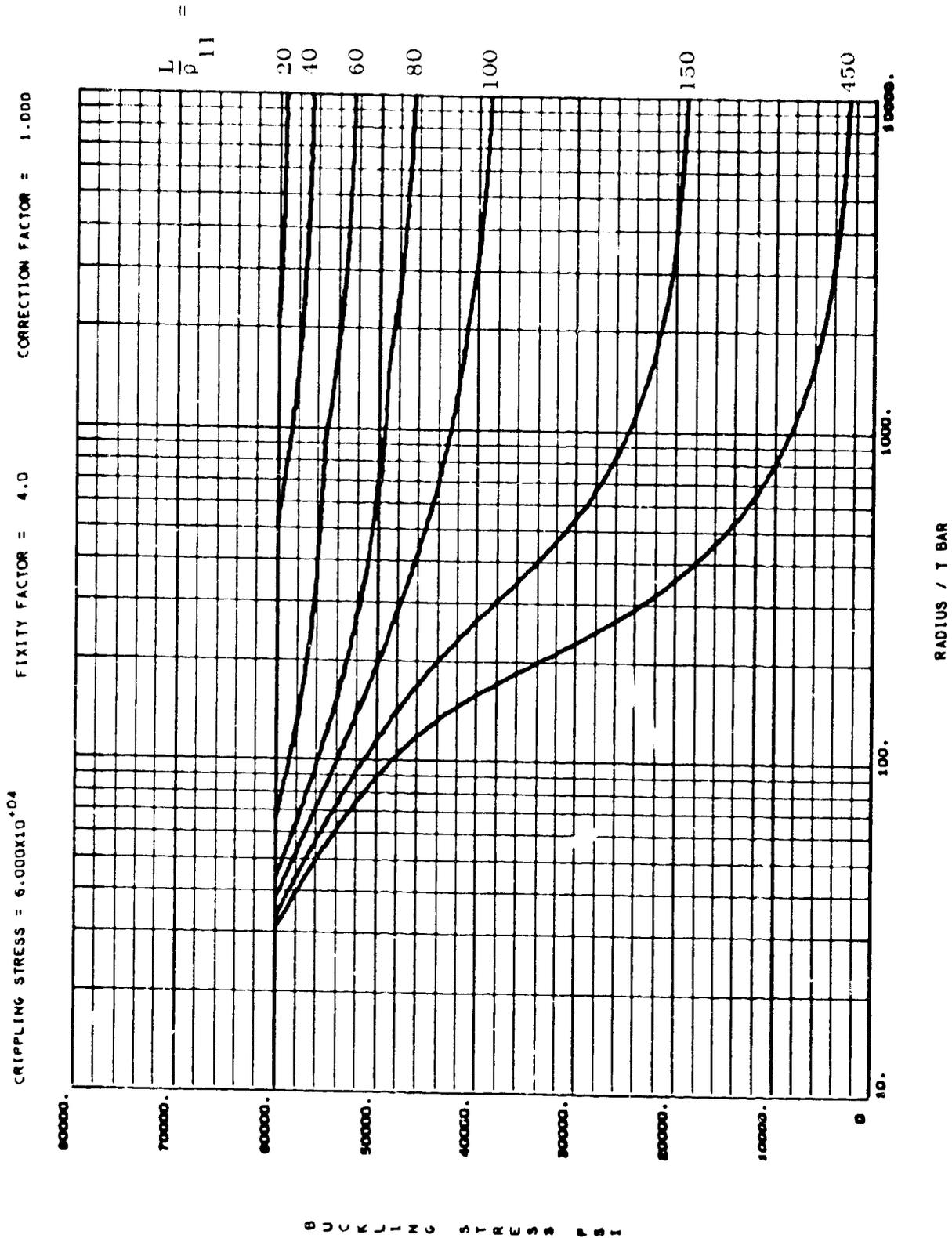


COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS  
 Figure 52(k) - (see Table XVII)



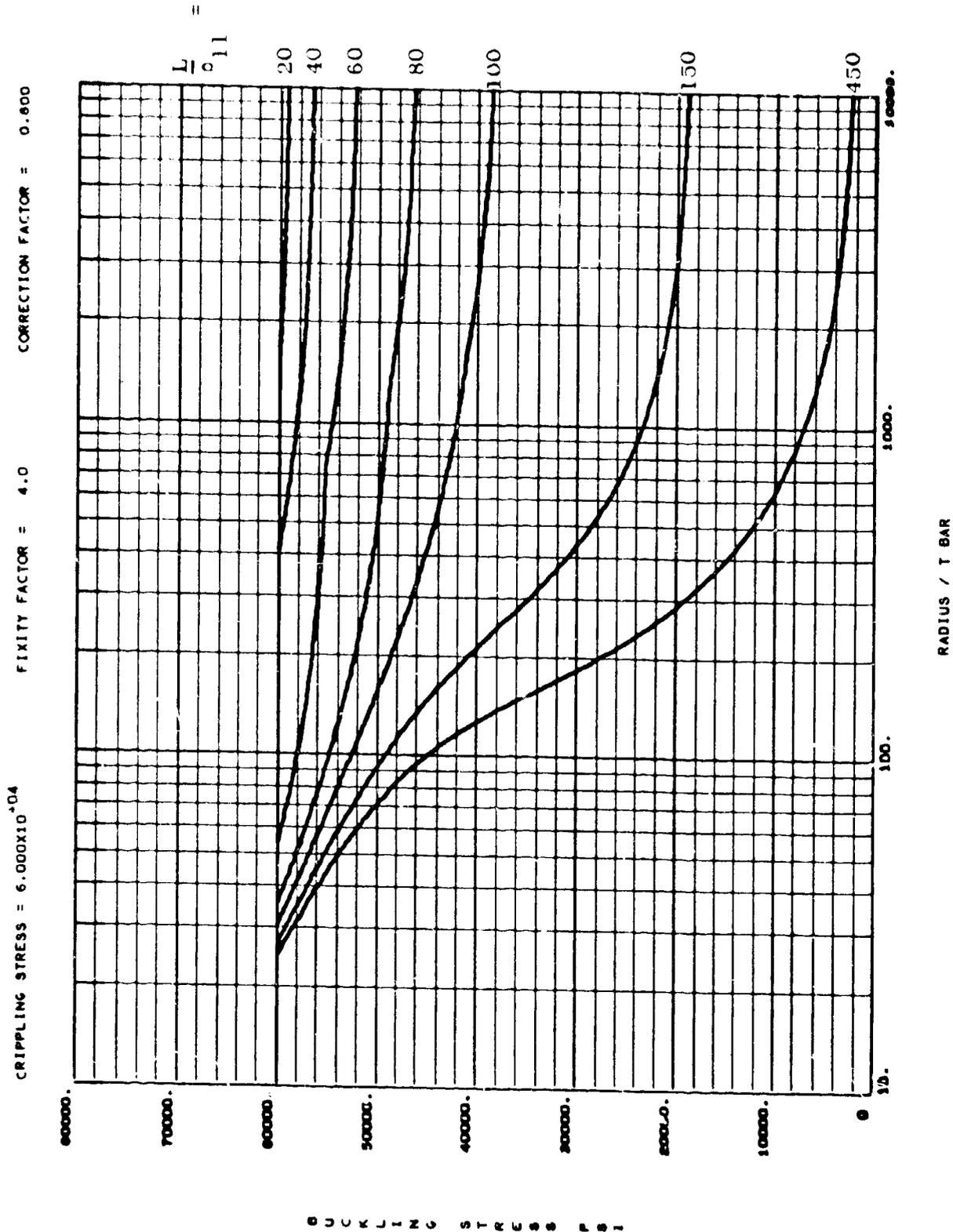
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 32(1) - (See Table XVII)

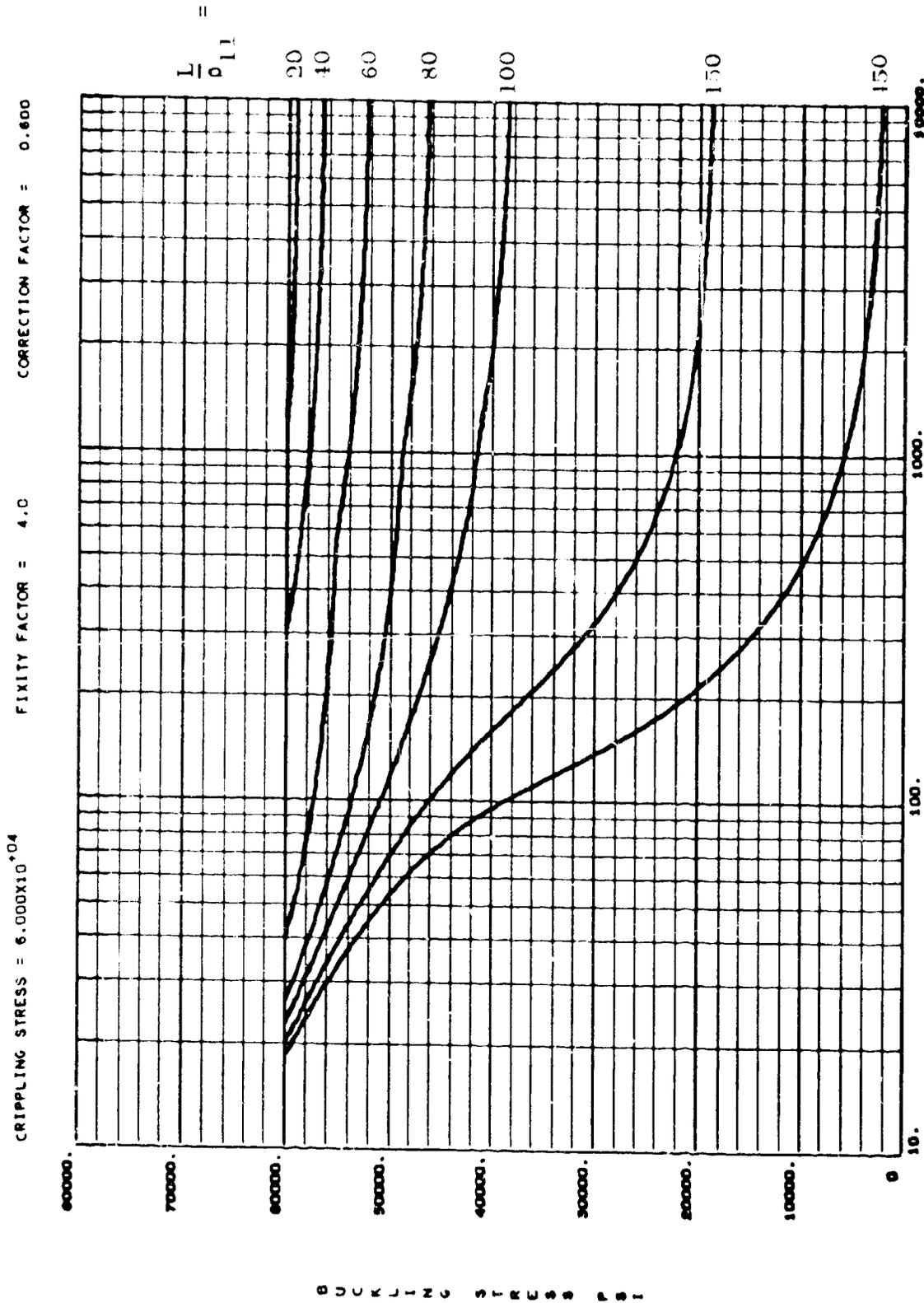


**COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS**  
 Figure 5.2(m) - (See Table XVII)

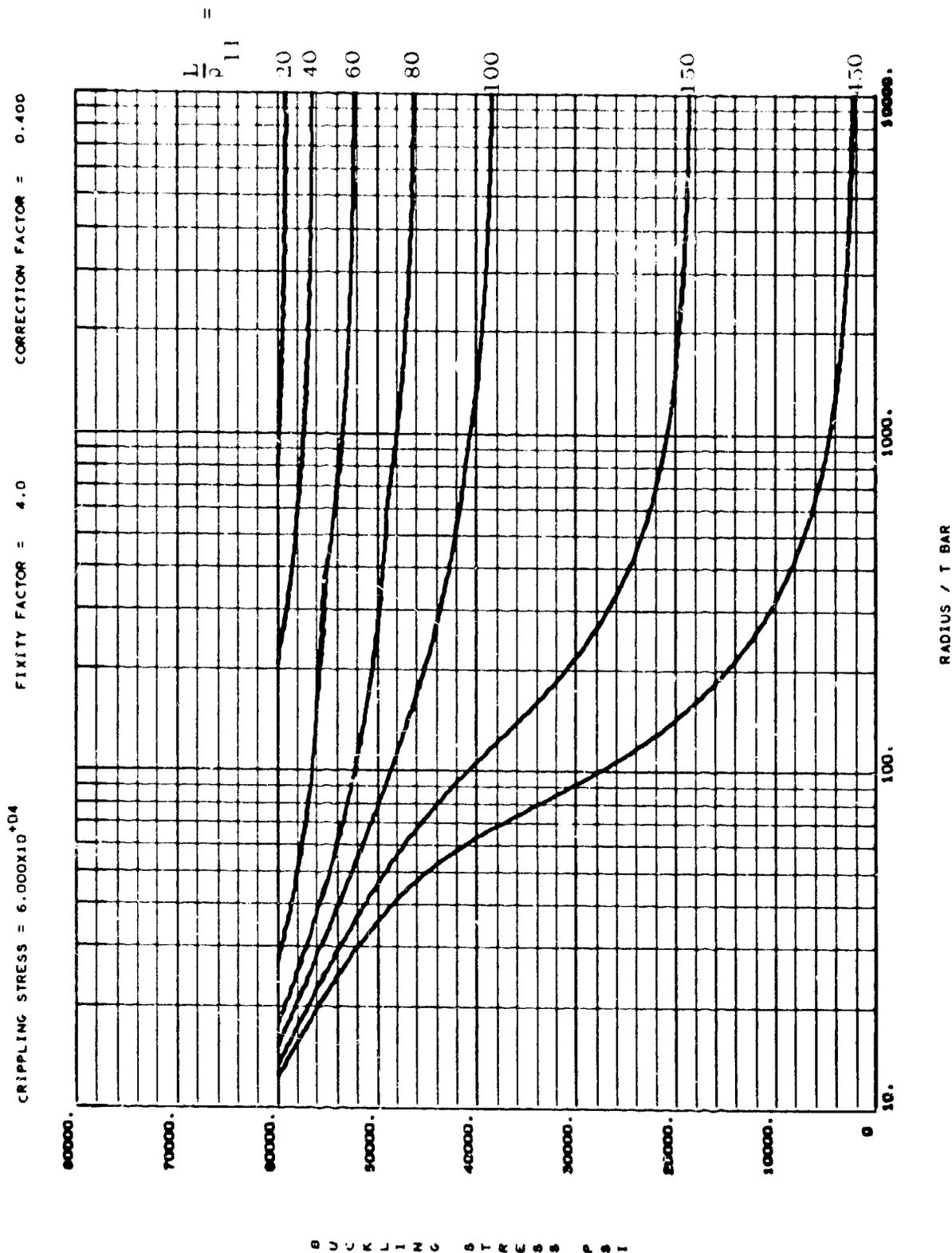
GENERAL DYNAMICS  
 Convair Division



COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 52(n) - (see Table XVII)

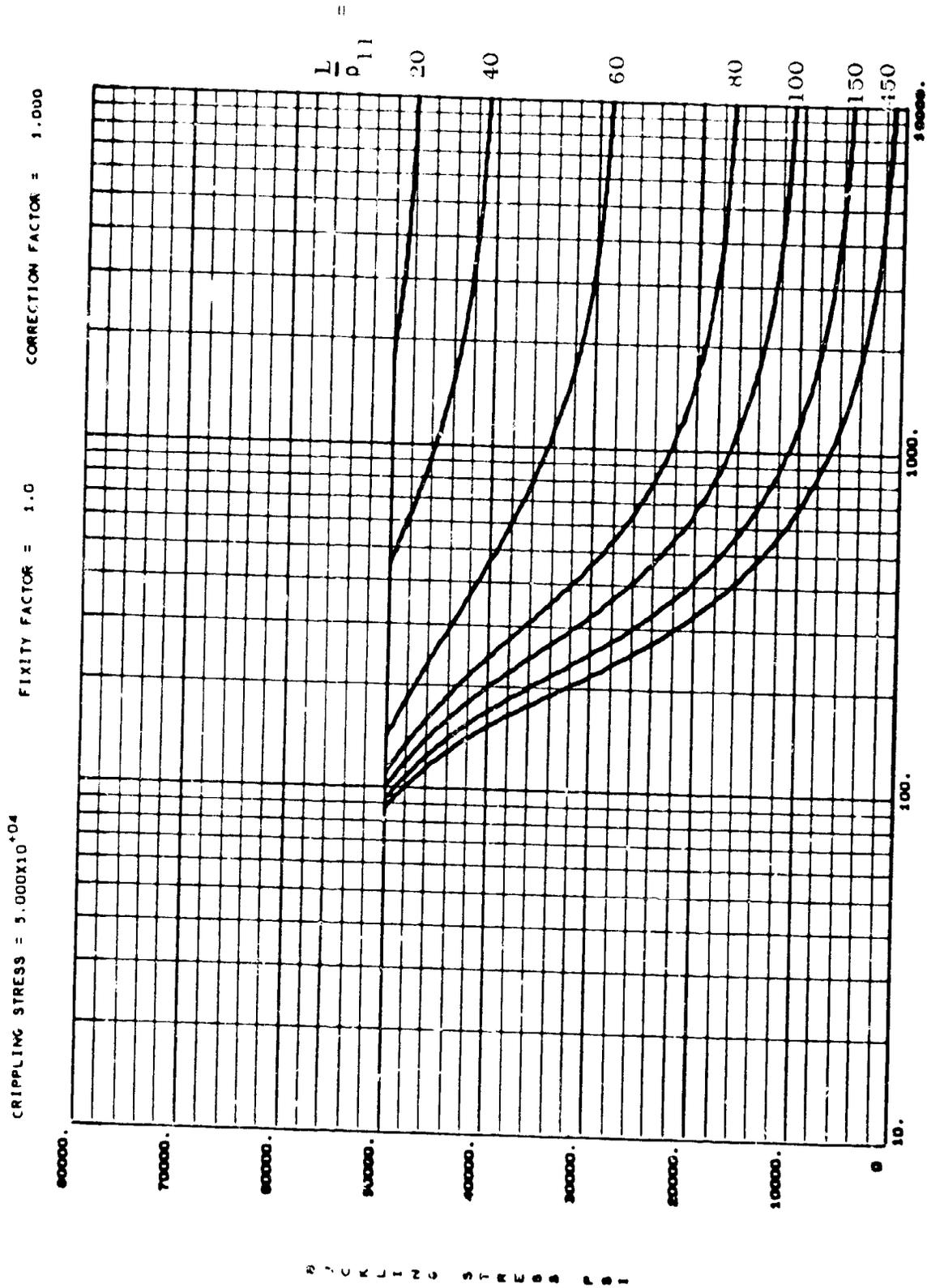


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 52(a) - (see Table XVII)

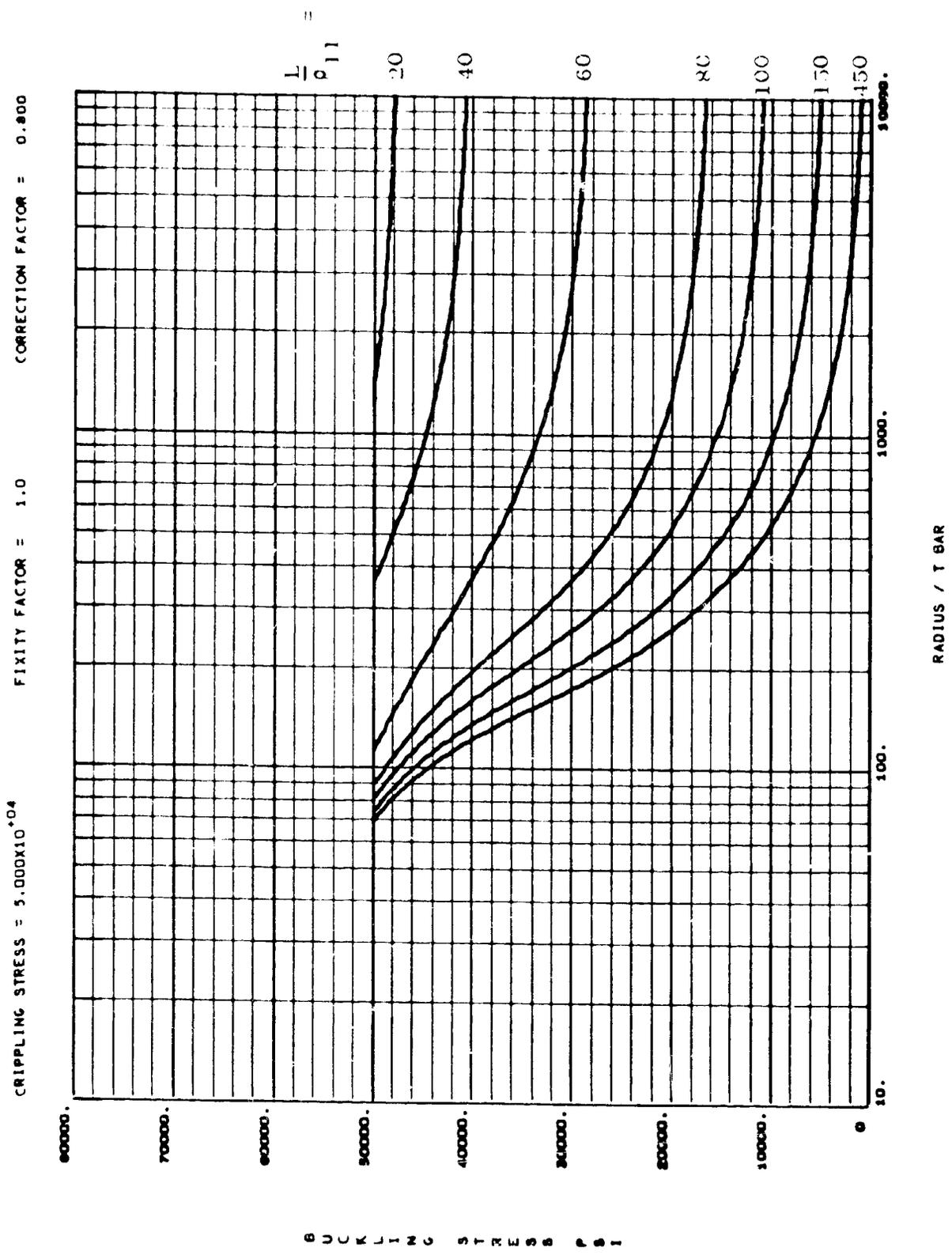


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 5.2(p) - (see Table VII)

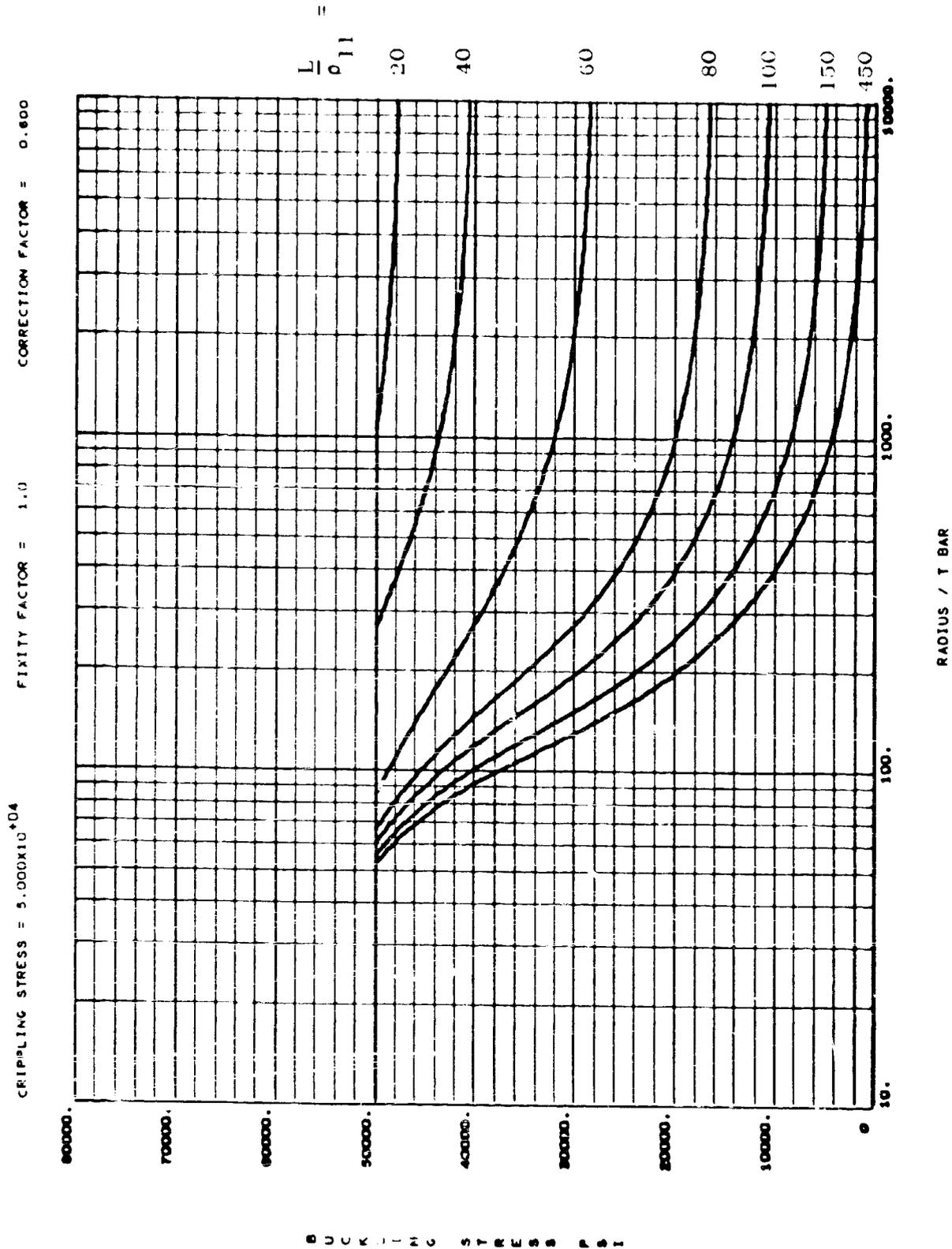


COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 33(a) - (see Table XVII)



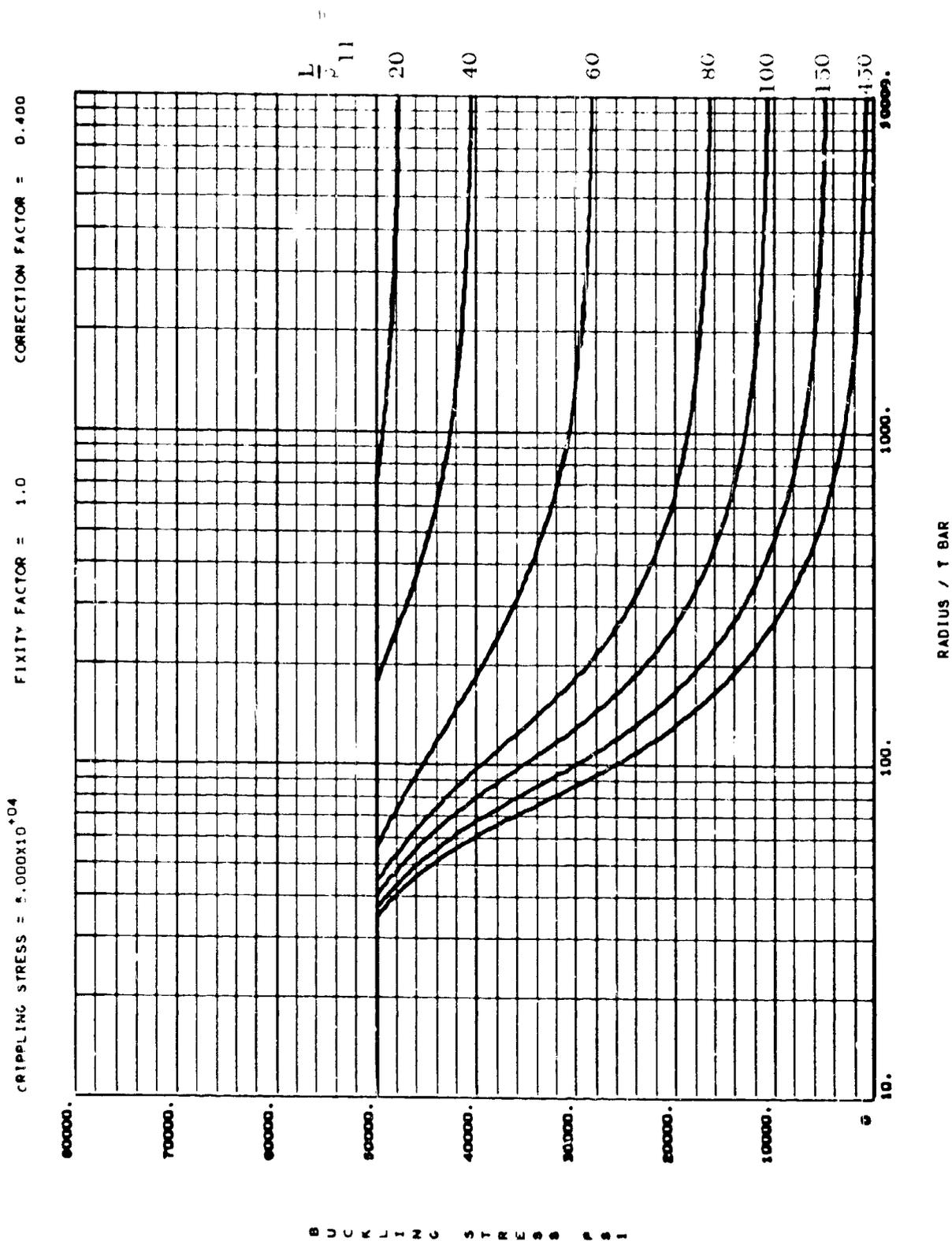
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 55(b) - (see Table XVII)

GENERAL DYNAMICS  
Convair Division



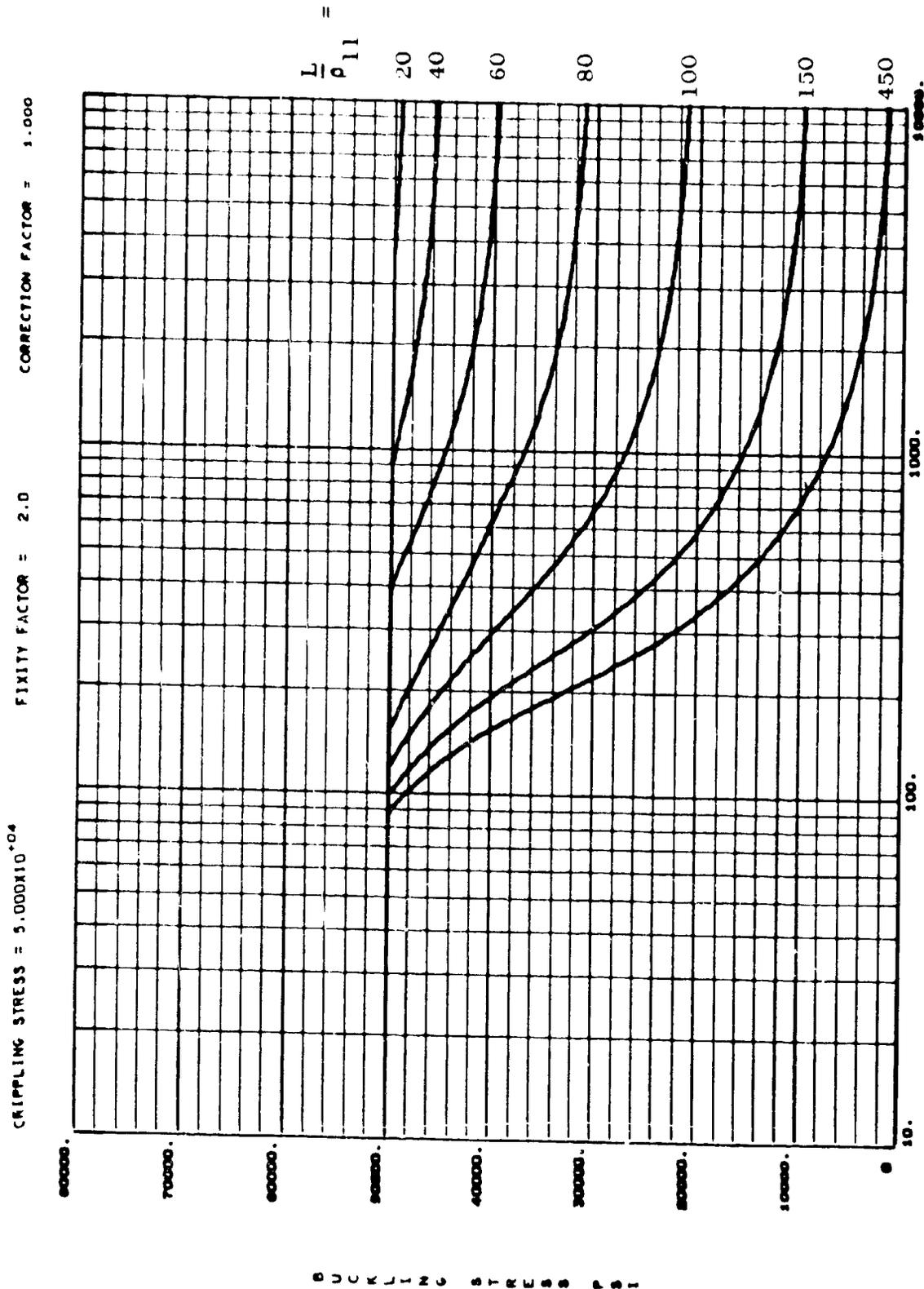
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 55(c) - (see Table XVII)



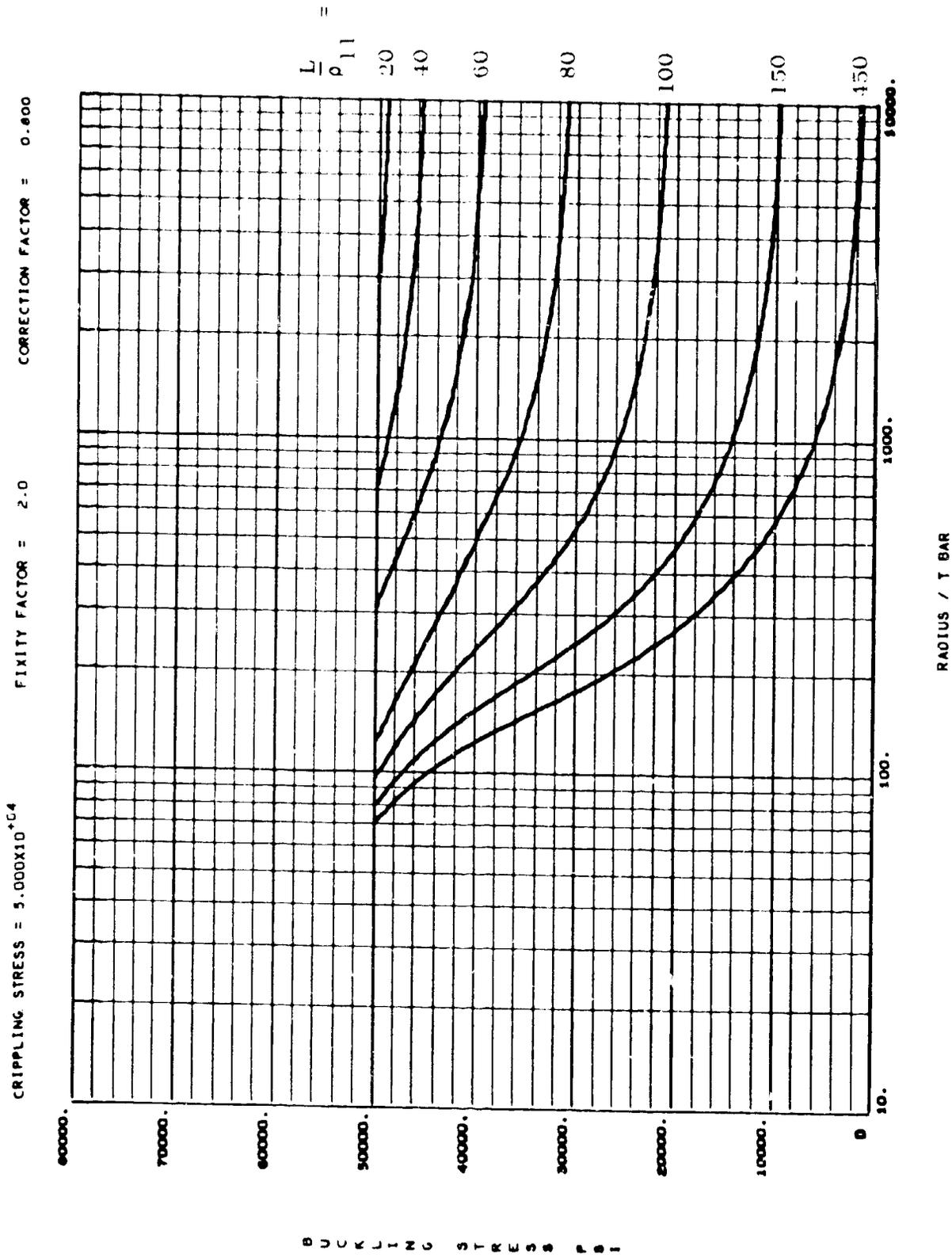
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 35(d) - (see Table VII)

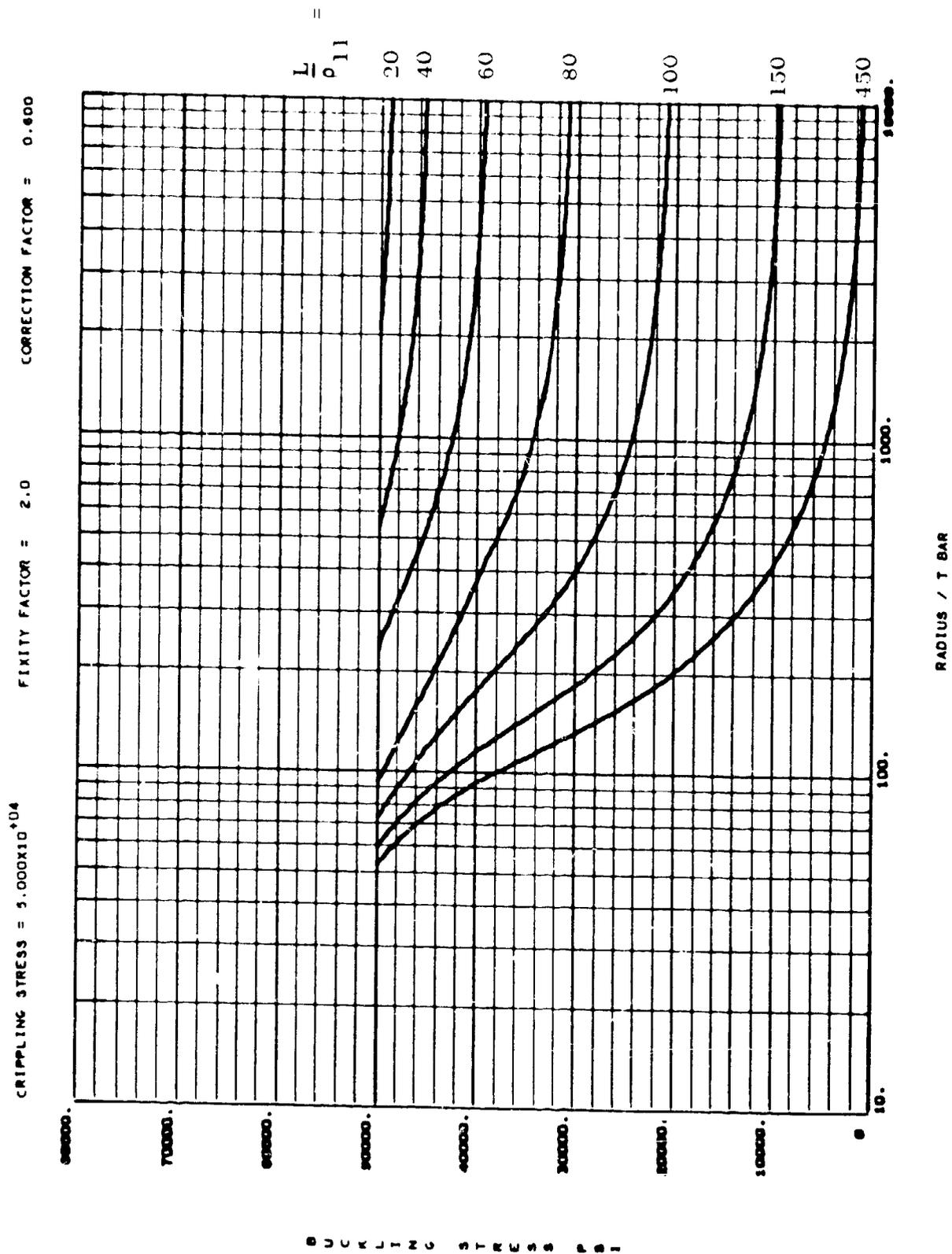


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

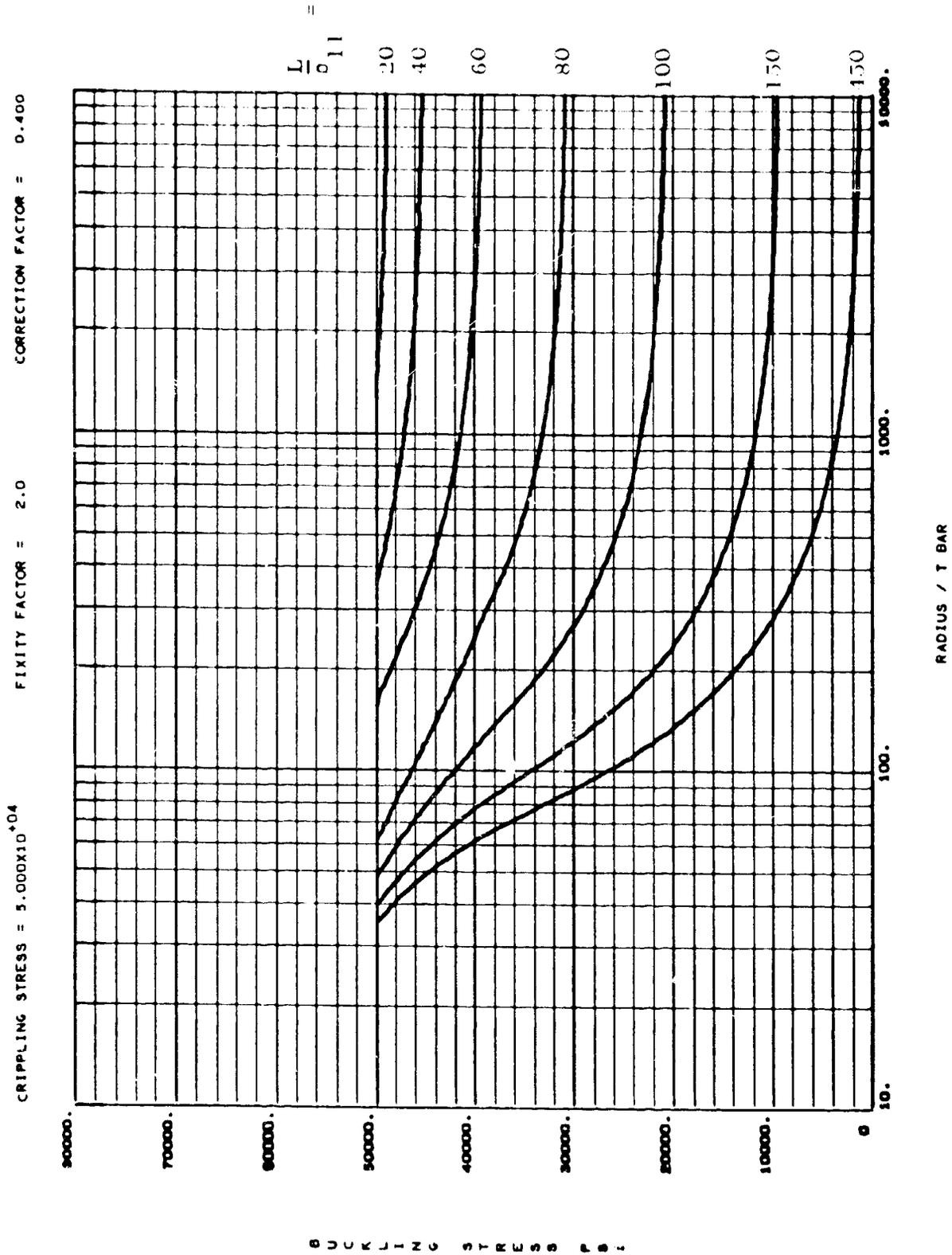
Figure 53(e) - (See Table XVII)



COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 25(C) - (See Table XVII)

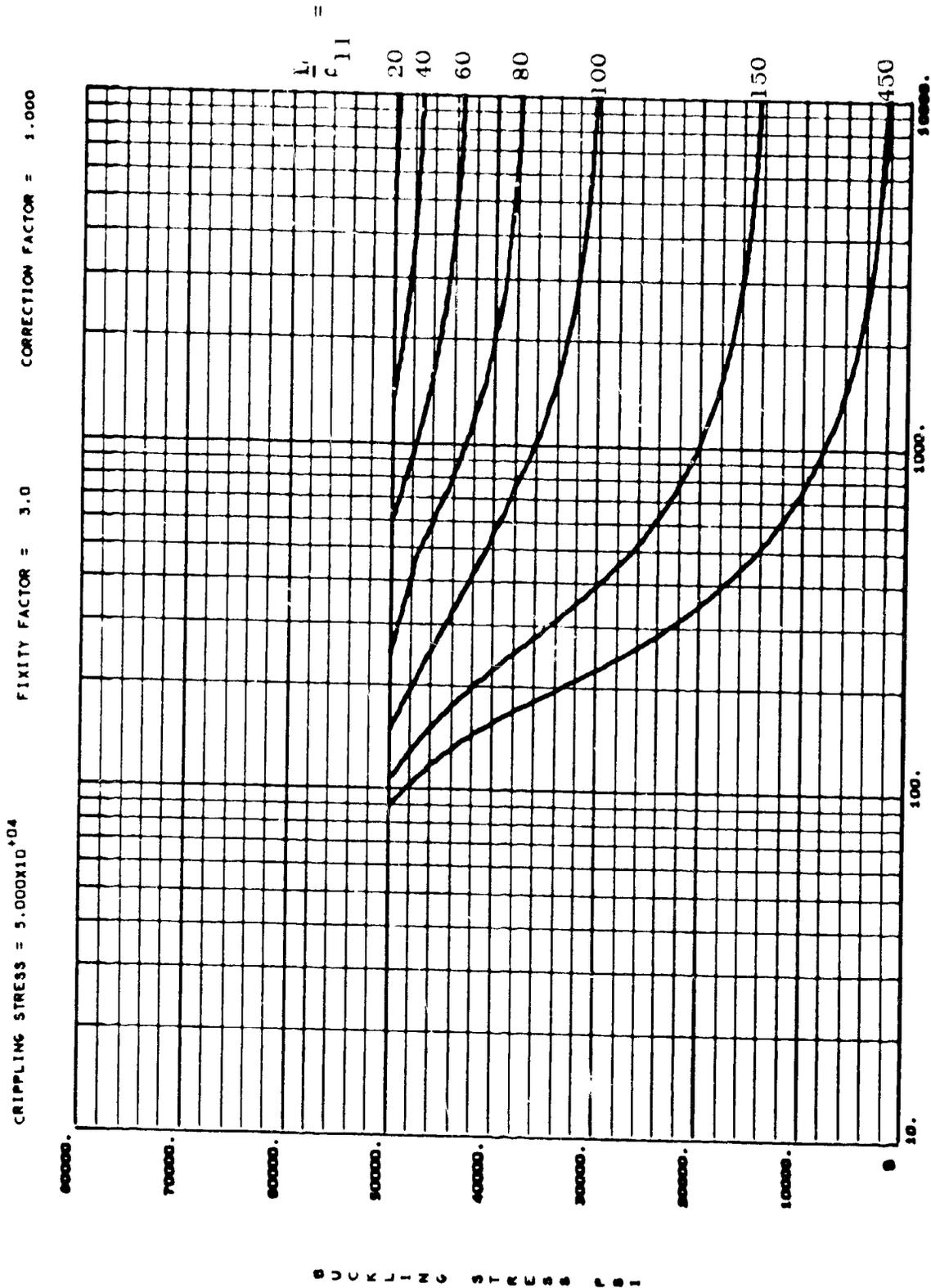


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
*Figure 33(g) - (See Table XVII)*

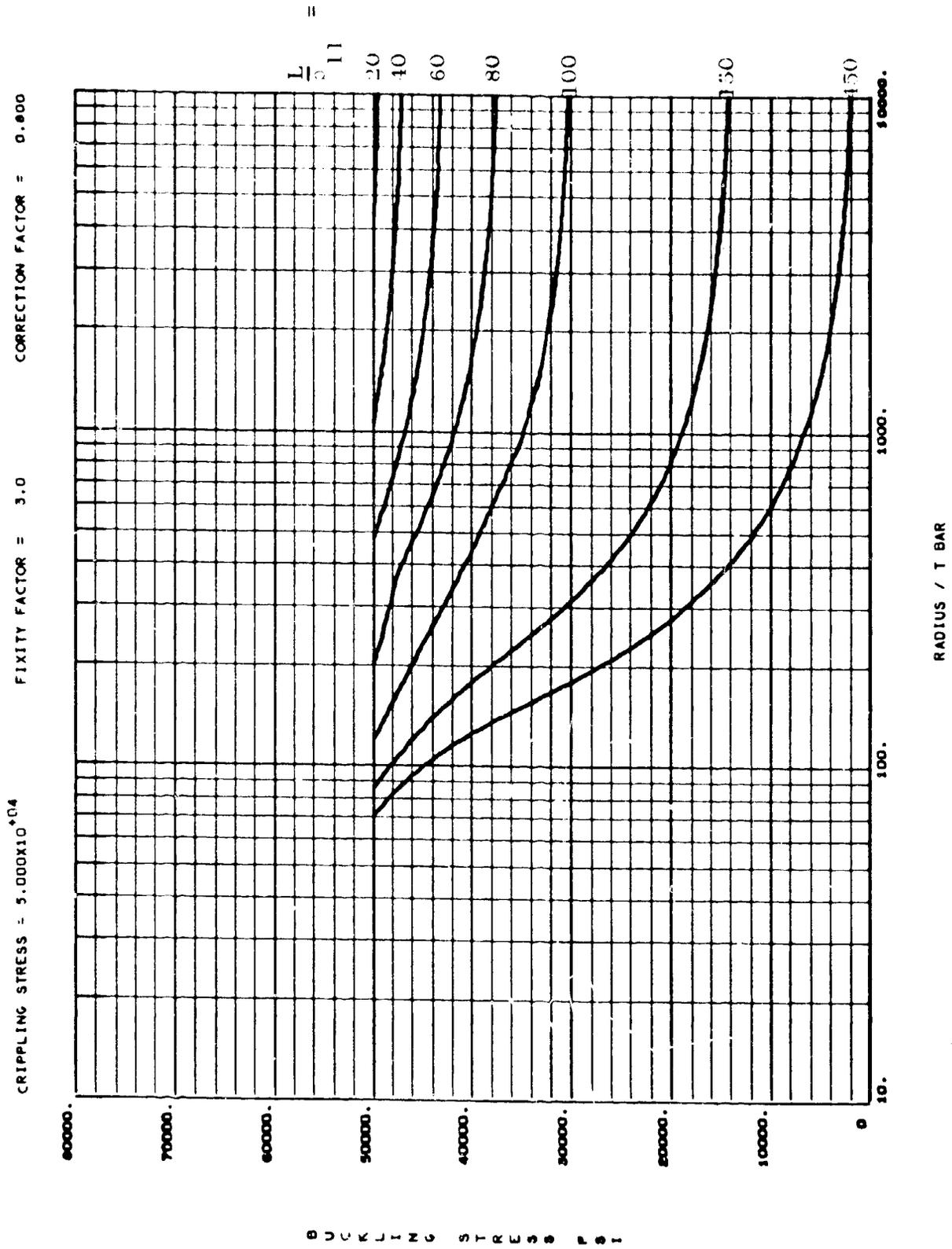


**COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS**

Figure 35(h) - (See Table XVII)

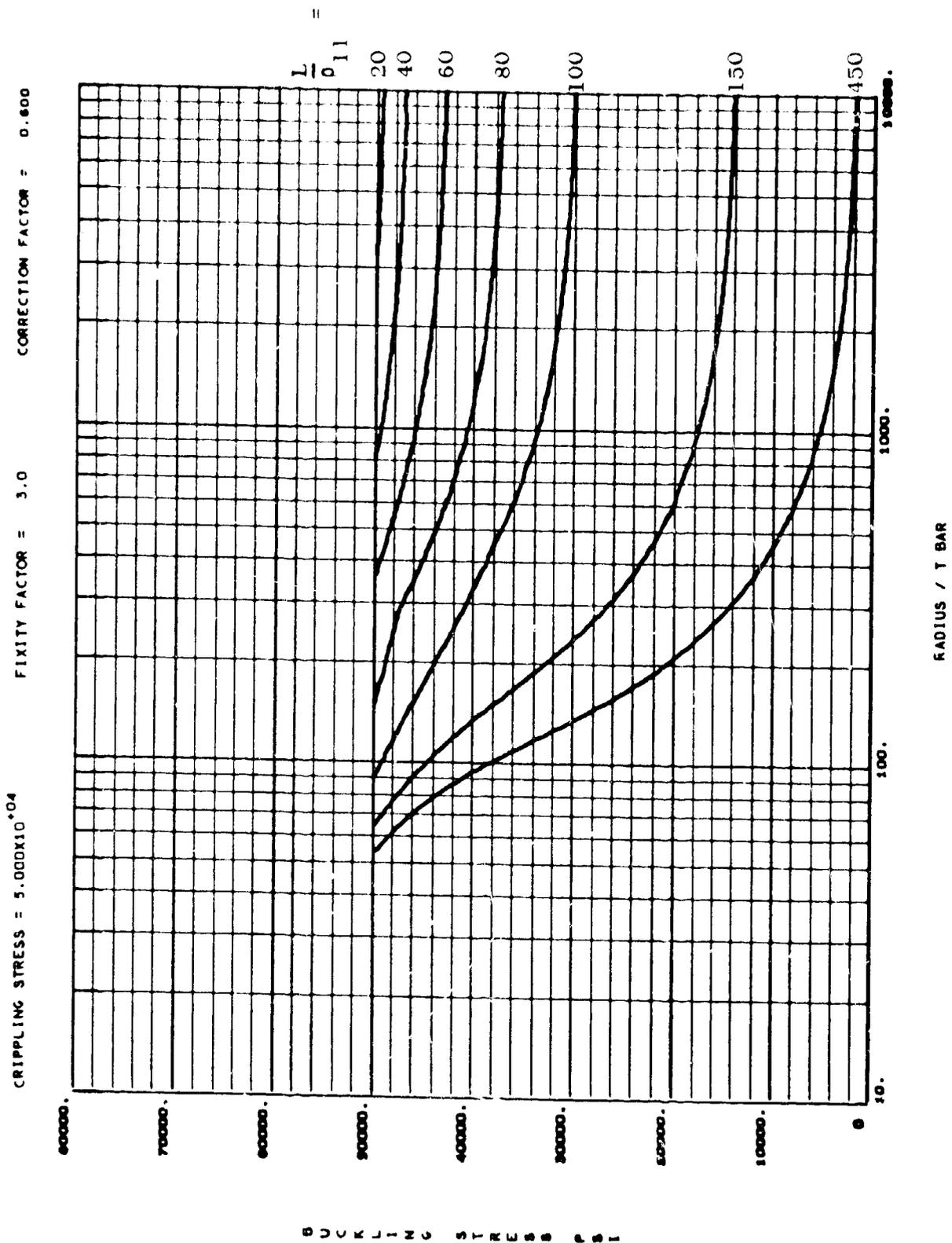


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 33(i) - (see Table XVII)



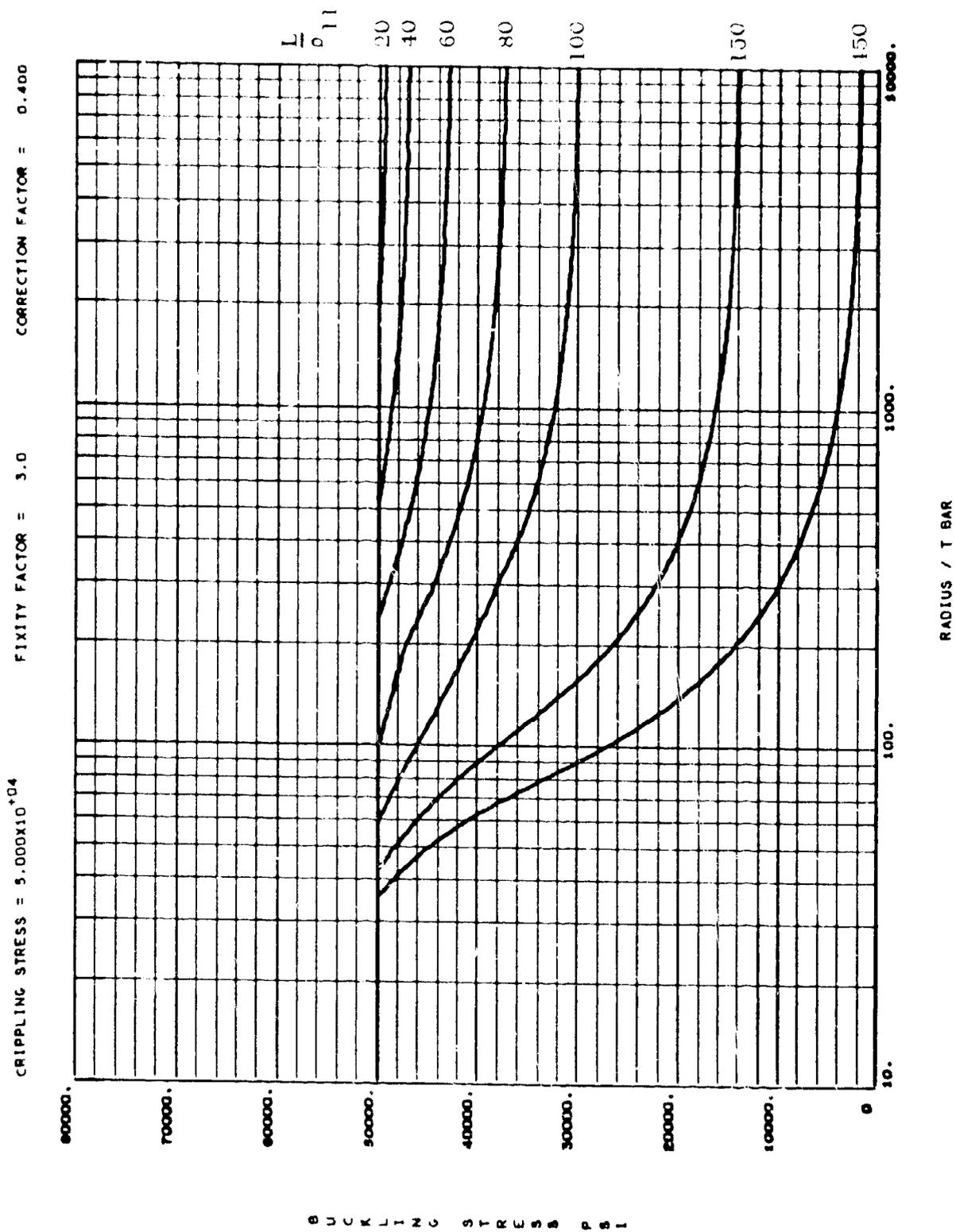
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 77(j) - (See Page VIII)



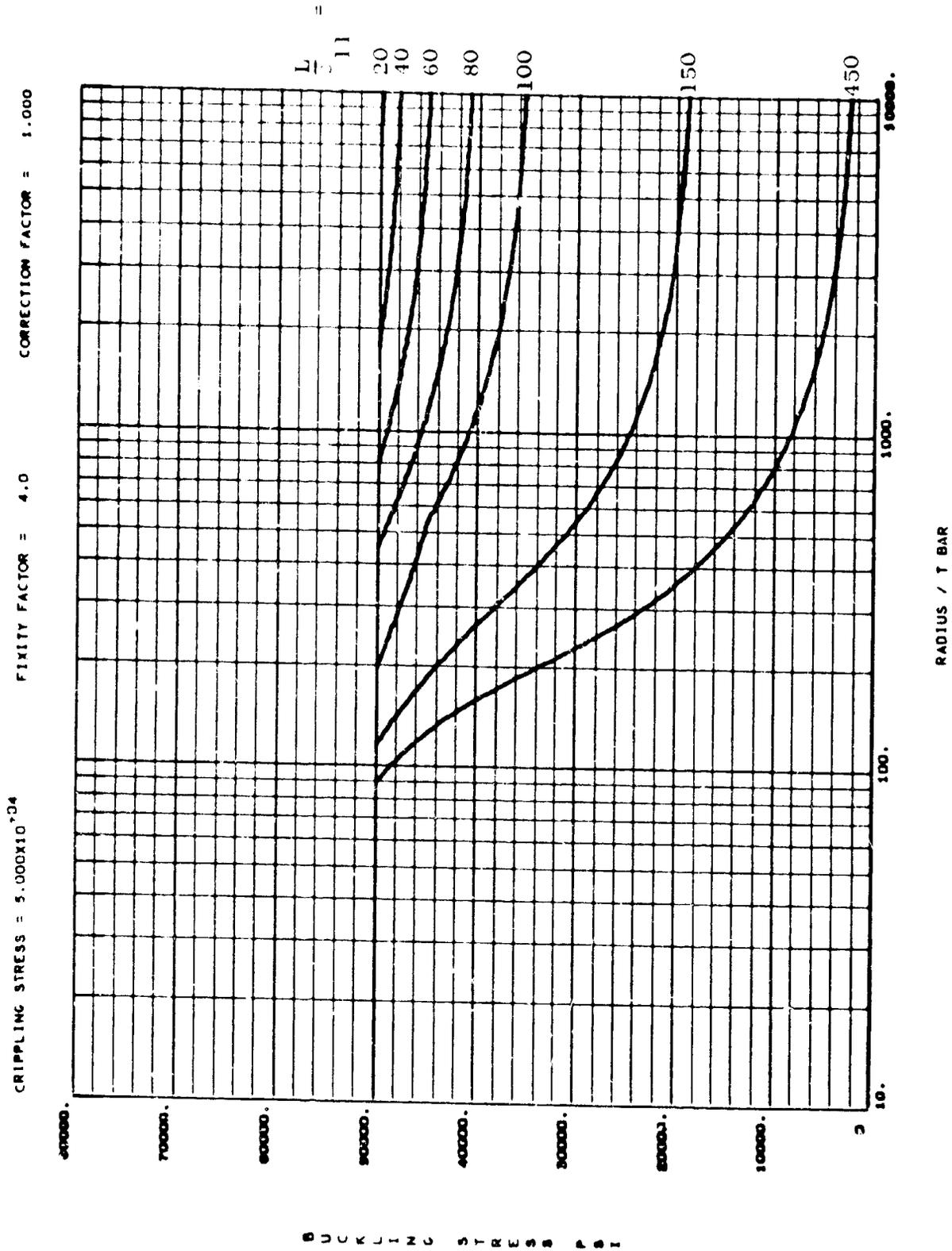
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 33(k) - (See Table XVII)



COMPRESSION TABLE (VII)  
 COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS

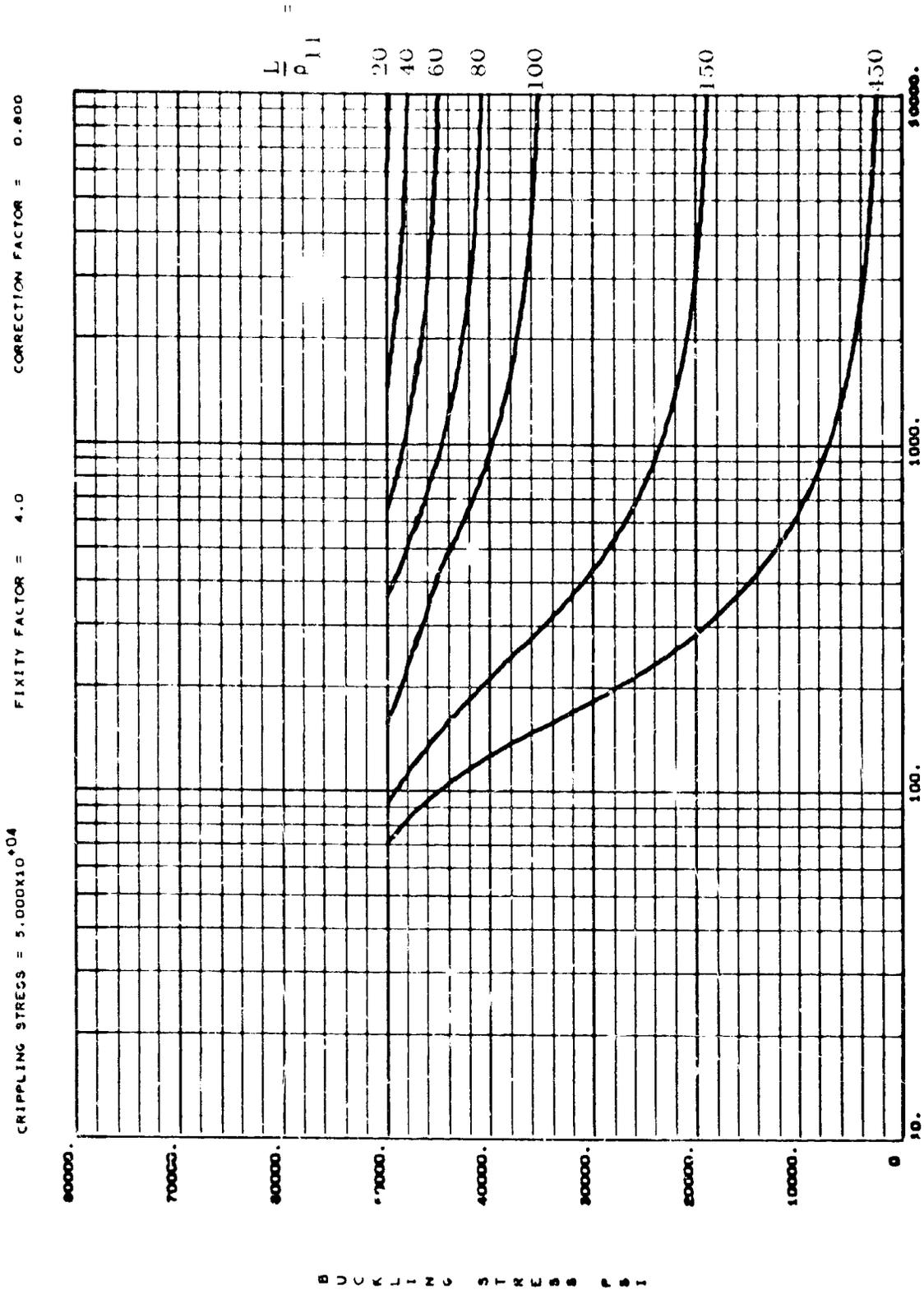
GENERAL DYNAMICS  
 Convair Division



RADIUS / T BAR

**COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS**

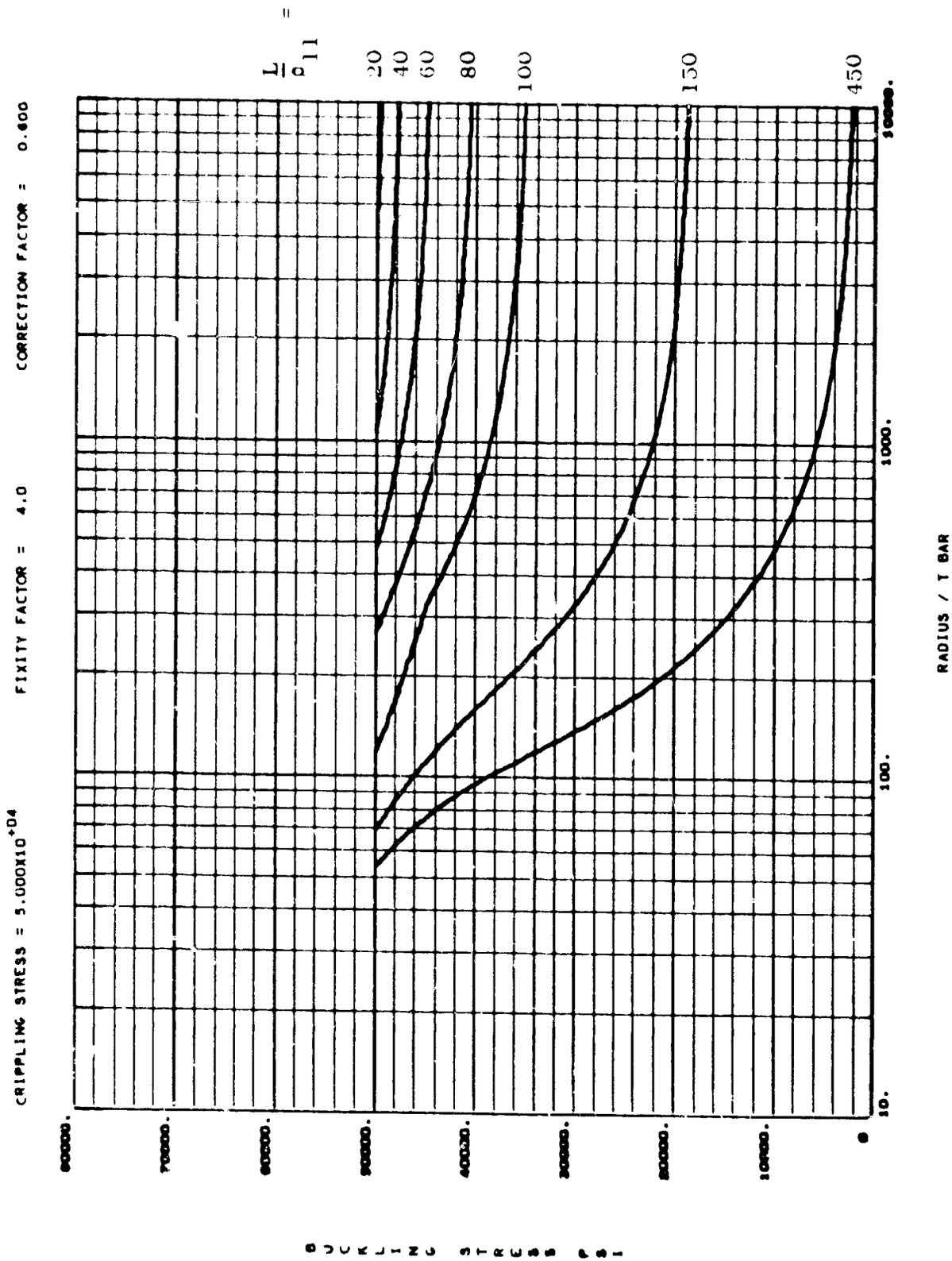
Figure 35(m) - (see Table XVII)



RADIUS / T BAR

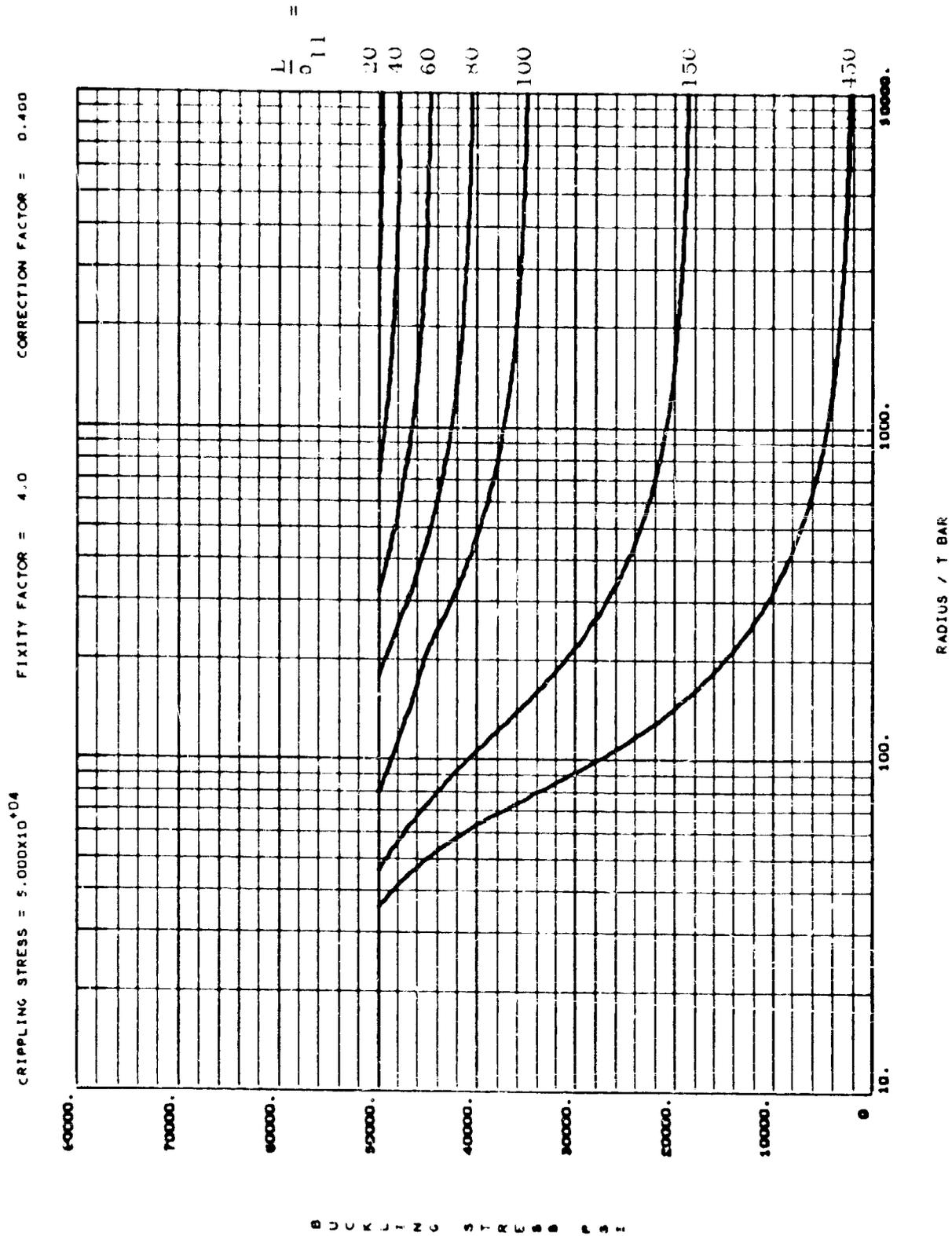
**COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS**

Figure 2100 - See Table VIII



COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

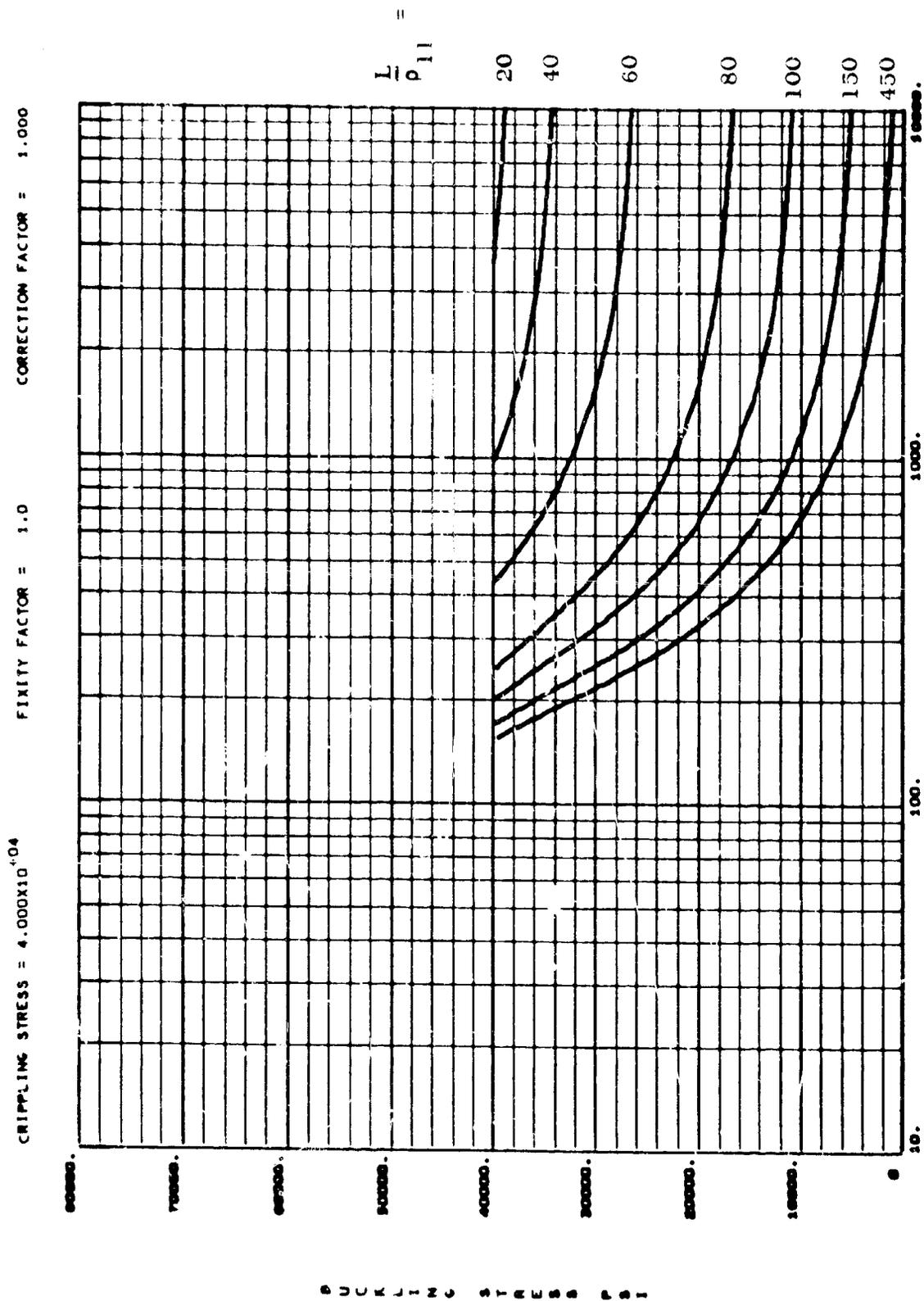
Figure 35(o) - (See Table XVII)



RADIUS / T BAR

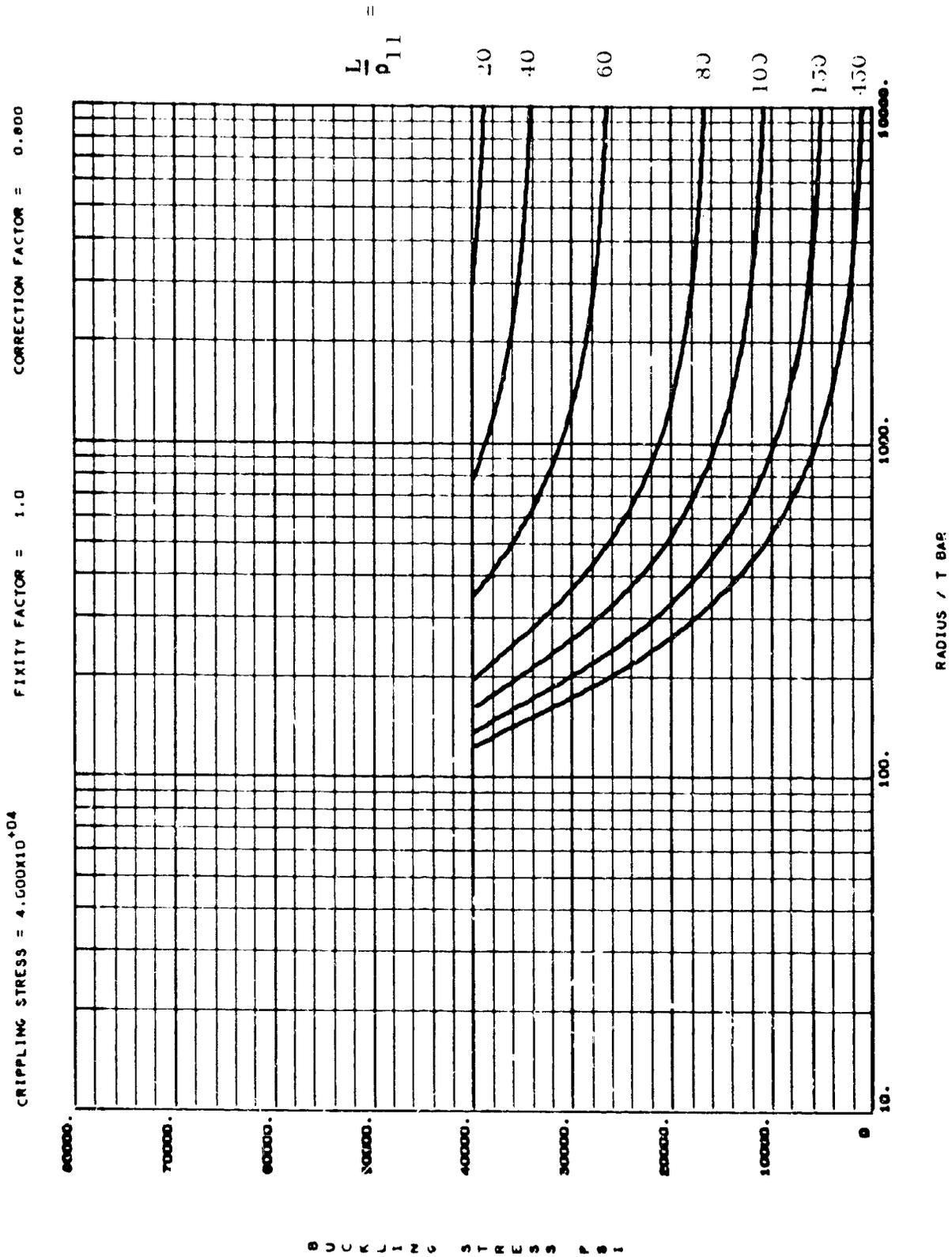
**COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS**

Figure 1791 - (See Table XVII)



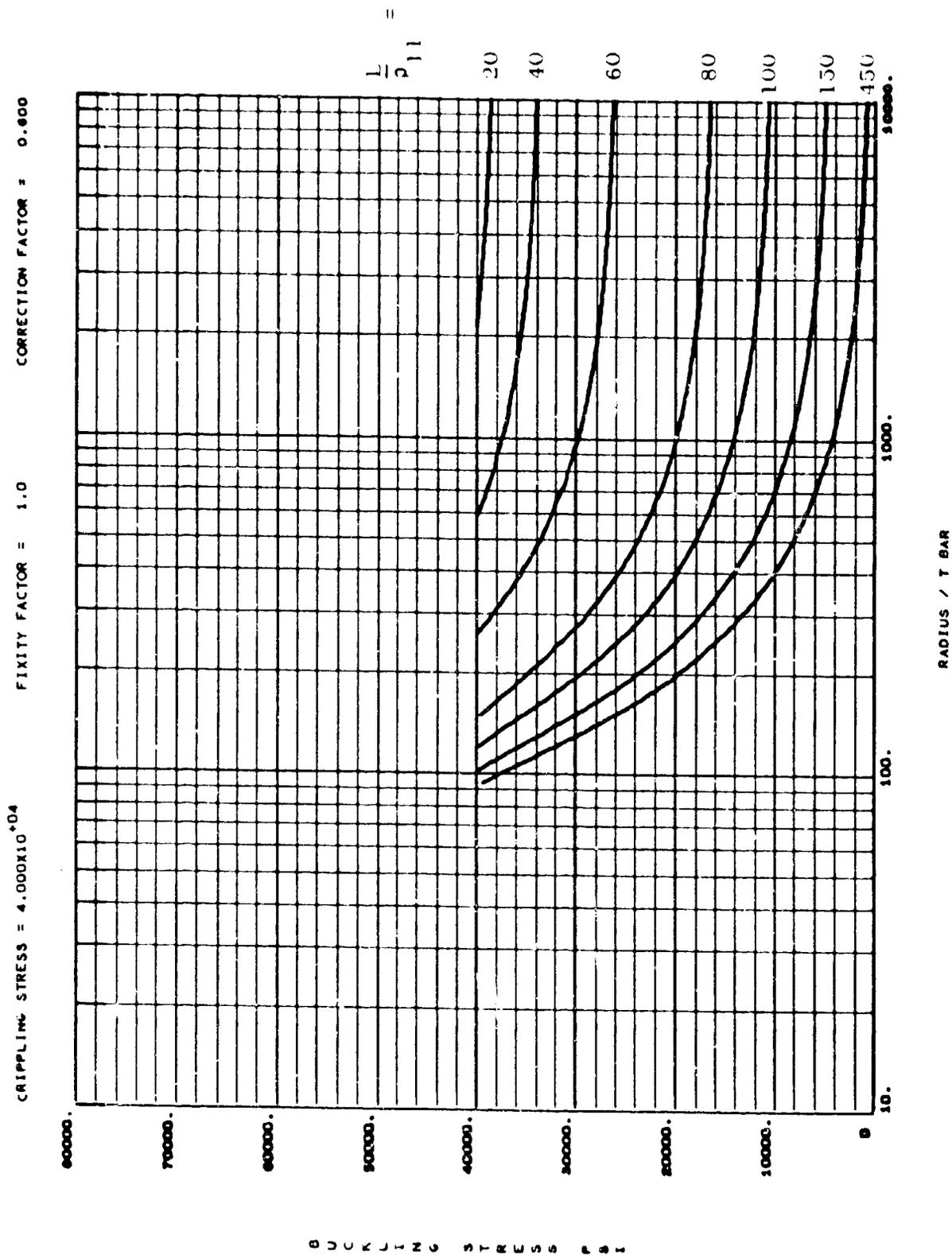
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 34(a) - (see Table XVII)



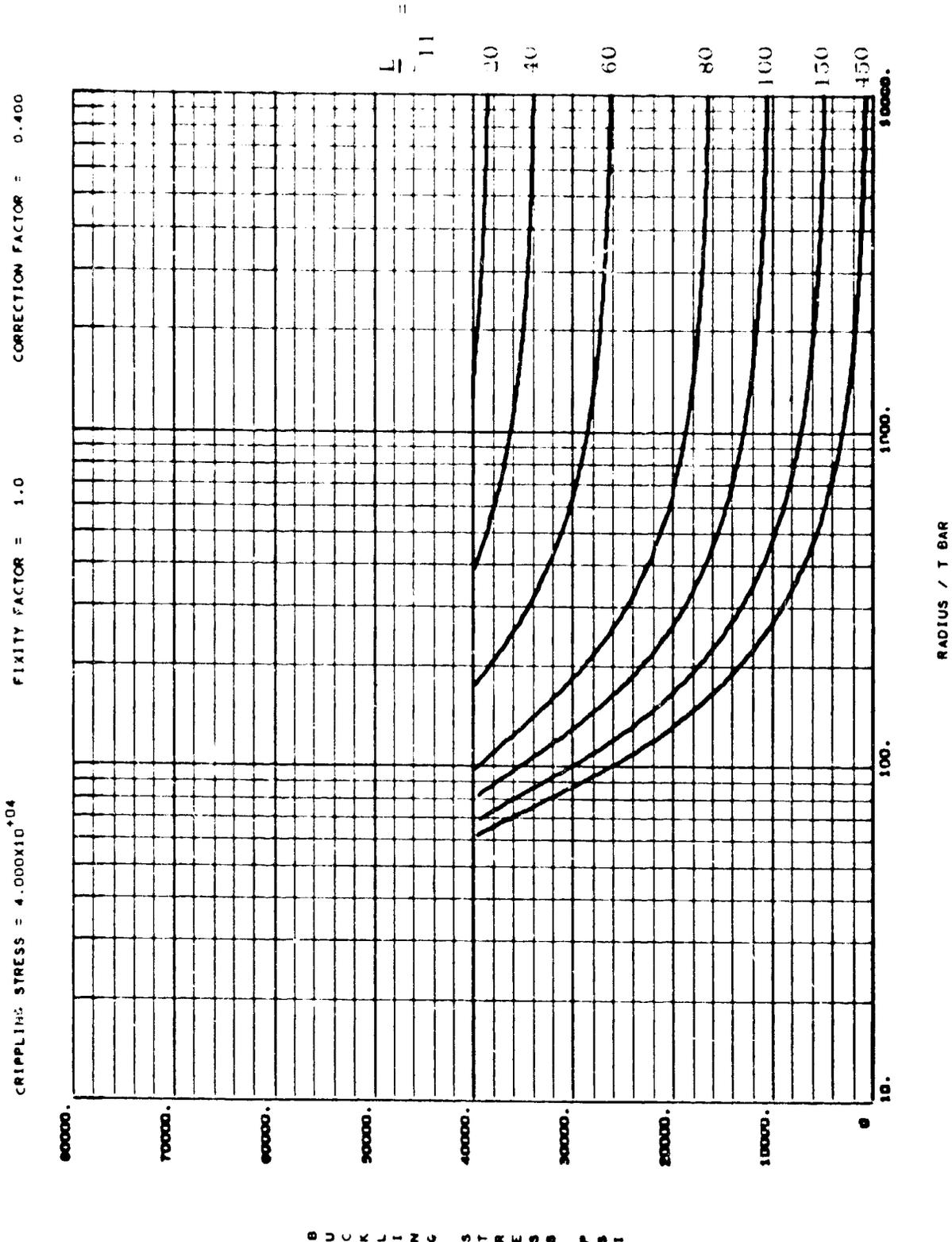
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 34(b) - (See Table XVII)



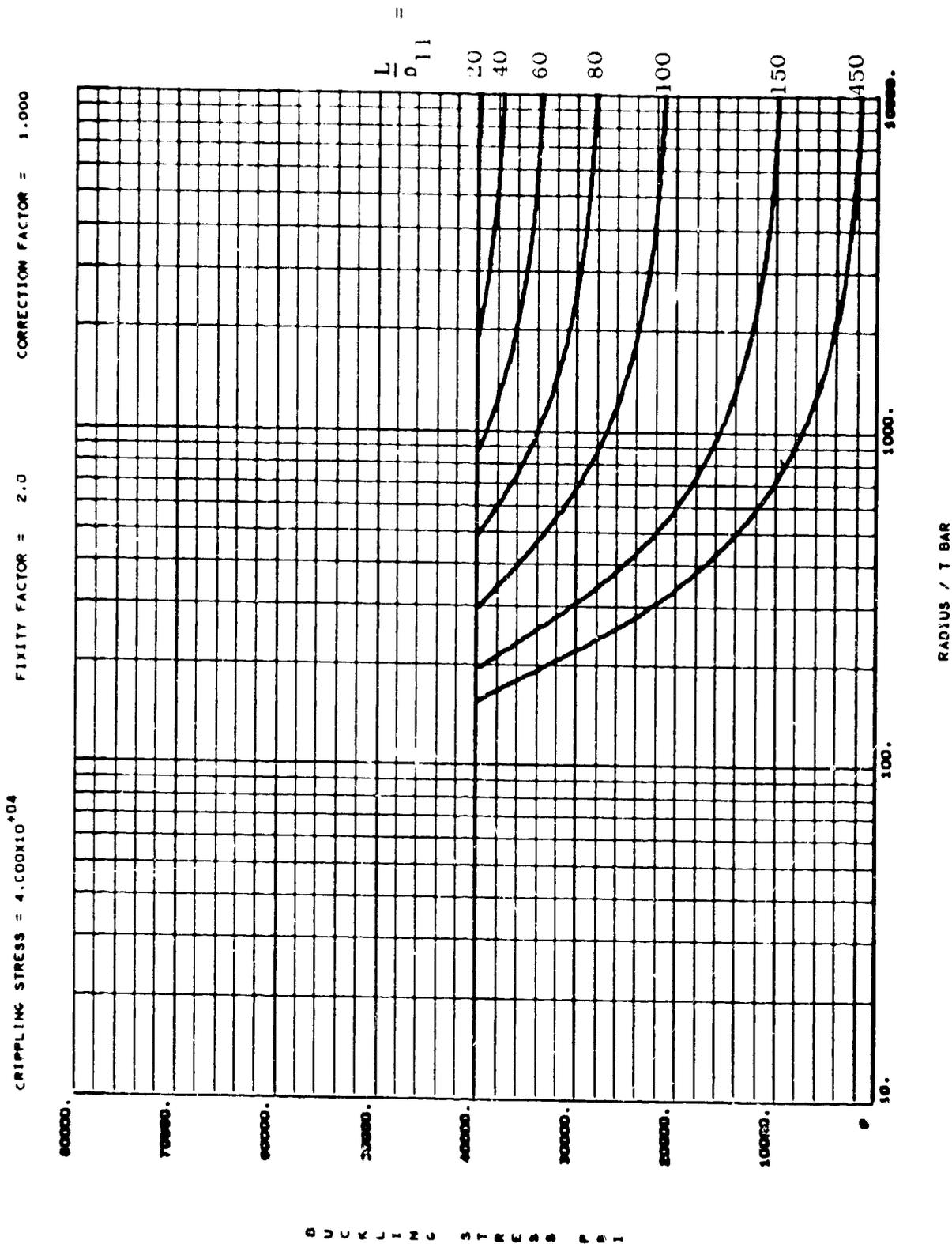
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 34(c) - (see Table XVII)



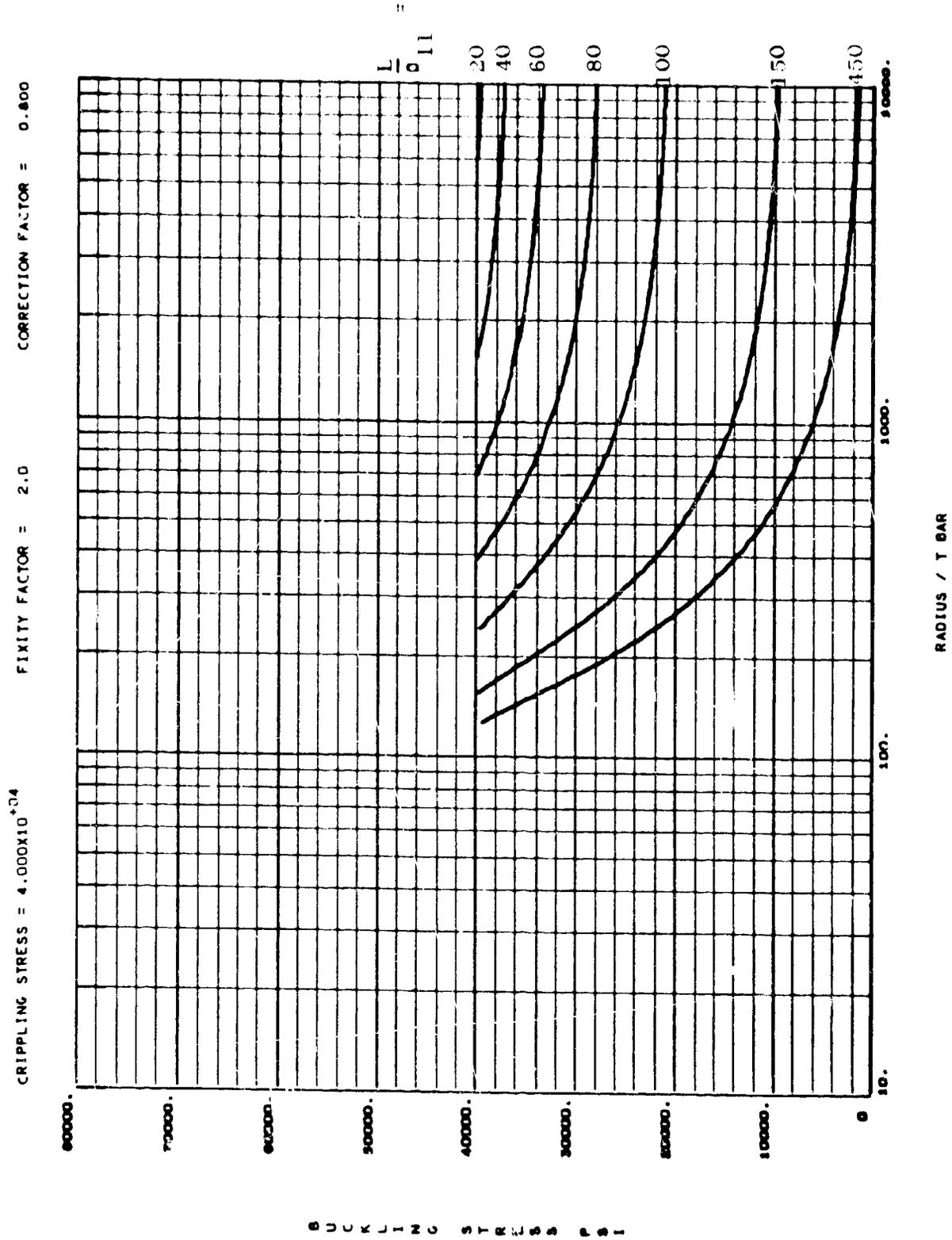
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 71(d) - (See Table VIII)



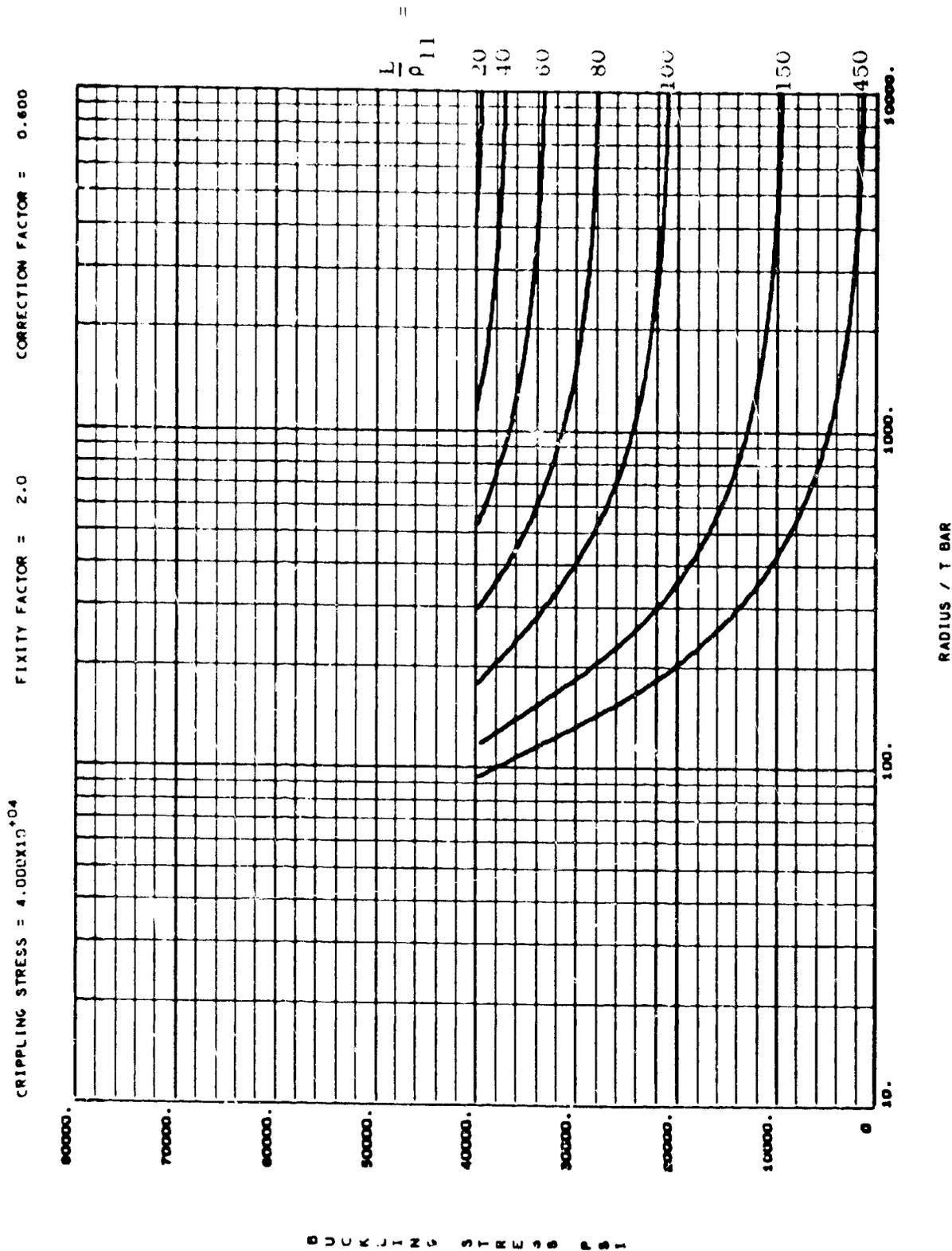
COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS

Figure 54(e) - (see Table XVII)



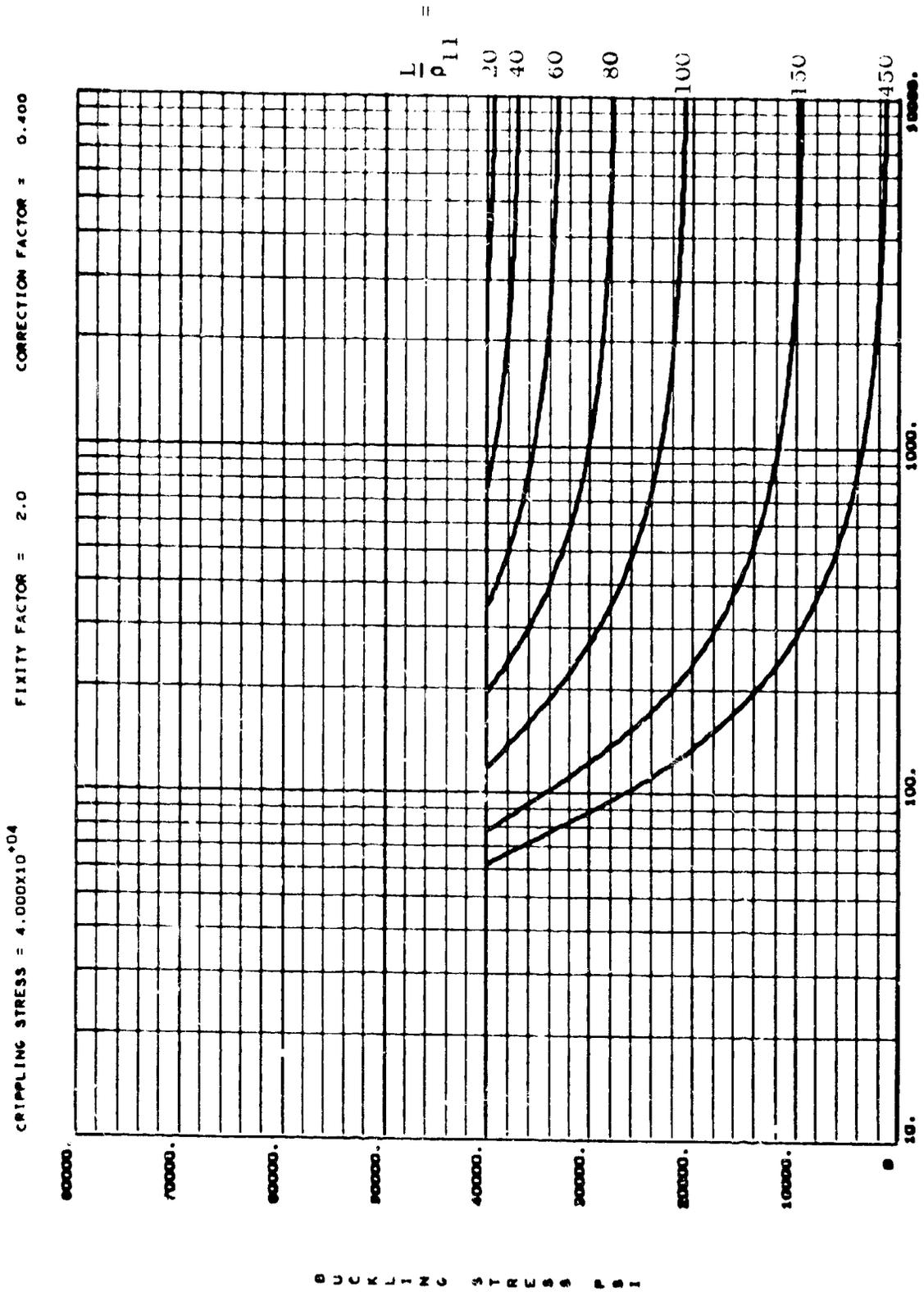
COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 34(F) - (see Table XVII)



COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS

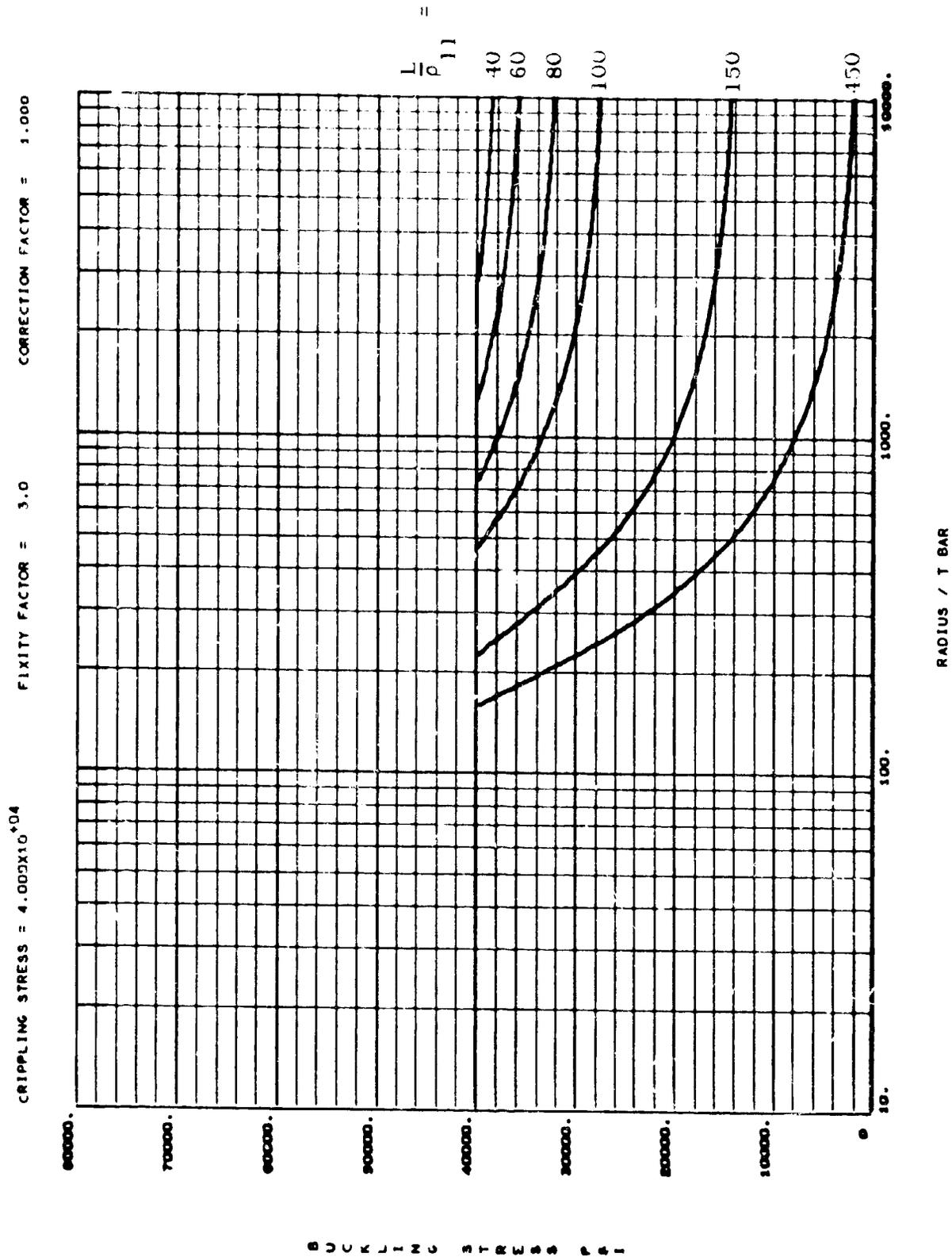
Figure 54(g) - (see Table XVII)



COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

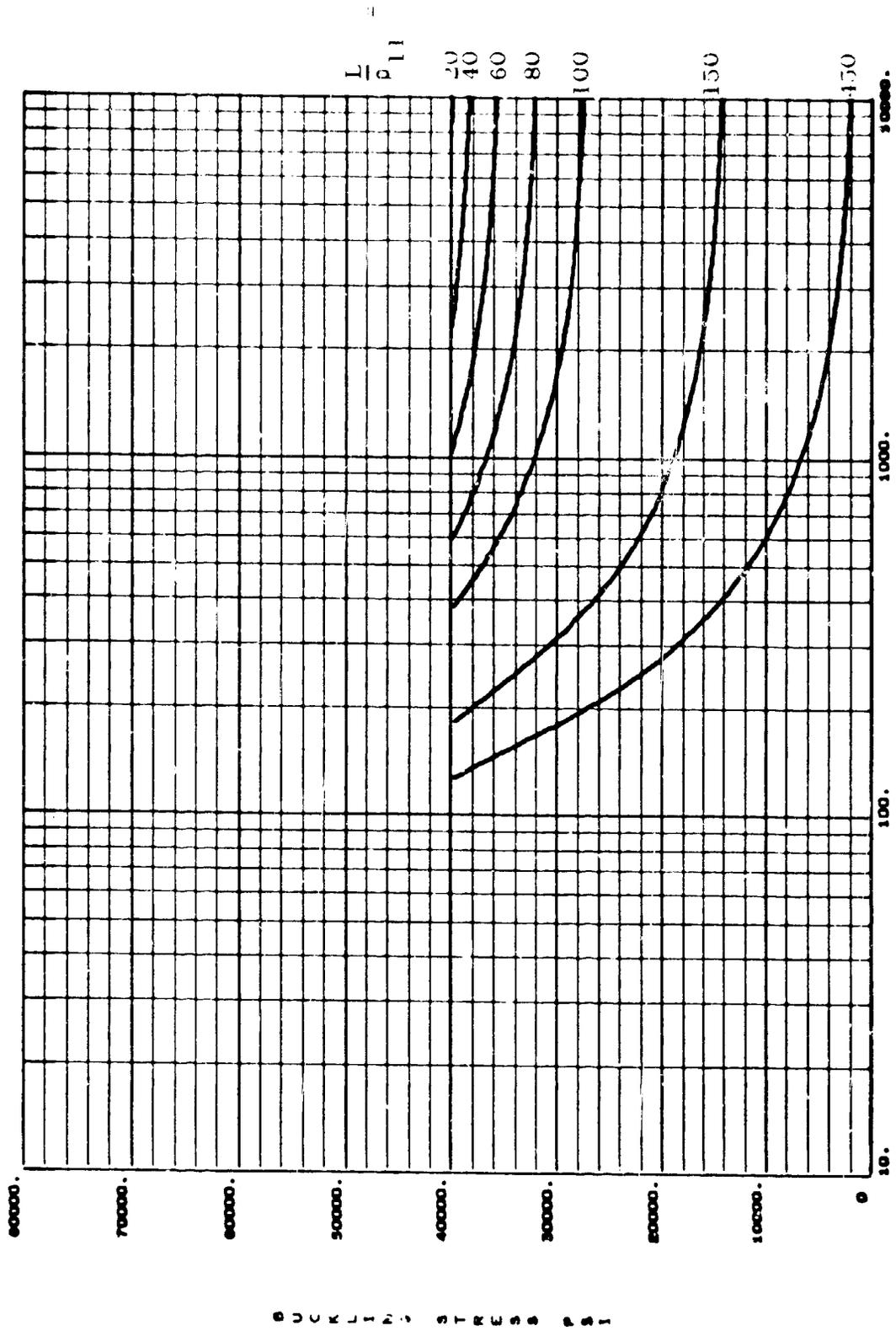
Figure 34(h) - (see Table XVII)

GENERAL DYNAMICS  
Convair Division



COMPRESSION BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 51(i) - (See Table XVII)

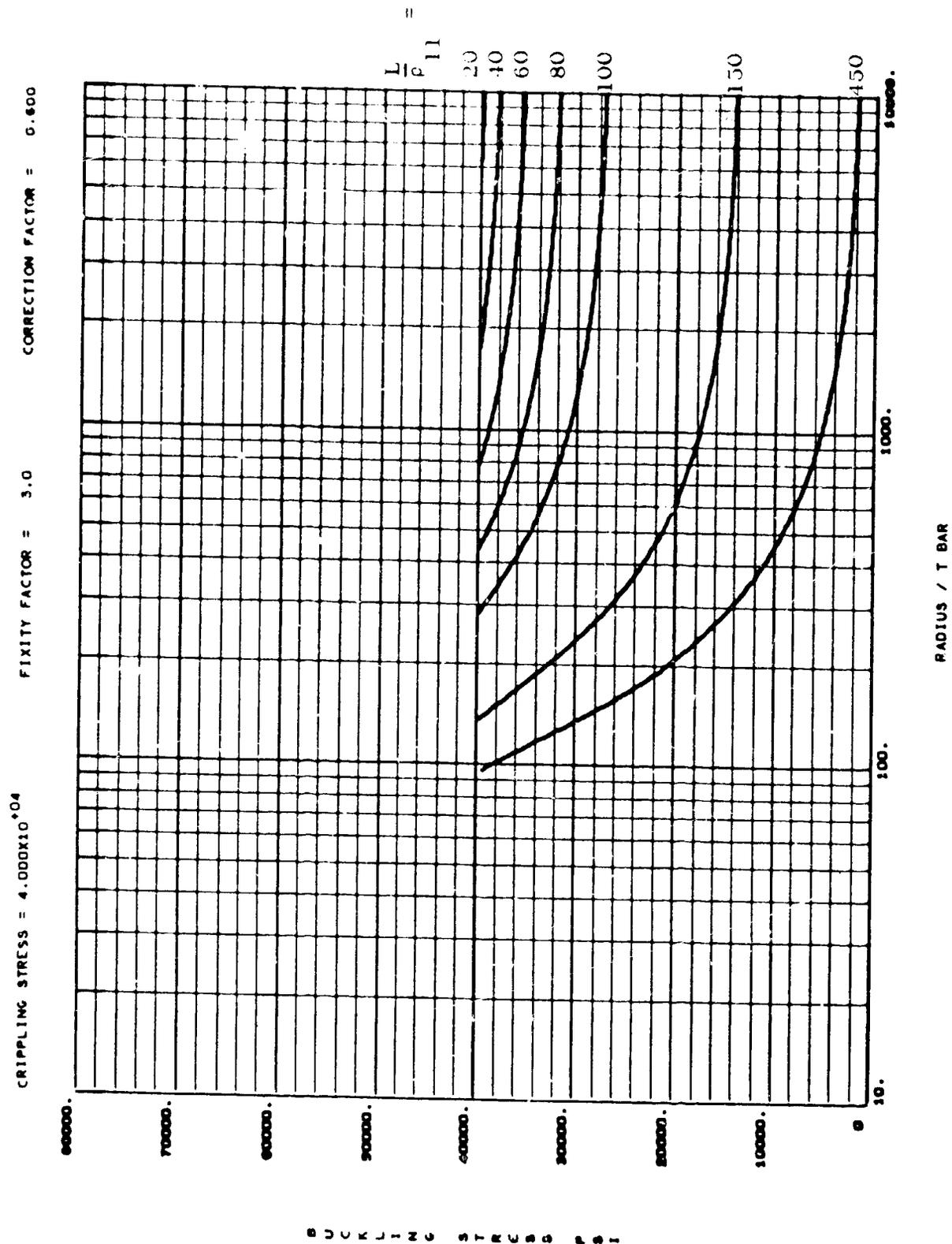
CRIPPLING STRESS =  $4.000 \times 10^{+04}$       FIXITY FACTOR = 3.0      CORRECTION FACTOR = 0.800



RADIUS / T BAR

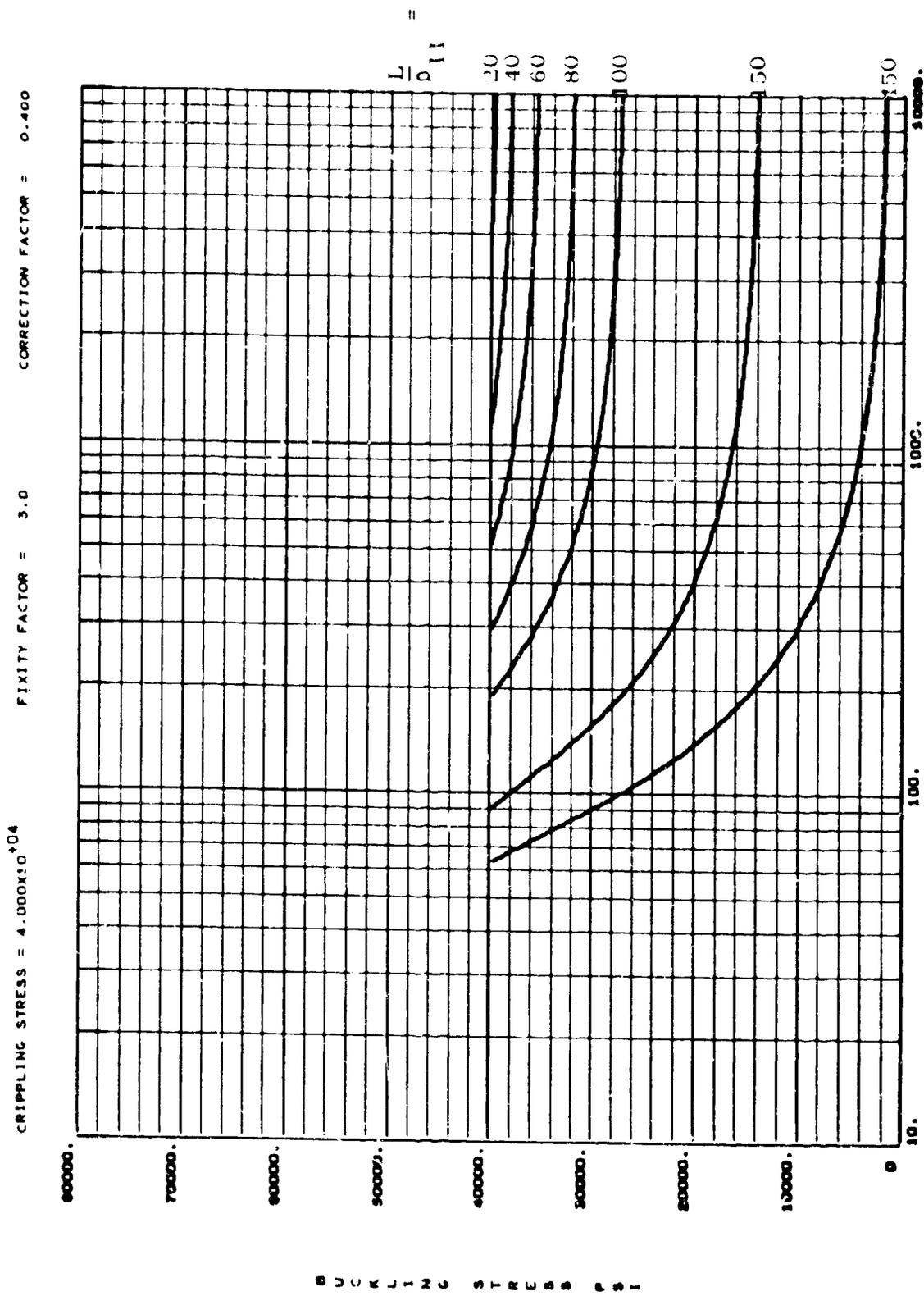
COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 51(j) - (see Table XVII)

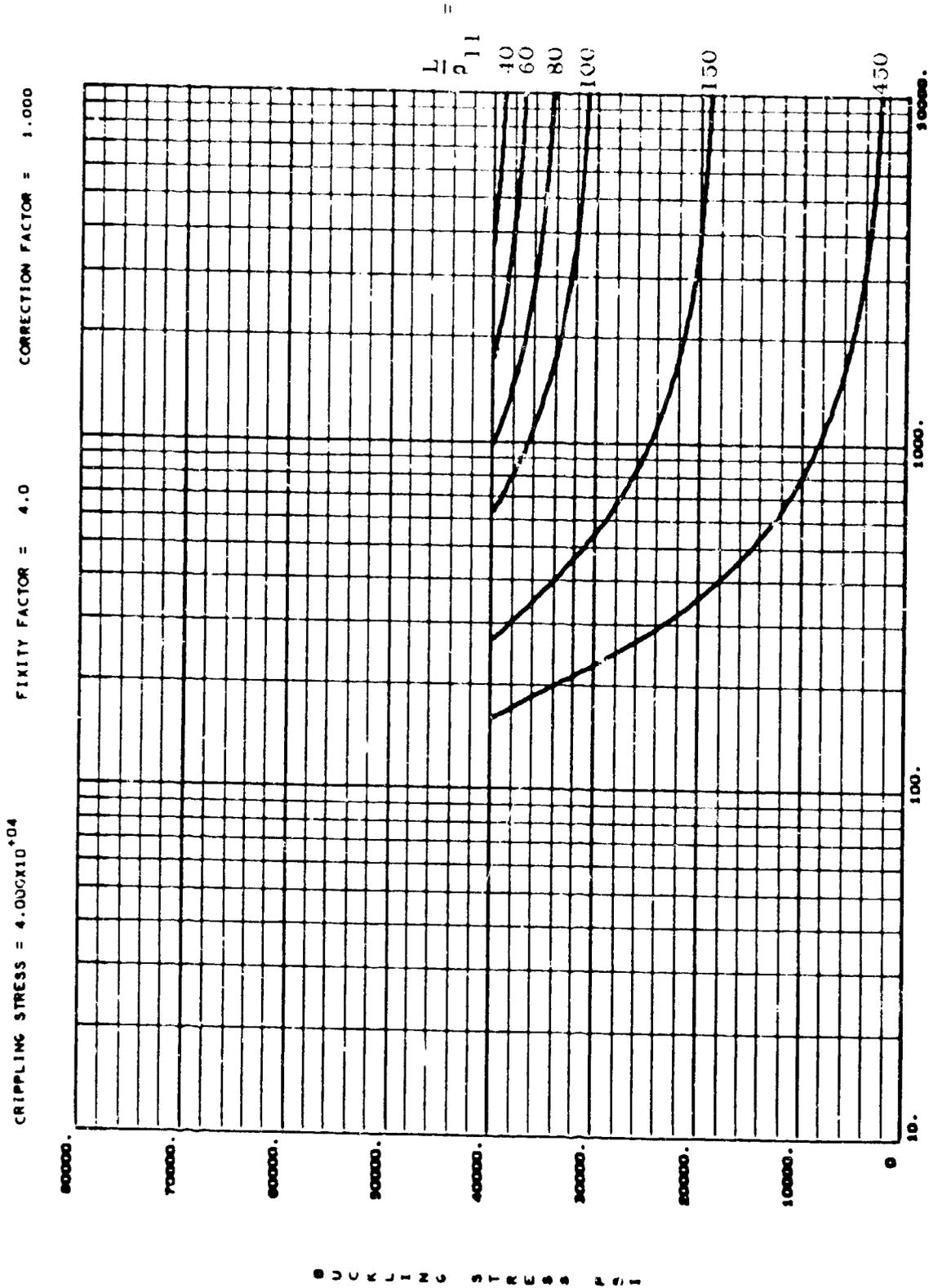


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 54(k) - (see Table XVII)

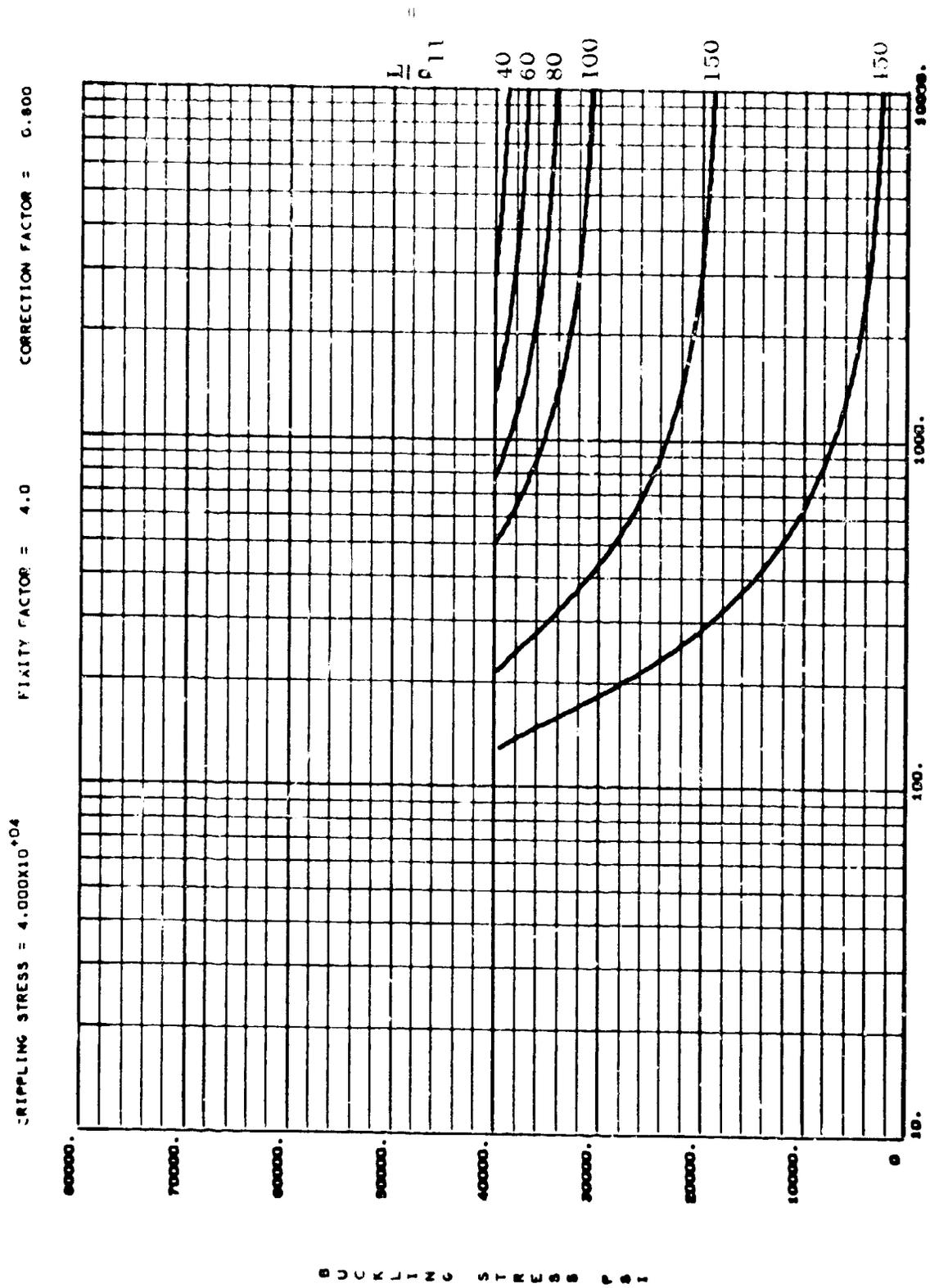


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS  
Figure 3(10) - (See Table XVII)



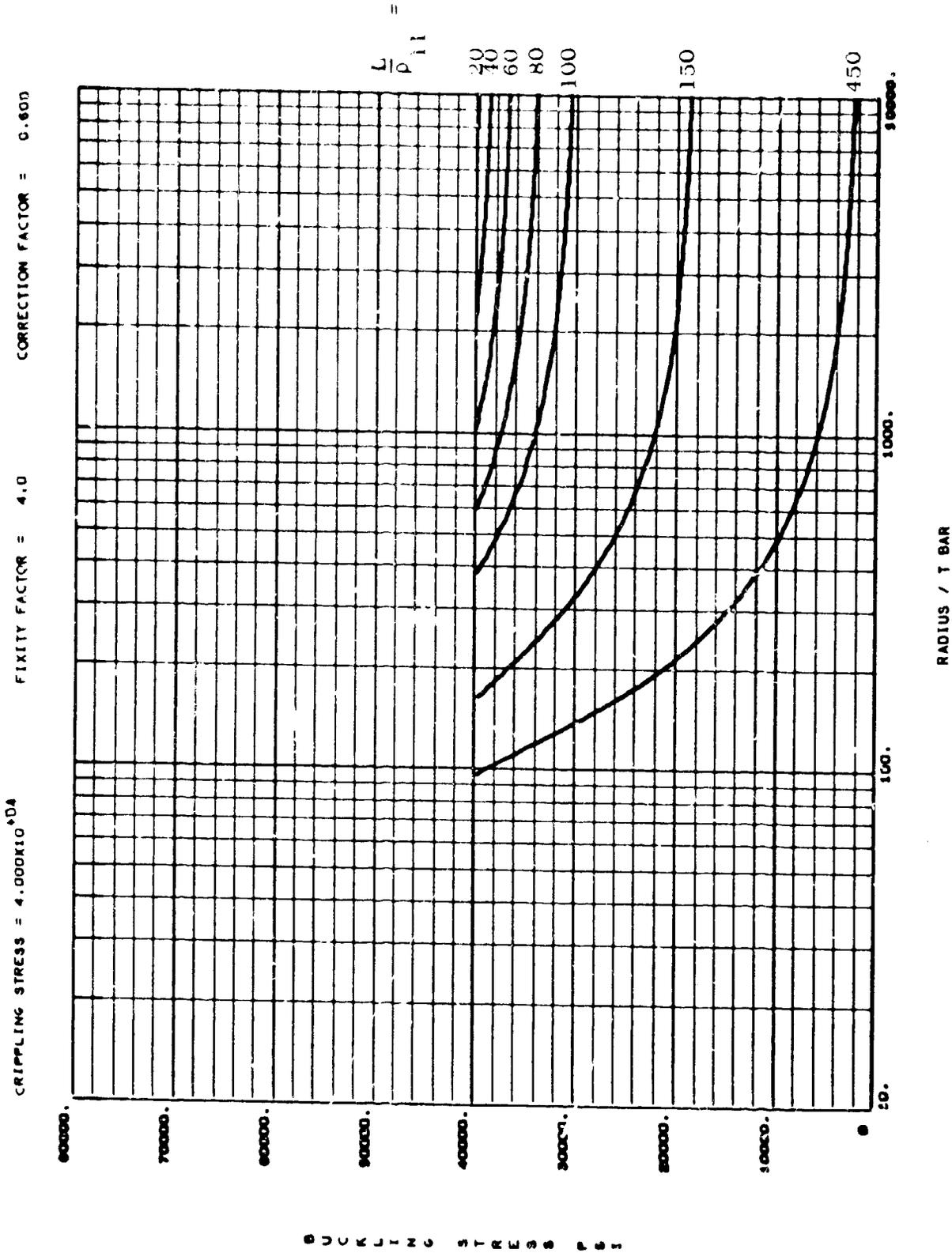
COMPRESSIVE BUCKLING STRESS FOR  
 LONGITUDINALLY STIFFENED 7075-T6  
 AL ALLOY CIRCULAR CYLINDERS

Figure 54(m) - (See Table XVII)

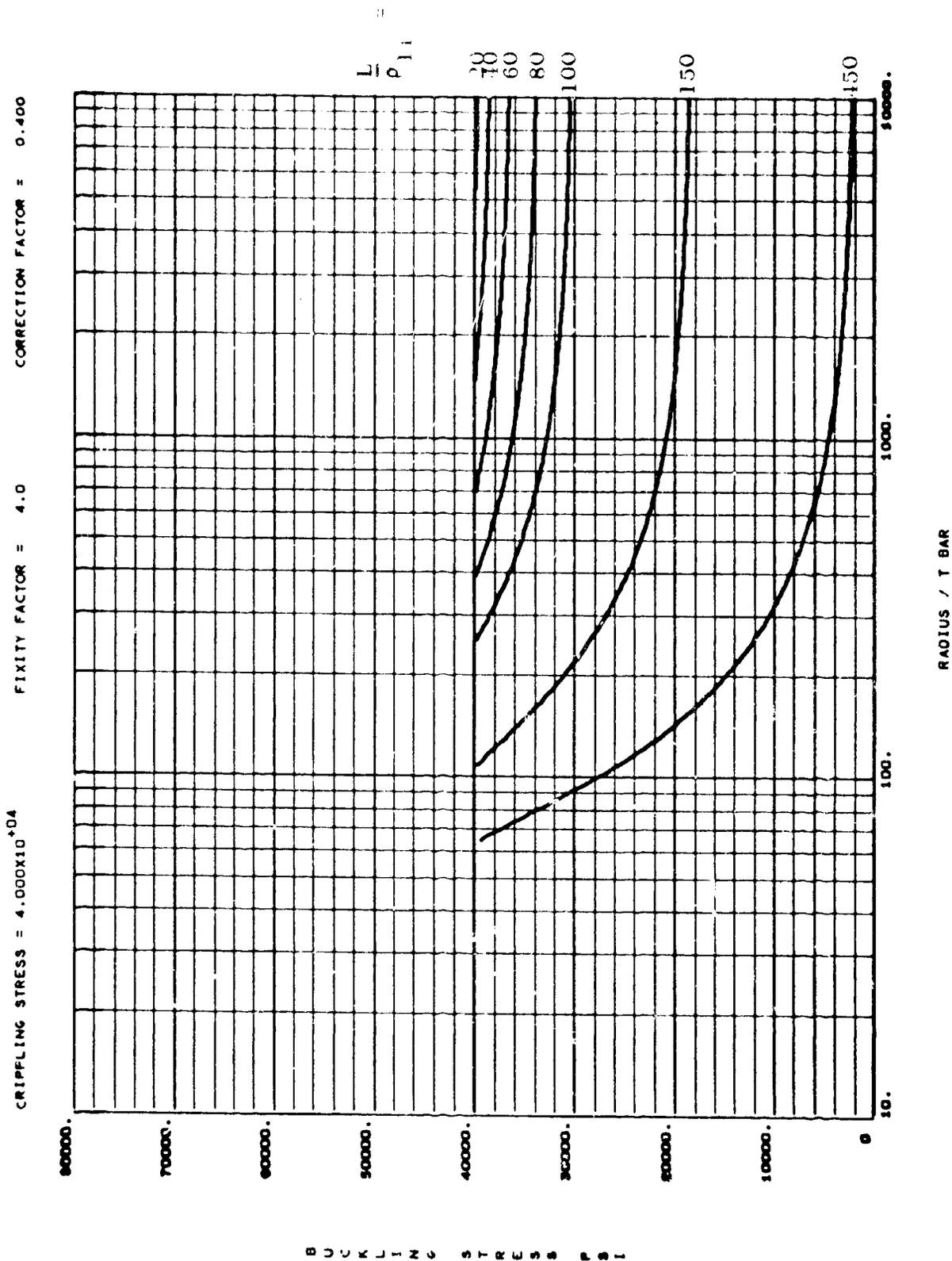


COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 1(n) - (See Table XVII)



**COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS**  
Figure 34(a) - (See Fible XVII)



COMPRESSIVE BUCKLING STRESS FOR  
LONGITUDINALLY STIFFENED 7075-T6  
AL ALLOY CIRCULAR CYLINDERS

Figure 34(p) - (see Table XVII)

13.0 - GENERAL INSTABILITY OF ORTHOTROPICALLY  
STIFFENED CIRCULAR CYLINDRICAL SHELLS  
SUBJECTED TO AXIAL COMPRESSION

13.1            Procedures

The procedures given here deal only with general instability (see Glossary) and give no consideration to the panel instability mode (see Glossary). The latter mode must be considered separately by means of the procedures given in Section 12.

These procedures employ the smearing-out technique whereby discrete stiffness values are averaged over the entire surface of the cylinder. One must therefore exercise engineering judgement to prevent misapplication to configurations having excessively large stiffener spacings.

Simple formulas for the  $A_{ij}$ 's and  $D_{ij}$ 's are given in Table XVIII. This table only considers the following two cases:

- (a) No pre-buckling occurs and all of the stringer and skin material is fully effective.
- (b) Buckling of the isotropic skin panels and/or local buckling of the stringers occurs, requiring the use of effective width concepts.

In some practical applications, one might encounter stringer spacings sufficiently large to justify the use of effective width concepts even in the absence of any pre-buckling. Aside from the effective width criterion, the approach would then be quite similar to that for case (b) above.

It is proposed that the following procedure be used for the approximate analysis of general instability in orthotropically stiffened circular cylindrical shells subjected to axial compression:

Step 1 - Using Table XVIII, compute the values  $A_{11}$ ,  $A_{22}$ ,  $A_{12}$ ,  $A_{33}$ ,  $D_{11}$ , and  $D_{22}$ .

Step 2 - Compute the ratio  $\frac{R^2 A_r}{I_r}$  where

$A_r$  = Cross-sectional area of ring excluding all basic cylindrical skin [see Figure 15 and equation (7-1).

$I_r$  = Centroidal moment of inertia of ring excluding all basic cylindrical skin [see Figure 15 and equation (7-13).]

$R$  = Radius to centroid of ring (excluding all basic cylindrical skin).

Step 3 - Using the curves of Figure 35 or the digital computer program of Section 18.3.3, determine the correction factor  $C_R$ . (Note that  $N_s$  = Number of Stringers).

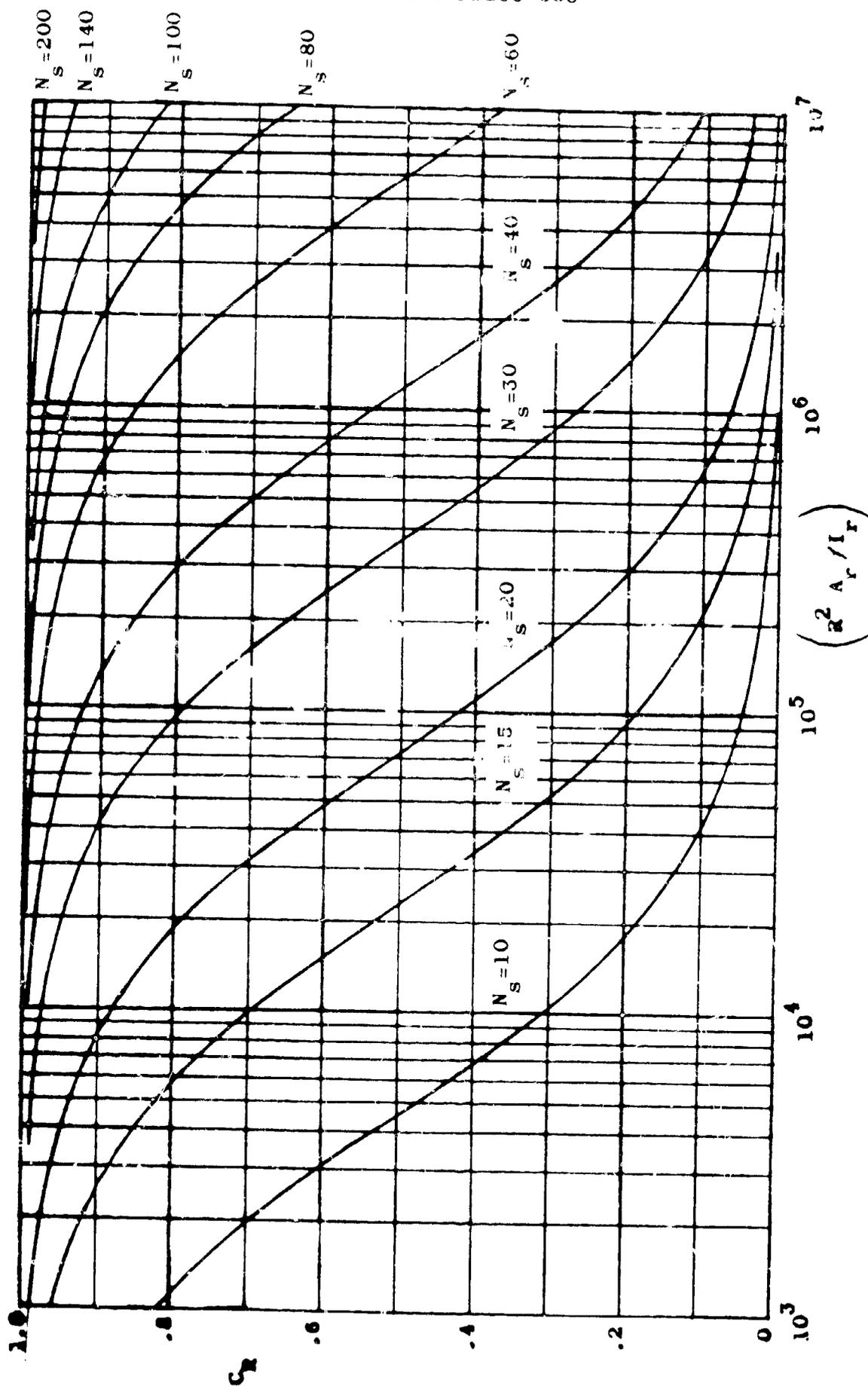


Figure 35 - Design Curves for the Correction Factor  $C_R$

Step 4 - Calculate the effective local longitudinal and circumferential radii of gyration ( $\rho_{11}$  and  $\rho_{22}$  respectively) as follows:

$$\rho_{11} = \sqrt{D_{11} A_{11}} \quad (13-1)$$

$$\rho_{22} = \sqrt{D_{22} A_{22}} \quad (13-2)$$

Step 5 - Compute the ratios  $\frac{a}{\rho_{11}}$ ,  $\frac{\sqrt{C_R} D}{\rho_{22}}$ , and  $\frac{a}{D}$  where

a = Spacing between rings.

D = Diameter of middle surface of basic cylindrical skin.

Step 6 - Compute the Thielemann parameter  $\eta_s$  from the following:

$$\eta_s = \frac{A_{12} + \frac{A_{33}}{2}}{\sqrt{A_{11} A_{22}}} \quad (13-3)$$

Step 7 - Enter the design curves of Section 13.2 to find the critical compressive loading coefficient  $\bar{N}$ . Section 18.3.1 gives the digital computer program which was used to obtain these curves. This program may be used to obtain additional plots or single-point solutions.

Step 8 - Compute the classical theoretical critical running load (lbs/inch) as follows:

$$(N_x)_{CL} = \frac{2\bar{N}}{R} \sqrt{\frac{D_{22}}{A_{11}}} \quad (13-4)$$

Step 9 - Use the procedure of Section 15 to establish the correlation (knock-down) factor  $\Gamma$ .

Step 10 - Compute the wide-column strength  $(N_x)_{wc}$  as follows:

$$(N_x)_{wc} = \frac{C_F \pi^2 D_{11}}{L^2} \quad (13-5)$$

where

$C_F$  = Fixity factor associated with ends of cylinder.

$L$  = Overall length of cylinder (The ring spacing,  $a$ , is NEVER substituted here).

Note that whenever

$$\frac{L}{\rho_{11}} < \left( \sqrt{2C_F} \right) \left( \pi \right) \left( \sqrt{\frac{E}{\sigma_{cc}}} \right) \quad (13-6)$$

one should consider the possibility for the Johnson parabola to give lower values than equation (13-5).

However, since condition (13-6) is not likely to be encountered in general instability problems, this case is not treated here. Also note that, since

$(N_x)_{wc}$  will usually be relatively small, one might choose to use the conservative assumption that

$$(N_x)_{wc} = 0.$$

Step 11 - Compute the predicted critical running load (lbs/in) as follows:

$$(N_x)_{cr} = (N_x)_{wc} + \Gamma \left[ (N_x)_{CL} - (N_x)_{wc} \right] \quad (13-7)$$

THIS IS THE FINAL STEP FOR THE METHOD.

The foregoing analysis procedure is based on the Thielemann [19] infinite-length orthotropic cylinder equation (see Sections 6 and 7) which assumes that the cylinder can accommodate longitudinal half-waves of any size. Finite-length considerations sometimes lead to the requirement that the longitudinal half-wavelengths be equal to the overall length  $L$  divided by an integer. To find solutions which satisfy this restriction, one may employ the digital computer program of Section 18.3.2. The print-out critical running load from this program is to be regarded as the classical theoretical value  $(N_x)_{CL}$ . This value is used together with Steps 9 through 11 above to establish the predicted critical running load  $(N_x)_{cr}$ .

The program of Section 18.3.2 should also be used whenever eccentricities of the longitudinal and/or circumferential stiffeners must be considered. Here again, the machine output is to be used in the same manner as the  $(N_x)_{CL}$  from Step 8 above.

Solutions obtained using the digital computer program of Section 18.3.2 include no consideration of the stiffness reduction covered by the correction factor  $C_R$ . However, even in these situations it is still useful to establish  $C_R$  since it provides some basis for evaluating the reliability of the predictions. That is, whenever  $C_R \geq .95$ , one might conclude that the discreteness mechanism depicted in Figure 15 is of negligible importance. On the other hand, lower  $C_R$  values would indicate the need to apply engineering judgement in evaluating the machine output.

TABLE XVIII - Recommended Formulas for the  $A_{ij}$ 's and  $D_{ij}$ 's of Circular Cylinders Having Both Longitudinal and Circumferential Stiffeners.

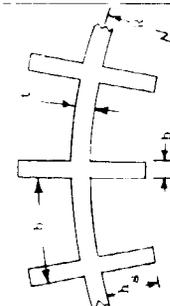
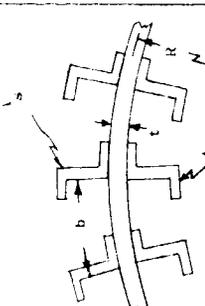
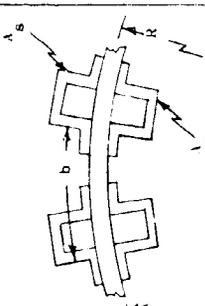
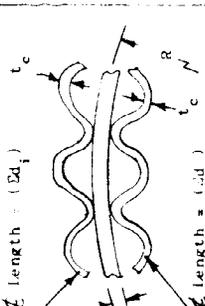
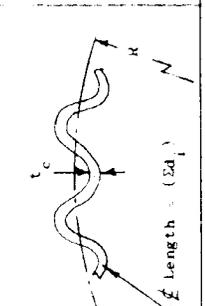
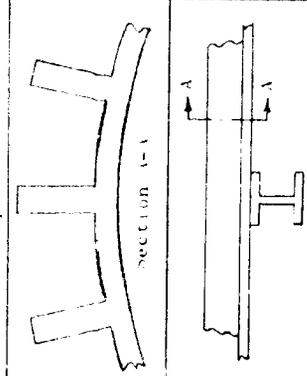
Case	Configuration	$t_k$	$\lambda_{11}$	$\lambda_{12}$	$A_{33}$	$D_{11}$	$D_{12}$	$D_{33}$	
A	Integral stringers, equally spaced rings. No eccentricities. Stringers and rings both symmetric with respect to skin. No buckling of isotropic skin panels. No local buckling of stiffeners.		$\left(\frac{2bh_s}{b} \cdot t\right)$	$\frac{1}{\pi t k}$	0	$\frac{1}{Gt}$	$EJ_k$	$\frac{Et_c^3}{a} + \frac{Et_c^3}{12(1-\nu^2)}$	0
B	Z-shaped non-integral stringers. Equally spaced rings. No eccentricities. Stringers and rings both symmetric with respect to skin. No buckling of isotropic skin panels. No local buckling of stiffeners.		$\left(\frac{2A_s}{b} \cdot t\right)$	"	"	"	"	"	"
C	Hat-shaped non-integral stringers. Equally spaced rings. No eccentricities. Stringers and rings both symmetric with respect to skin. No buckling of isotropic skin panels. No local buckling of stiffeners.		"	"	"	"	"	"	"
D	Longitudinal corrugations attached to continuous cylindrical skin. Equally spaced rings. No eccentricities. Corrugations and rings both symmetric with respect to skin. No local buckling.		$\left[\frac{(2A_1)}{\pi R} t_c \cdot t\right]$	"	"	$\frac{1}{G \left[ \frac{4\pi R}{(2A_1)} t_c \cdot t \right]}$	$\frac{Et_c^3}{a} + \frac{E}{12(1-\nu^2)} \left[ 2t_c^3 \left(\frac{6a}{A_1}\right) \cdot t^3 \right]$	"	
E	Simple corrugated wall. Equally spaced rings. No eccentricities. Corrugation and rings both symmetric about reference cylindrical surface of radius R. No local buckling.		$\left[ \frac{(2A_1) t_c}{2\pi R} \right]$	"	"	$\frac{1}{G \left[ \frac{2\pi R}{(2A_1)} t_c \right]}$	$\frac{Et_c^3}{a} \cdot \left[ \frac{6a}{12(1-\nu^2)} \left(\frac{6a}{A_1}\right) \right]$	"	

TABLE XVIII - Recommended Formulas for the  $A_{ij}$ 's and  $D_{ij}$ 's of Circular Cylinders Having Both Longitudinal and Circumferential Stiffeners (Continued)

CASE	CONFIGURATION	$t_x'$	$A_{11}$	$A_{22}$ Note (1)	$A_{12}$	$A_{33}$	$D_{11}$	$D_{22}$ [See note (1)]	$D_{12}$	$D_{33}$	
F	Same as cases A, B, and/or C, except that buckling of isotropic skin panels occurs.	see Note (d) below	$\frac{1}{Gt_x}$	$\frac{1}{E} \left( \frac{t}{a} \right)$	0	$\frac{1}{Gt}$	$EI_x'$	$\frac{EI_x'}{a} + \frac{Et^3}{12(1-\nu^2)}$	0	0	
G	Same as cases A, B, and/or C, except that local buckling of longitudinal stiffeners occurs.	"	"	"	"	$\frac{1}{Gt}$	"	"	"	"	
H	Same as cases A, B, and/or C, except that both buckling of isotropic skin panels and local buckling of longitudinal stiffeners occur.	"	"	"	"	$\frac{1}{Gt}$	"	"	"	"	
I	Any of cases A through H, except stiffeners and/or rings eccentric with respect to skin.			Digital computer program of Section 18.3.2 may be used to obtain classical critical running load.							

## Notes for TABLE XVIII:

- (a) The tabulated formulas constitute simplifications suitable for practical engineering purposes. To be rigorous, much more complicated expressions would be required.
- (b) For cases where the buckling stress exceeds the proportional limit of the stress-strain curve,  $E_{\tan}$  and  $G_{\tan}$  must be respectively substituted for  $E$  and  $G$  in all formulas except for  $A_{22}$  and the  $EI_r/a$  term for  $D_{22}$ .
- (c) The  $A_{ij}$ 's and  $D_{ij}$ 's arise out of mathematical integrations involving the distribution of the composite wall material about the cylindrical centroidal surface. When either buckling of the isotropic skin panels or local buckling of the stiffeners occurs, only effective widths are considered. Note that the centroidal surface has curvature of its own. Therefore, the related material distribution is equivalent to that which exists about the centroidal plane of the flat plate obtained by unfolding the composite circular shell wall into a flat configuration. All influences of curvature, in this regard, are inherent in the basic shell equations into which the  $A_{ij}$ 's and  $D_{ij}$ 's are substituted.
- (d) The quantities  $A_s$ ,  $I_x$ ,  $I_x'$ ,  $t_x$ ,  $t_x'$ ,  $t_x''$ ,  $t_x'''$ ,  $\left(\frac{\partial \theta}{\partial \theta}\right)$ , and  $(\sum d_i)$  are defined in the notes for Table XVI.
- (e) The quantity  $A_r$  is the cross-sectional area of a single ring (no basic cylindrical skin included).
- The quantity  $I_r$  is the centroidal moment of inertia of a single ring (no basic cylindrical skin included).
- The cross section referred to here is obtained by passing a radial plane, containing the axis of revolution, through the ring.
- (f) The quantity  $a$  is the spacing between rings (assumed to be uniform throughout the structure).
- (g) The term  $(1-\nu^2)$  has been omitted from the  $D_{11}$  formulas and the  $EI_r/a$  term for  $D_{22}$  since the specified configurations provide incomplete restraint to the related anticlastic bending (see Glossary). The second term in the  $D_{22}$  formulas retain the  $(1-\nu^2)$  factor

Notes for TABLE XVIII:(Continued)

since the usually broad axial extent of the skin panels affords anticlastic restraint in the same manner as that customarily recognized for flat plates.

- (h) The quantities  $D_{12}$  and  $D_{33}$  are assumed equal to zero in the interest of simplicity. This is a conservative practice.
- (i) The analyst might sometimes choose to consider an effective width ( $a_e$ ) of the basic cylindrical skin in the computation of the ring section properties (both area and moment of inertia). Indeed, when no buckling of the isotropic skin panels occurs and the circumferential stiffeners are exceptionally close together, the entire skin may be considered fully effective ( $a_e = a$ ). On the other hand, when the stiffeners are widely spaced or when buckling of the isotropic skin panels occurs, one might choose to select  $a_e$  equal to the effective width by which a single stringer cross section is augmented. The formulas given in the table assume  $a_e = 0$  as a practical conservative expediency, since a rigorous determination of this value introduces complexities beyond the scope of the report. However, where engineering judgement or other means can be employed to select a reasonable  $a_e$  value, the symbols  $A_r'$  and  $I_r'$  may be used to respectively denote the augmented cross-sectional area and moment of inertia values. For cases A, B, C, F, G, and H shown in the table, the following formulas should then be used for the constants  $A_{22}$  and  $D_{22}$  when the linear portion of the stress-strain curve applies:

$$A_{22} = \frac{1}{E} \left( \frac{A_r'}{a} \right) \quad D_{22} = \frac{EI_r'}{a} + \left( \frac{a - a_e}{a} \right) \frac{Et^3}{12(1-\nu^2)}$$

For cases when the buckling stress exceeds the proportional limit, these formulas should be modified to reflect tangent modulus effects in the skin material only.

13.2

Design Curves

Table XIX lists the families provided here.

TABLE XIX - Table of Contents for the Design Curves  
"Critical Compressive Loading Coefficient  
for the General Instability of Stiffened  
Circular Cylinders

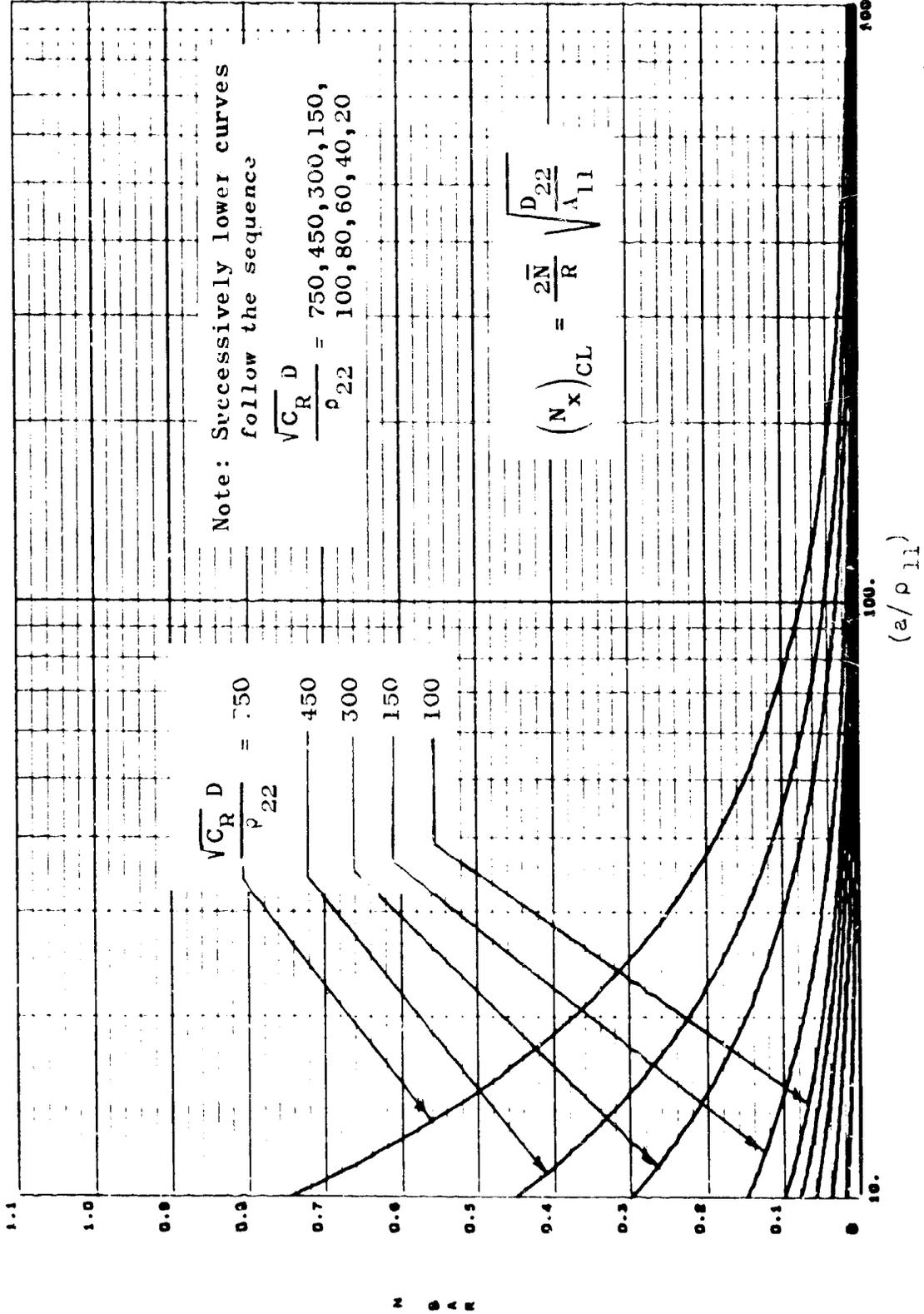
<u>Figure</u> <u>Number</u>	<u><math>\frac{a}{D}</math></u>	<u><math>\eta</math></u> <u><math>p</math></u>	<u><math>\eta</math></u> <u><math>s</math></u>	<u>Page</u>
36(a)	0.01	0	0.1	287
36(b)	0.01	0	0.5	288
36(c)	0.01	0	1.0	289
36(d)	0.01	0	5.0	290
36(e)	0.01	0	10.0	291
36(f)	0.05	0	0.1	292
36(g)	0.05	0	0.5	293
36(h)	0.05	0	1.0	294
36(i)	0.05	0	5.0	295
36(j)	0.05	0	10.0	296
36(k)	0.1	0	0.1	297
36(l)	0.1	0	0.5	298
36(m)	0.1	0	1.0	299
36(n)	0.1	0	5.0	300
36(o)	0.1	0	10.0	301
36(p)	0.5	0	0.1	302
36(q)	0.5	0	0.5	303
36(r)	0.5	0	1.0	304

TABLE XIX - Table of Contents for the Design Curves  
"Critical Compressive Loading Coefficient  
for the General Instability of Stiffened  
Circular Cylinders (Continued)

<u>Figure</u> <u>Number</u>	<u><math>\frac{a}{D}</math></u>	<u><math>\eta_p</math></u>	<u><math>\eta_s</math></u>	<u>Page</u>
36(a)	0.5	0	5.0	305
36(t)	0.5	0	10.0	306
36(u)	1.0	0	0.1	307
36(v)	1.0	0	0.5	308
36(w)	1.0	0	1.0	309
36(x)	1.0	0	5.0	310
36(y)	1.0	0	10.0	311

ETA SUB 5 = 0.100

RING SPACING/DIAMETER = 0.010

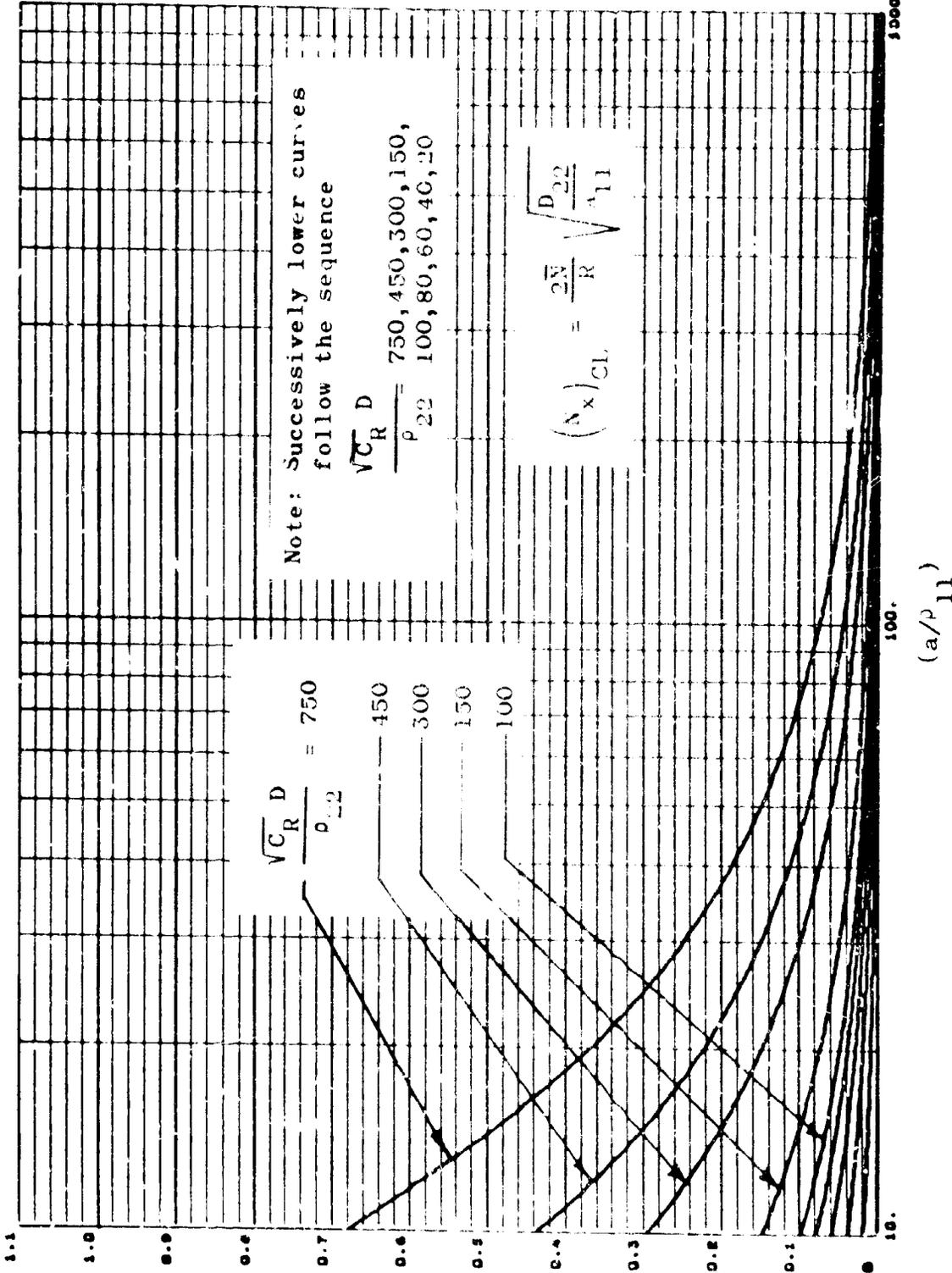


CRITICAL COMPRESSIVE LOADING COEFFICIENT FOR THE GENERAL INSTABILITY OF STIFFENED CIRCULAR CYLINDERS

Figure 36(a) - (See Table XIX)

ETA SUB 5 = 0.500

RING SPACING/DIAMETER = 0.010

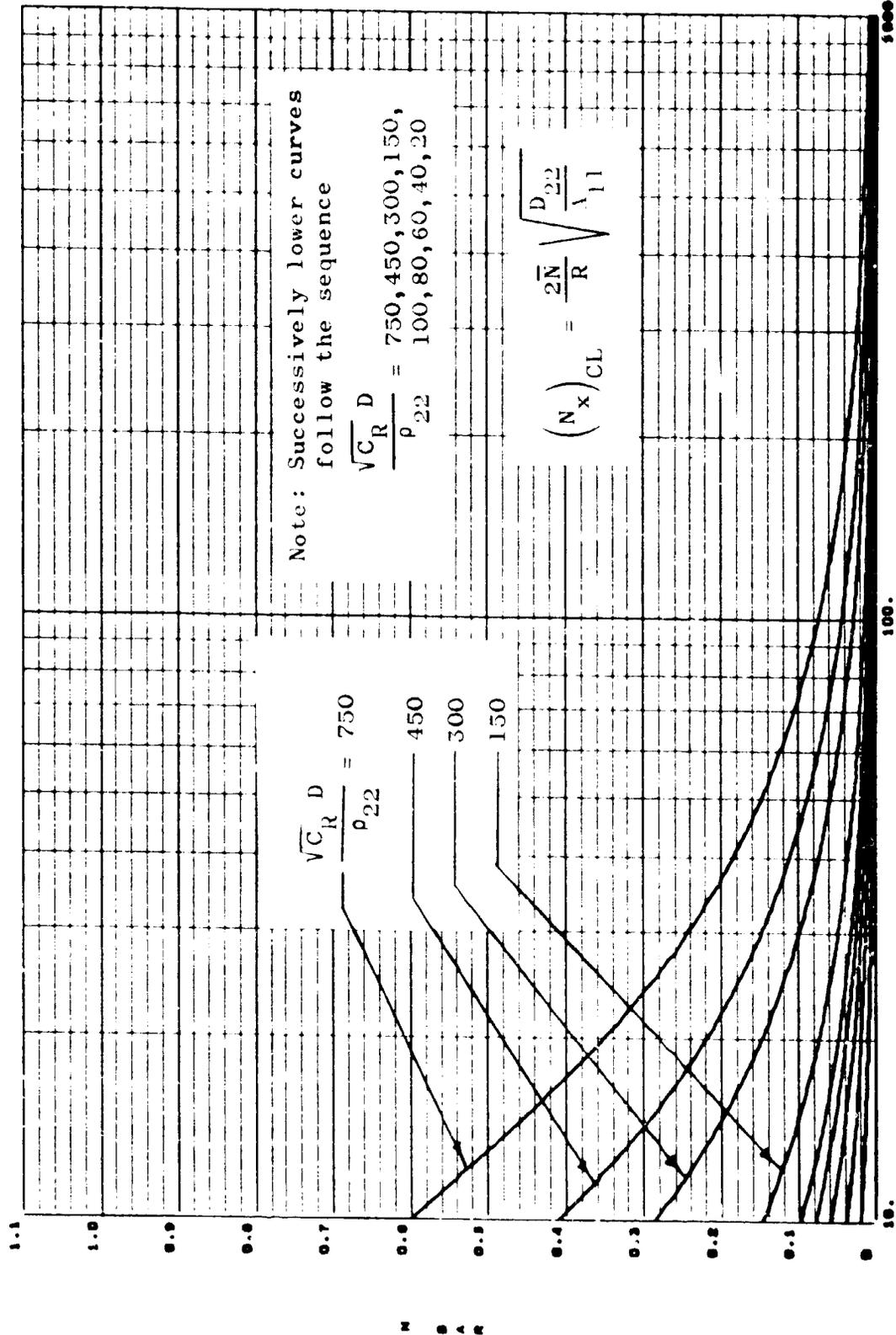


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(b) - (See Table XIX)

ETA SUB 3 = 1.000

RING SPACING/DIAMETER = 0.010



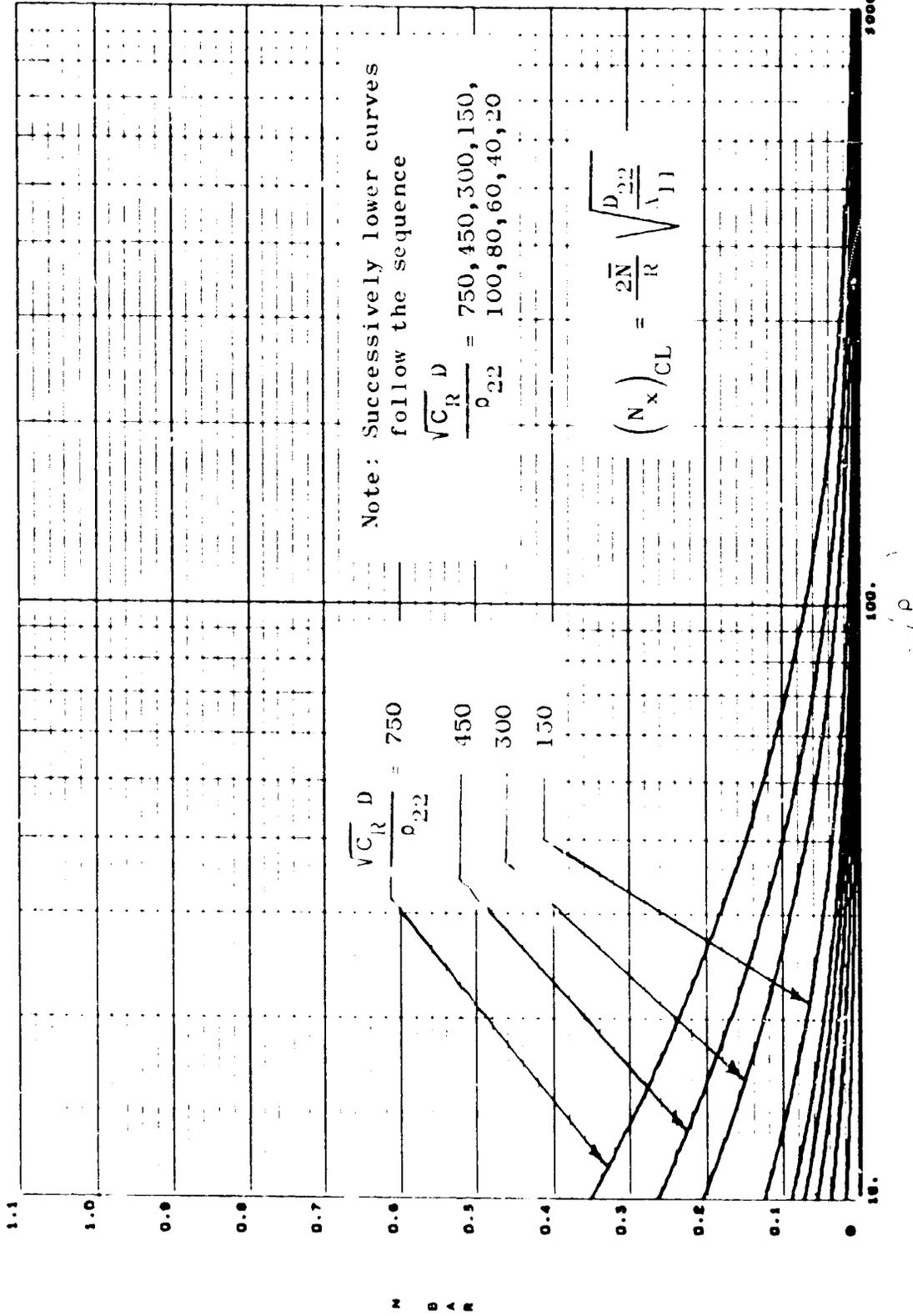
(cont.)

CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS

Figure 36(c) - (see Table IX)

ETA SUB 5 = 5.000

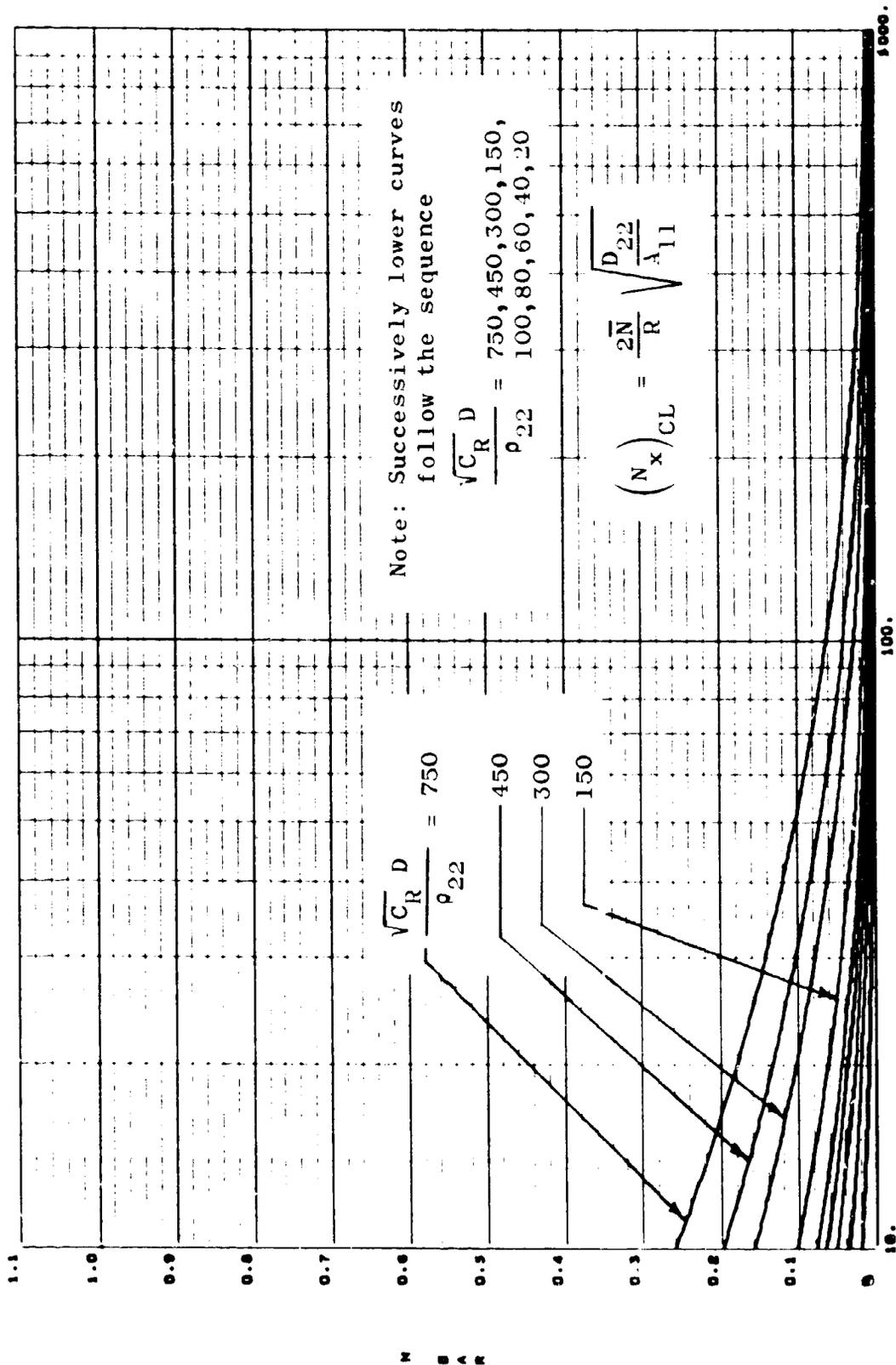
RING SPACING/DIAMETER = 0.010



CRITICAL COMPRESSIVE LOADING COEFFICIENT FOR THE GENERAL INSTABILITY OF STIFFENED CIRCULAR CYLINDERS  
 Figure 36(d) - (see Table XIX)

ETA SUB 8 = 10.000

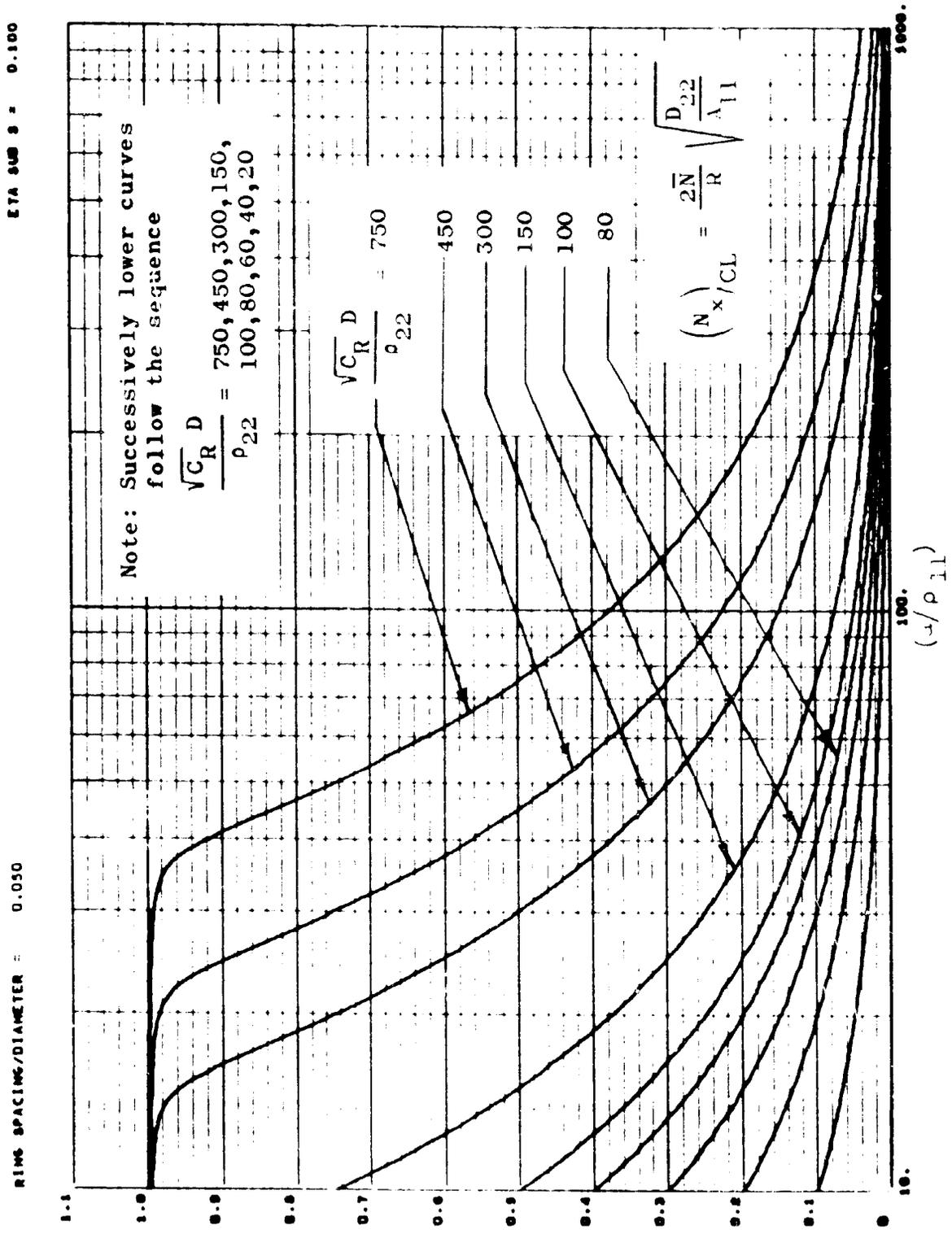
RING SPACING/DIAMETER = 0.010



(1/2 p 11)

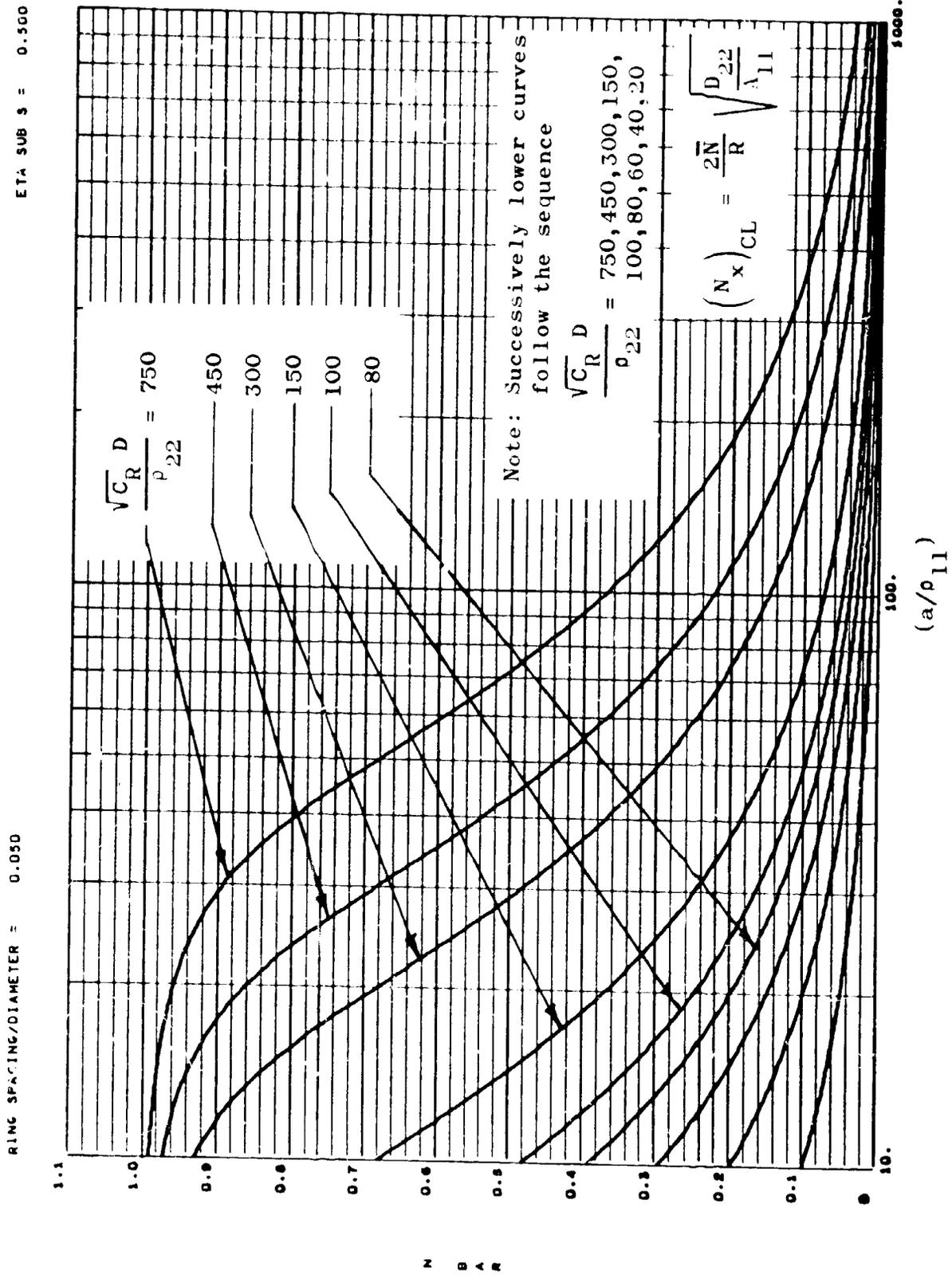
CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(e) - (See Table XIX)

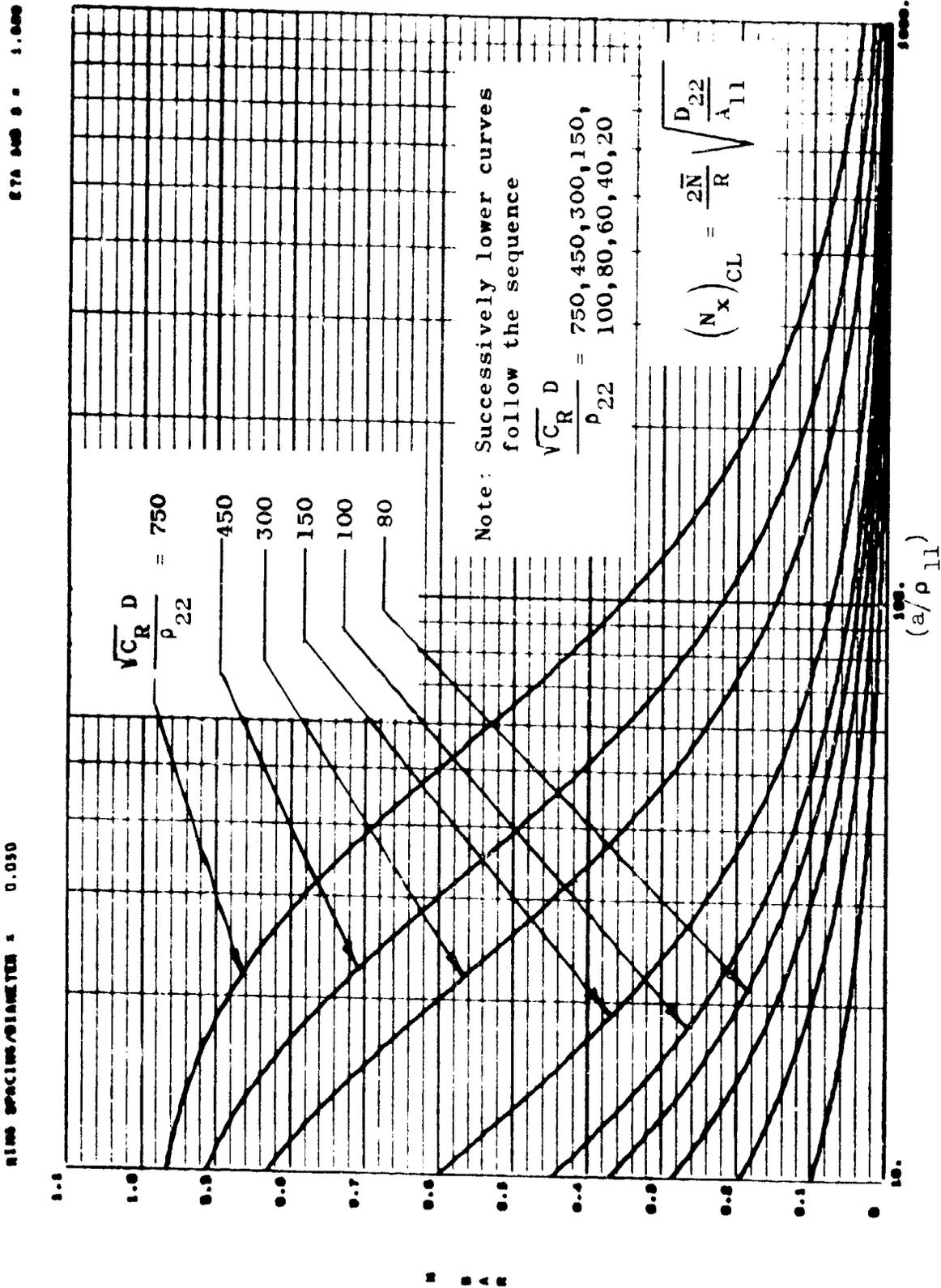


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(f) - (See Table XIX)

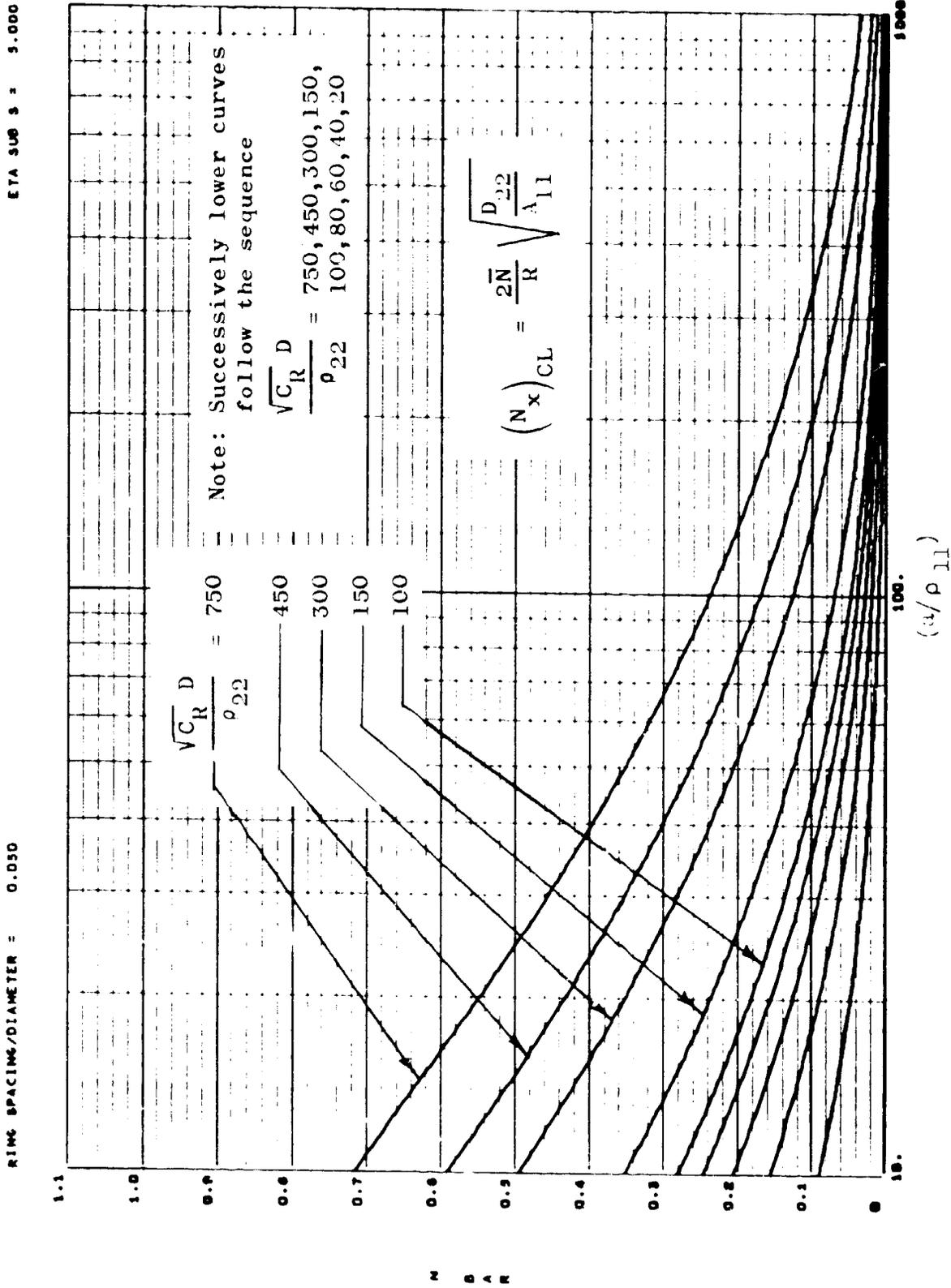


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS  
 Figure 36(g) - (See Table XIX)

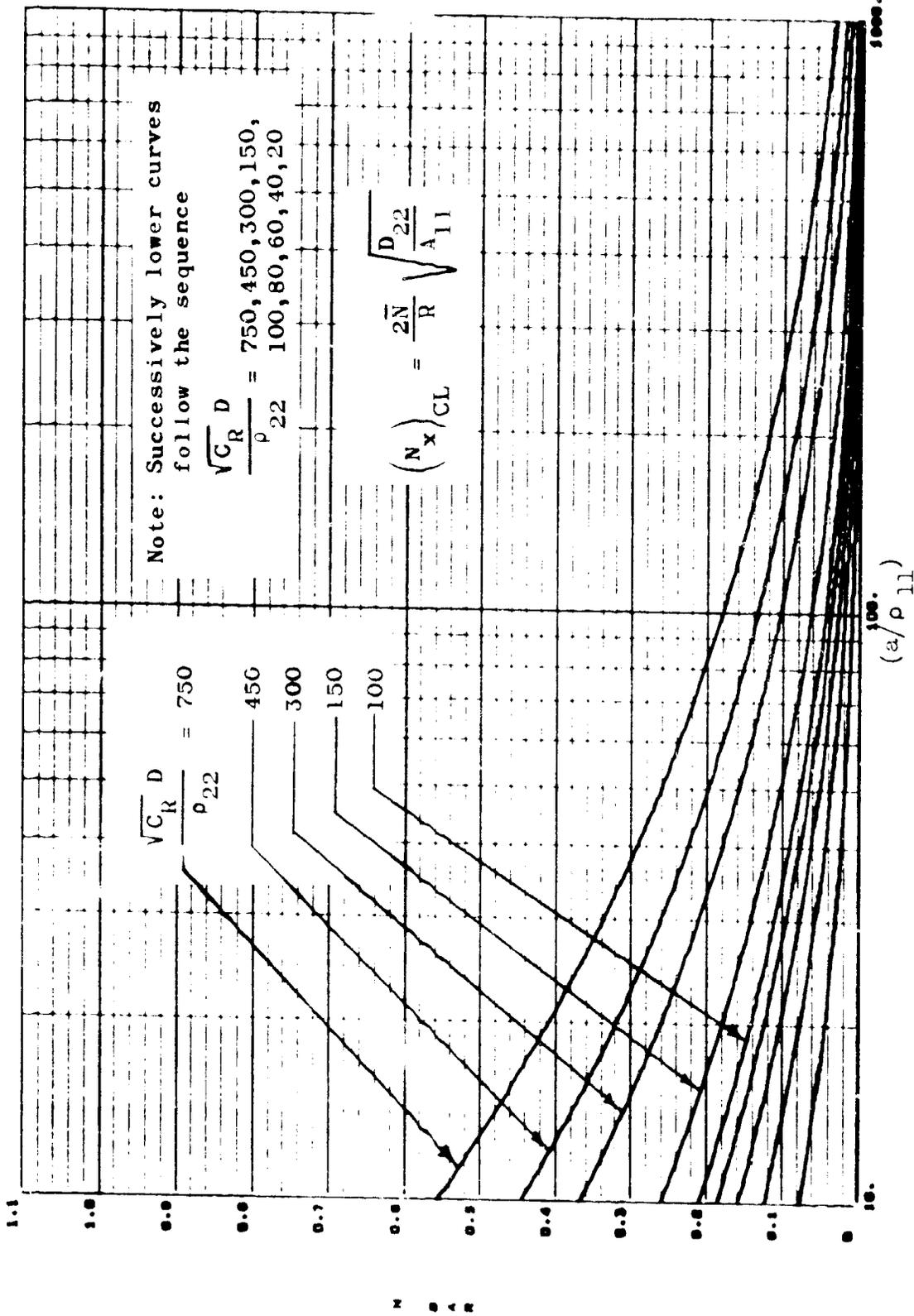


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS

Figure 36(h) - (See Table XIX)



CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS  
 Figure 36(i) - (See Table XIX)

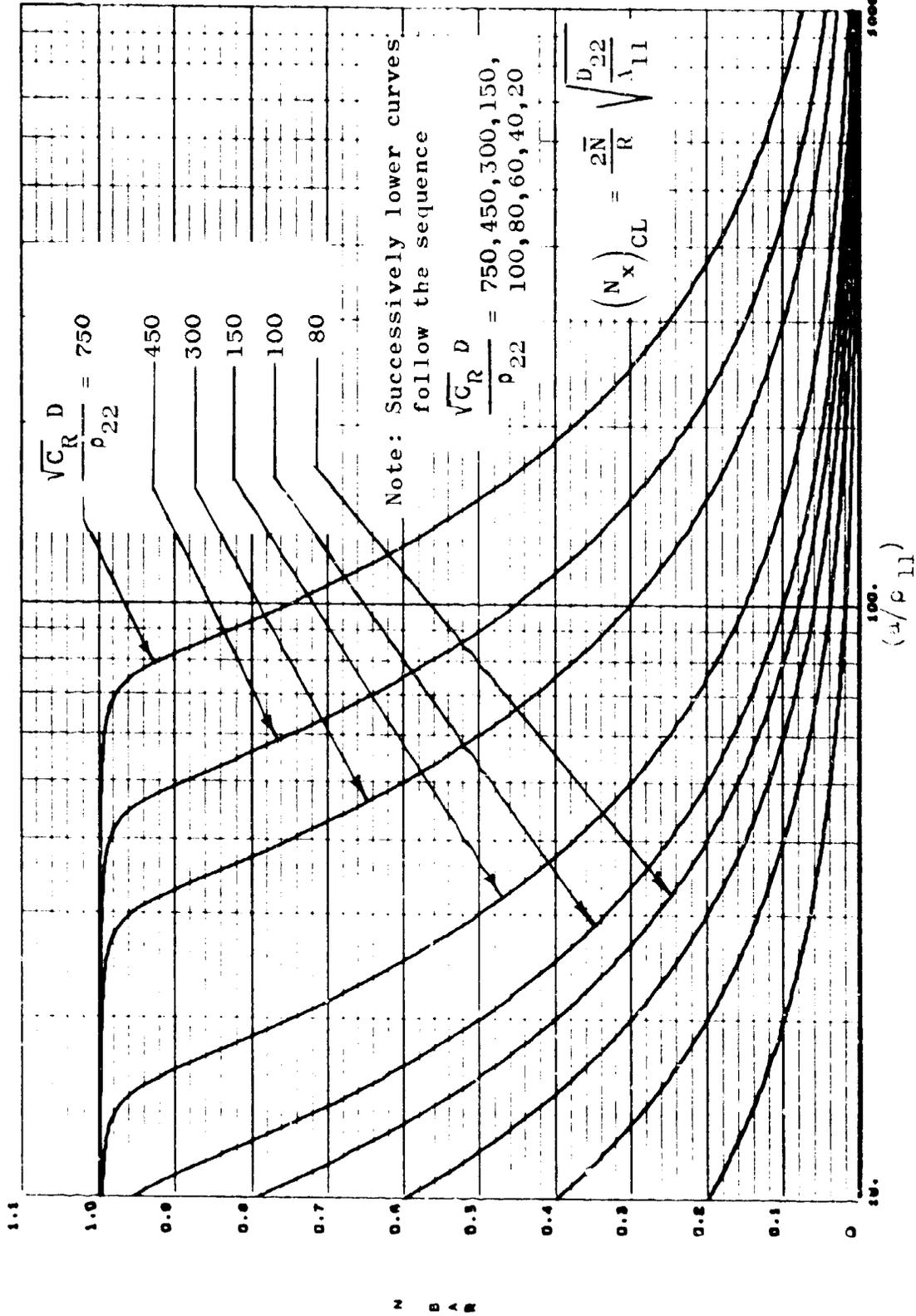


CRITICAL COMPRESSIVE LOADING COEFFICIENT FOR THE GENERAL INSTABILITY OF STIFFENED CIRCULAR CYLINDERS

Figure 36(j) - (See Table XIX)

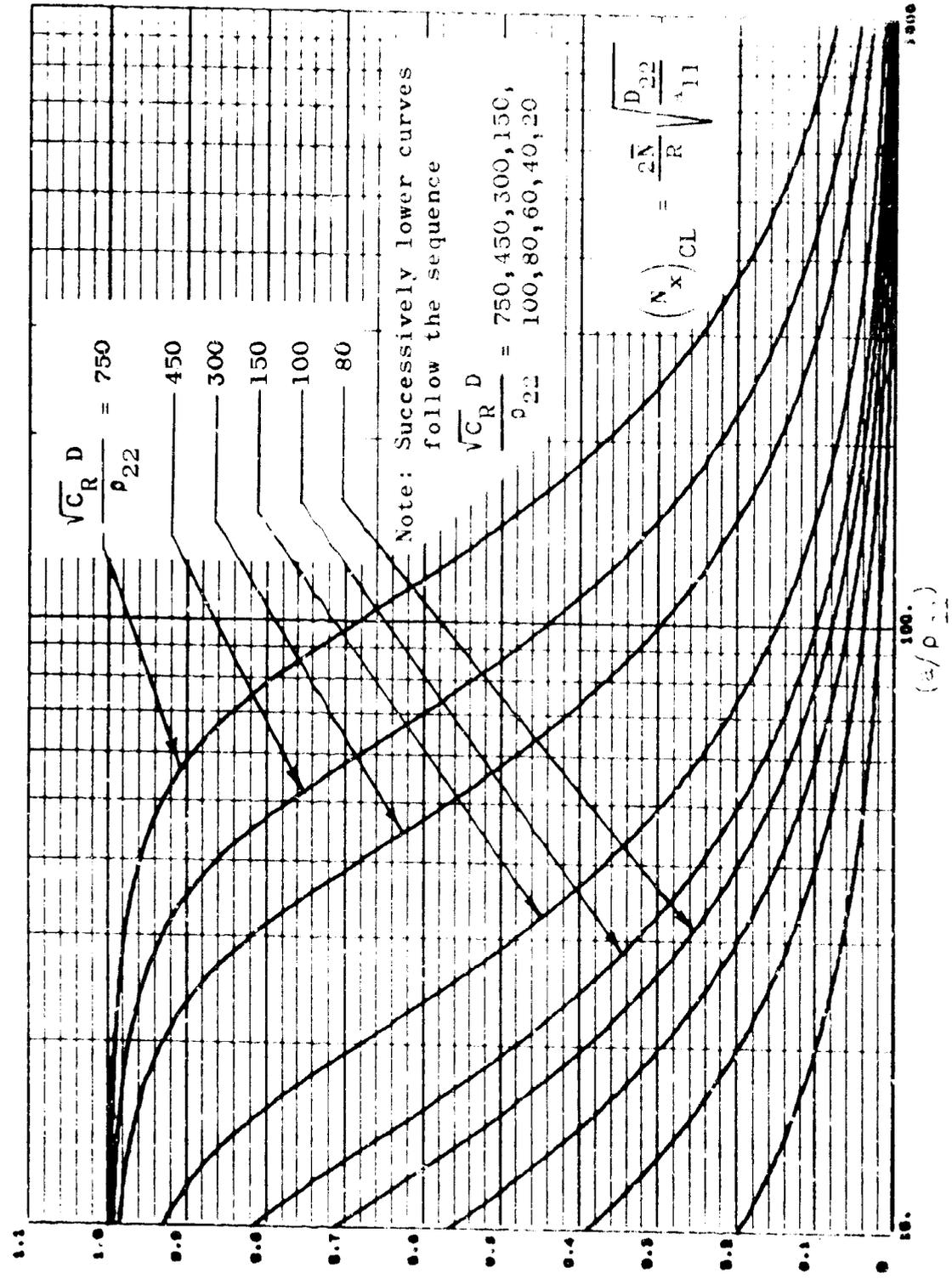
ETA SUB 3 = 0.100

RING SPACING/DIAMETER = 0.100

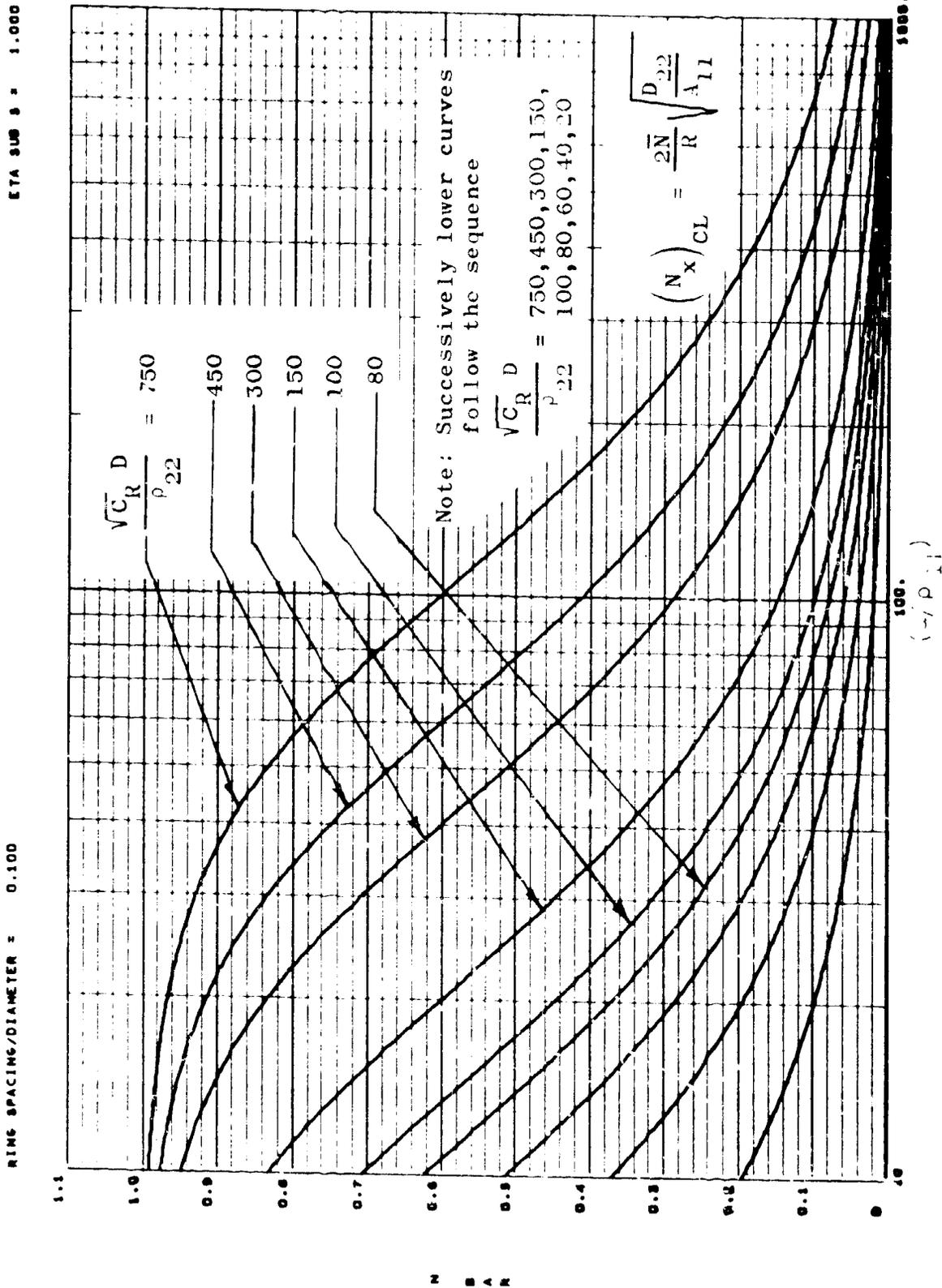


CRITICAL COMPRESSIVE LOADING COEFFICIENT FOR THE GENERAL INSTABILITY OF STIFFENED CIRCULAR CYLINDERS

Figure 36(k) - (See Table XIX)

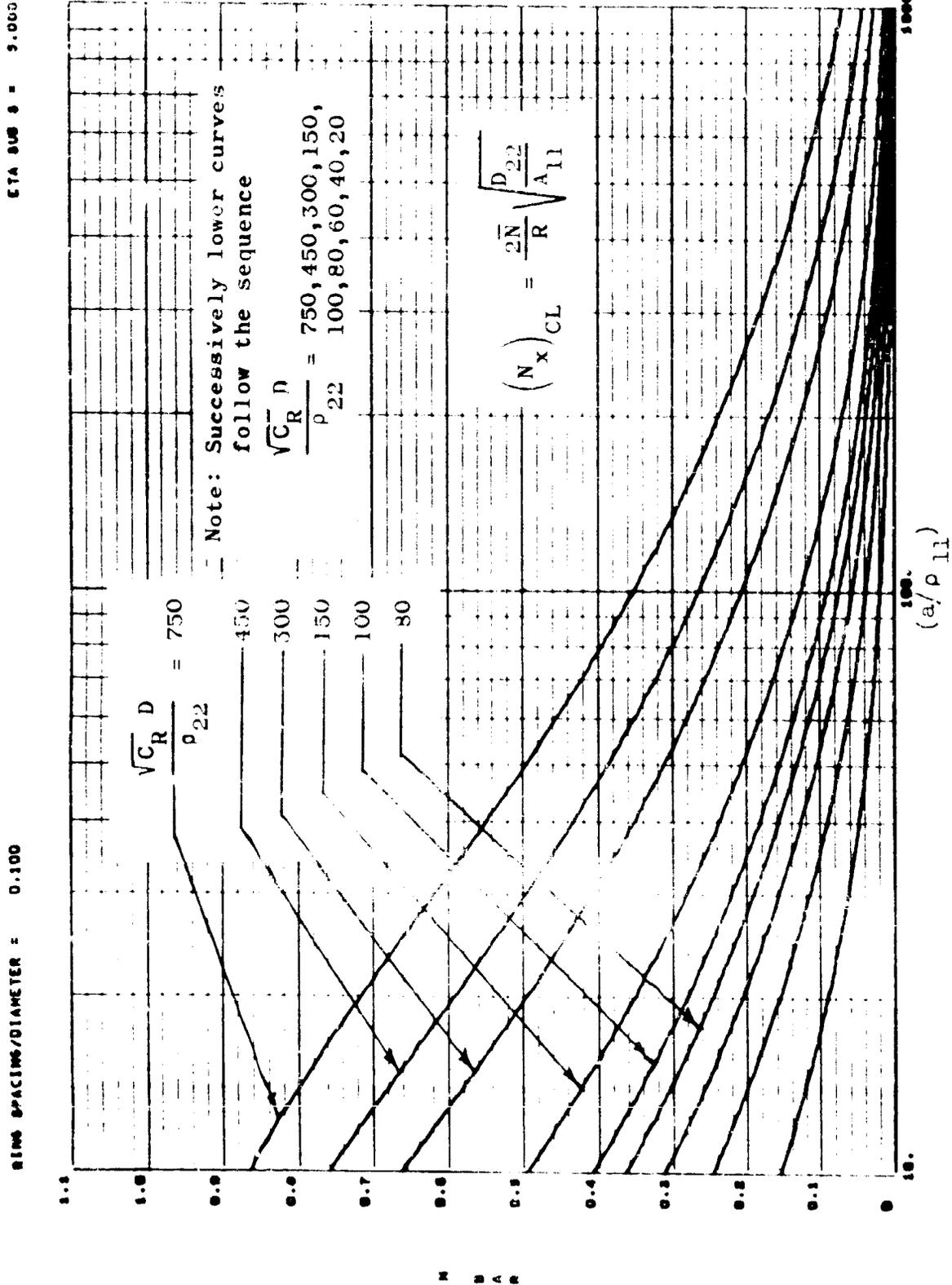


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS  
 (See Table XIX)

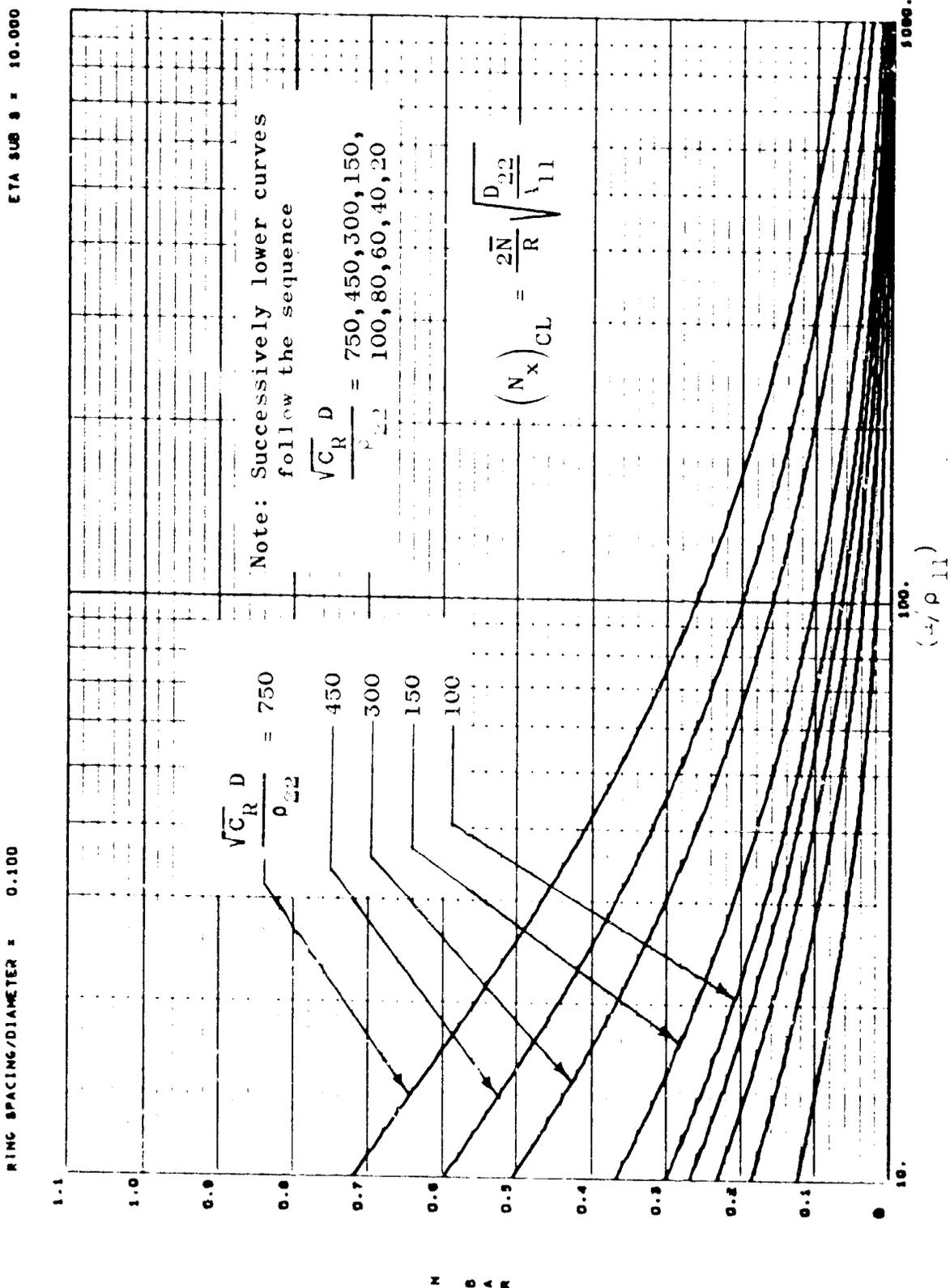


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS

Figure 35(m) - (See Table XIX)

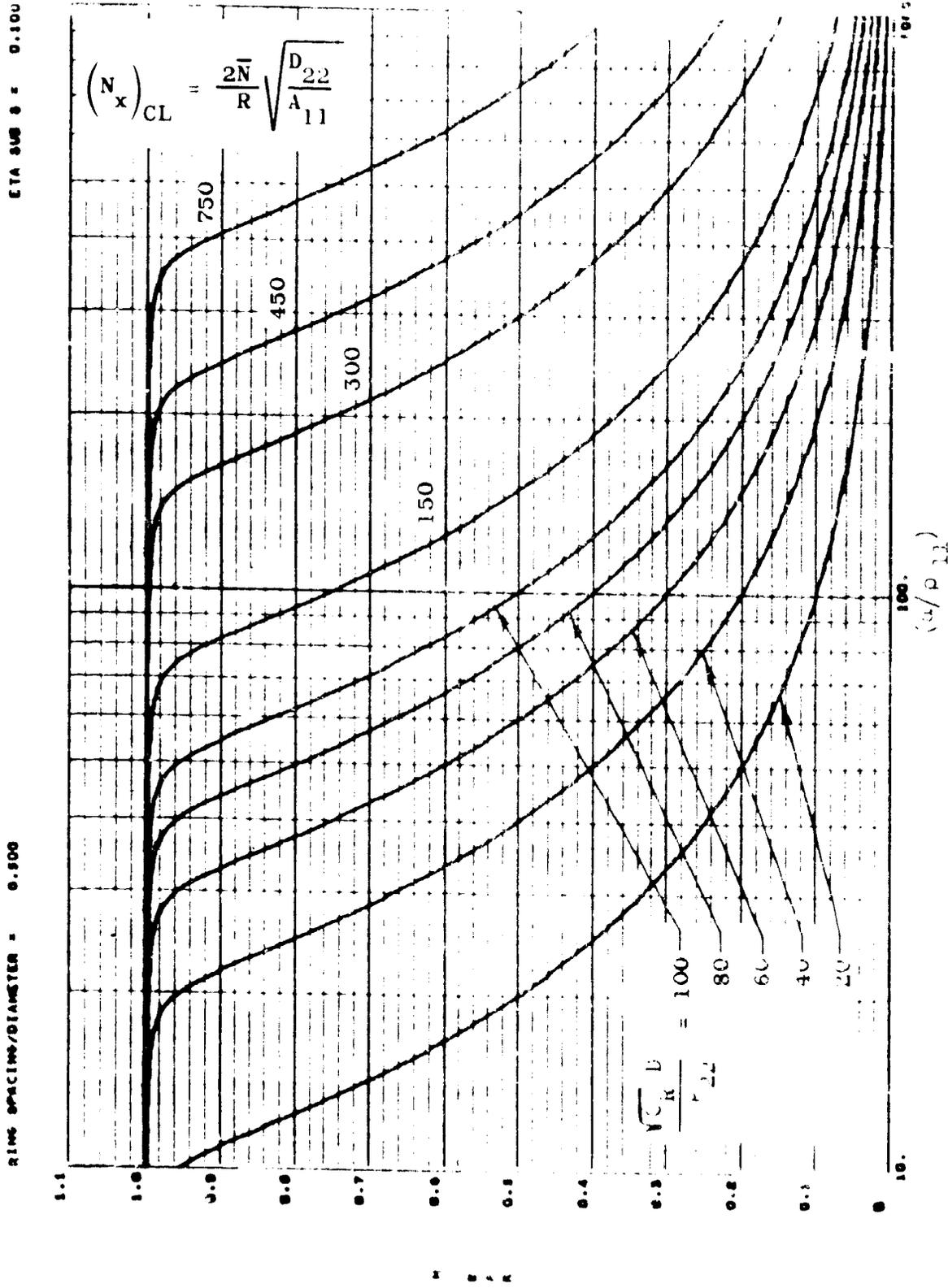


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS  
 Figure 36(n) - (See Table XIX)



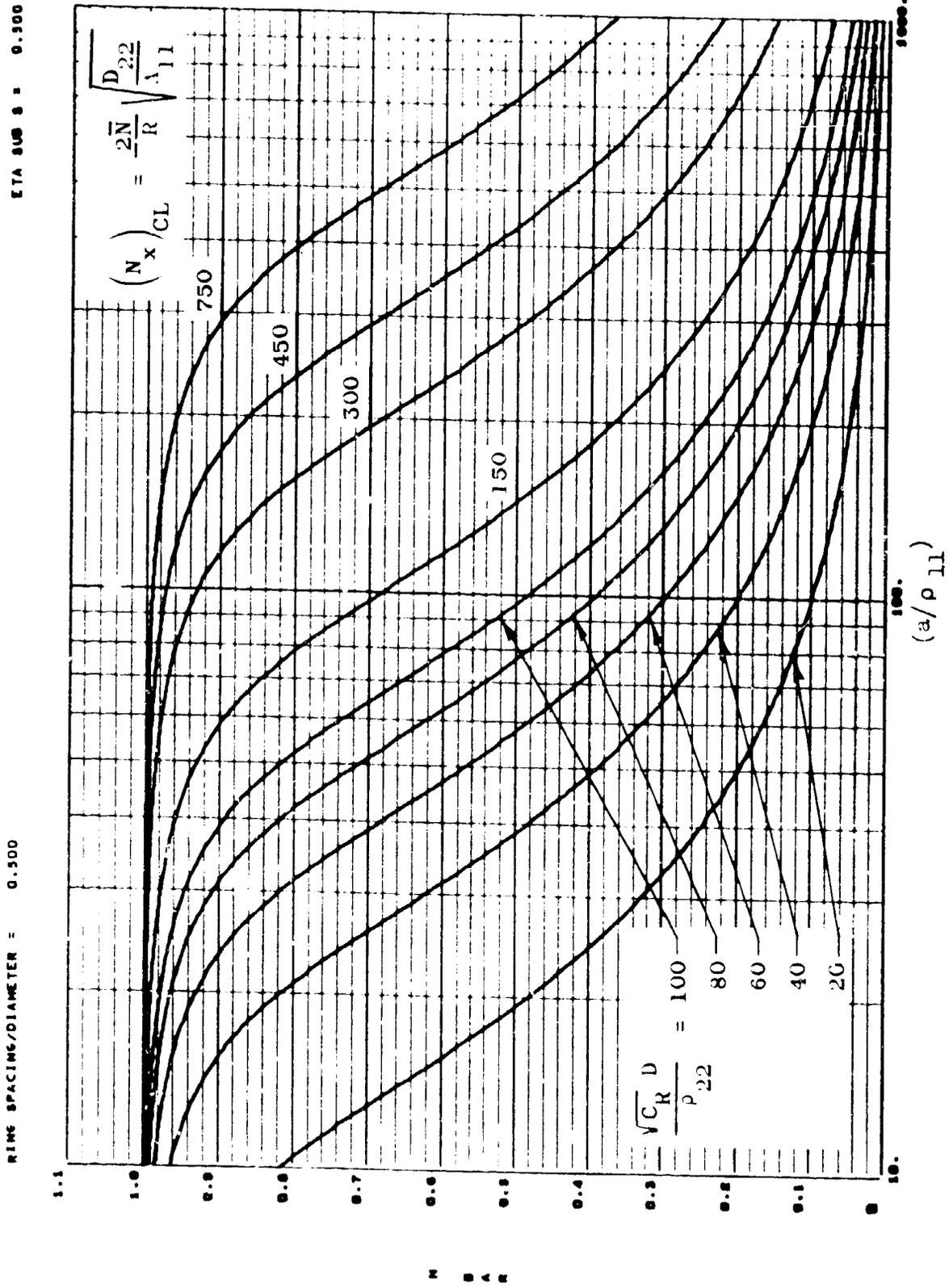
CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(o) - See Table XIX



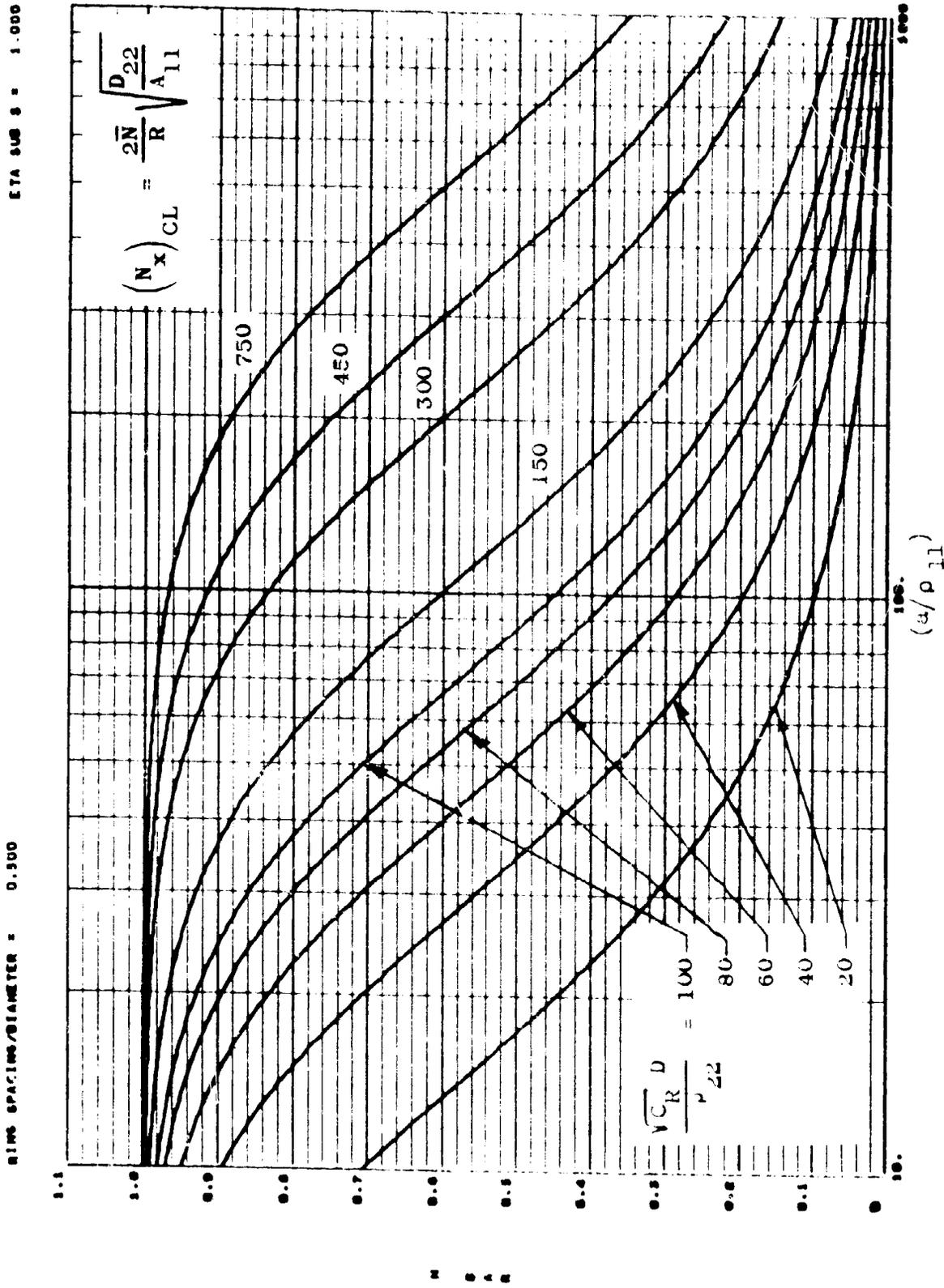
CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 3.12 - (See Table XIX)



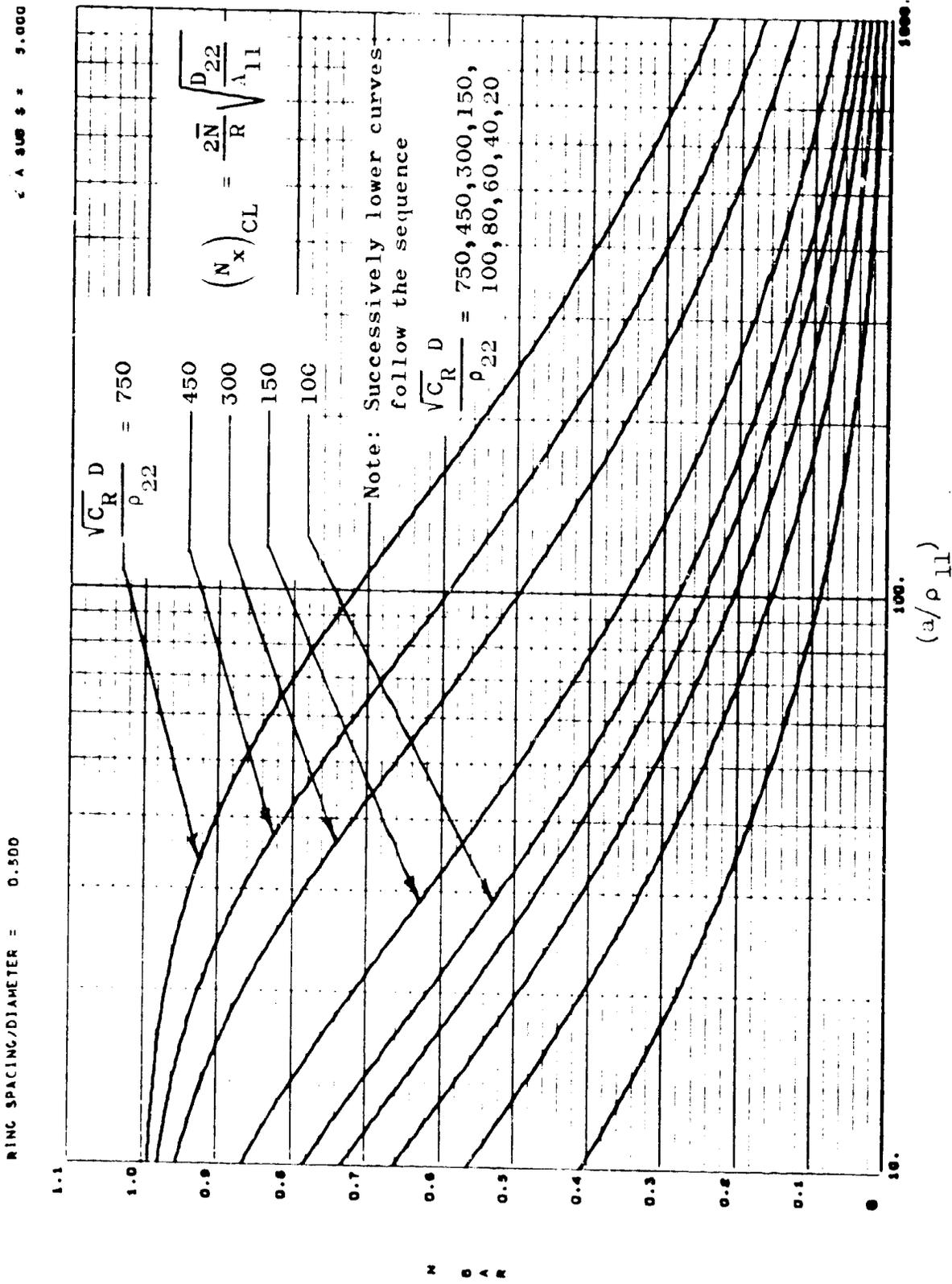
CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(g) - (See Table XIX)



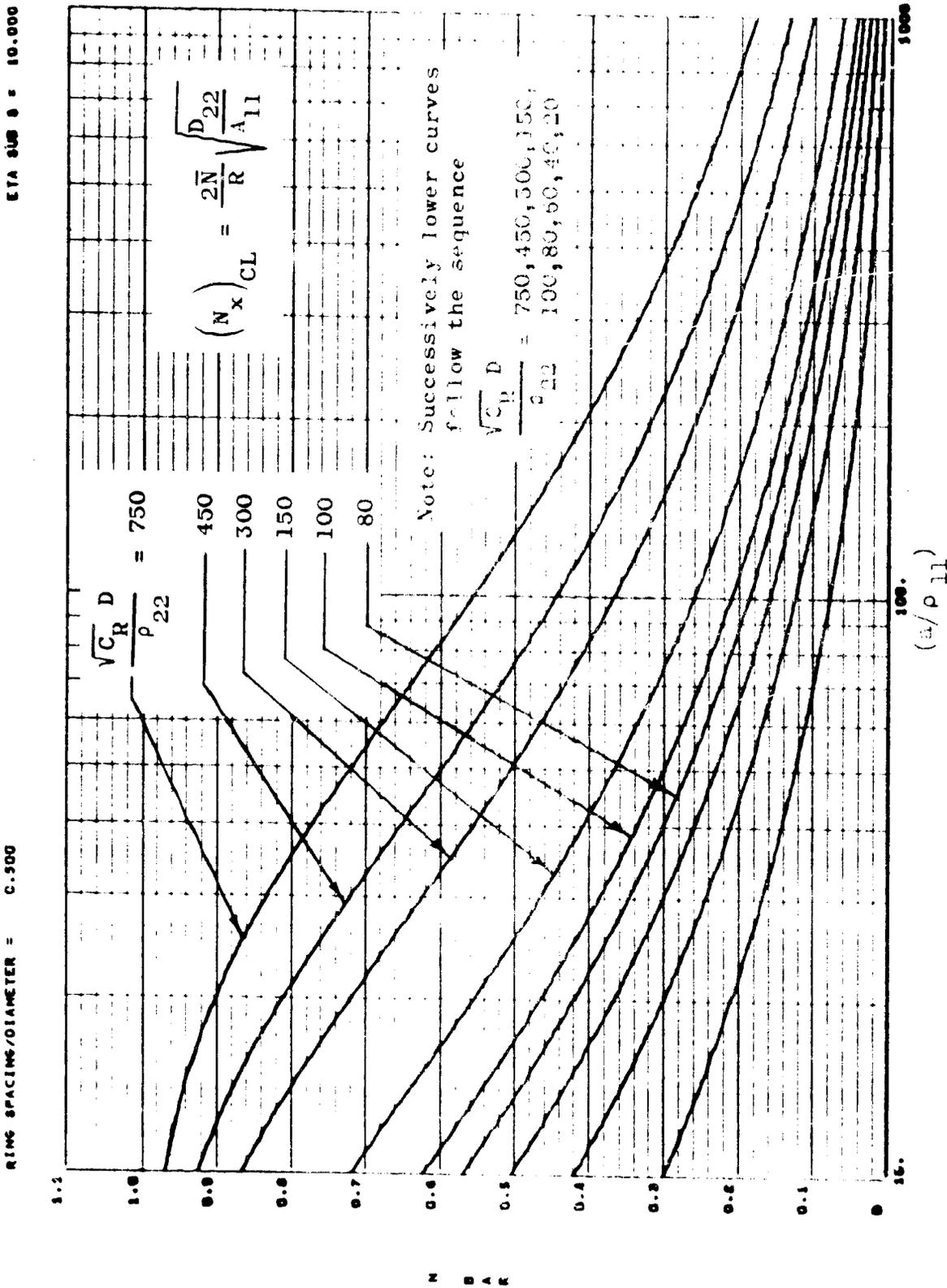
CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(r) - (See Table XIX)



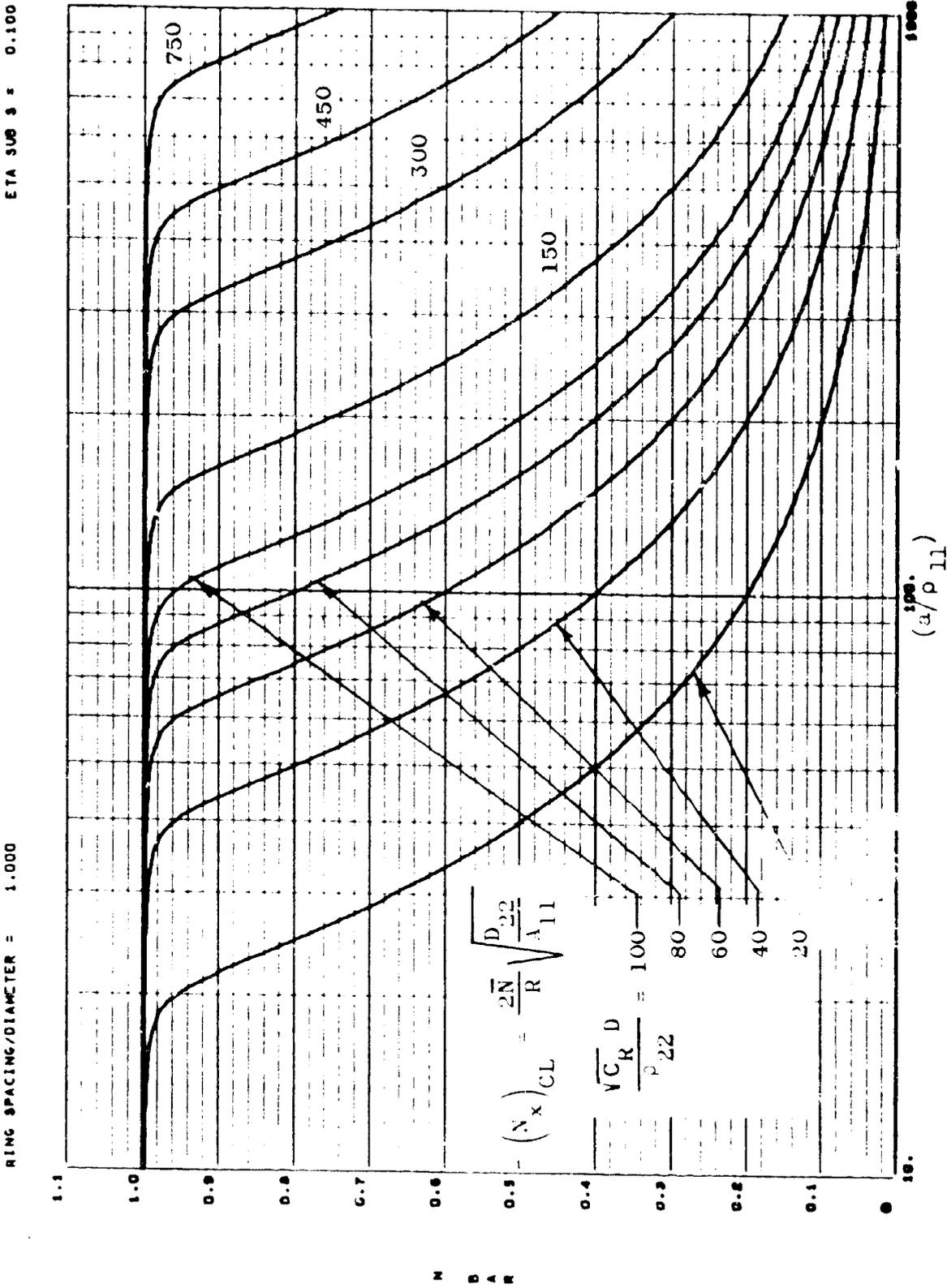
CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS

Figure 36(s) - (See Table XIX)

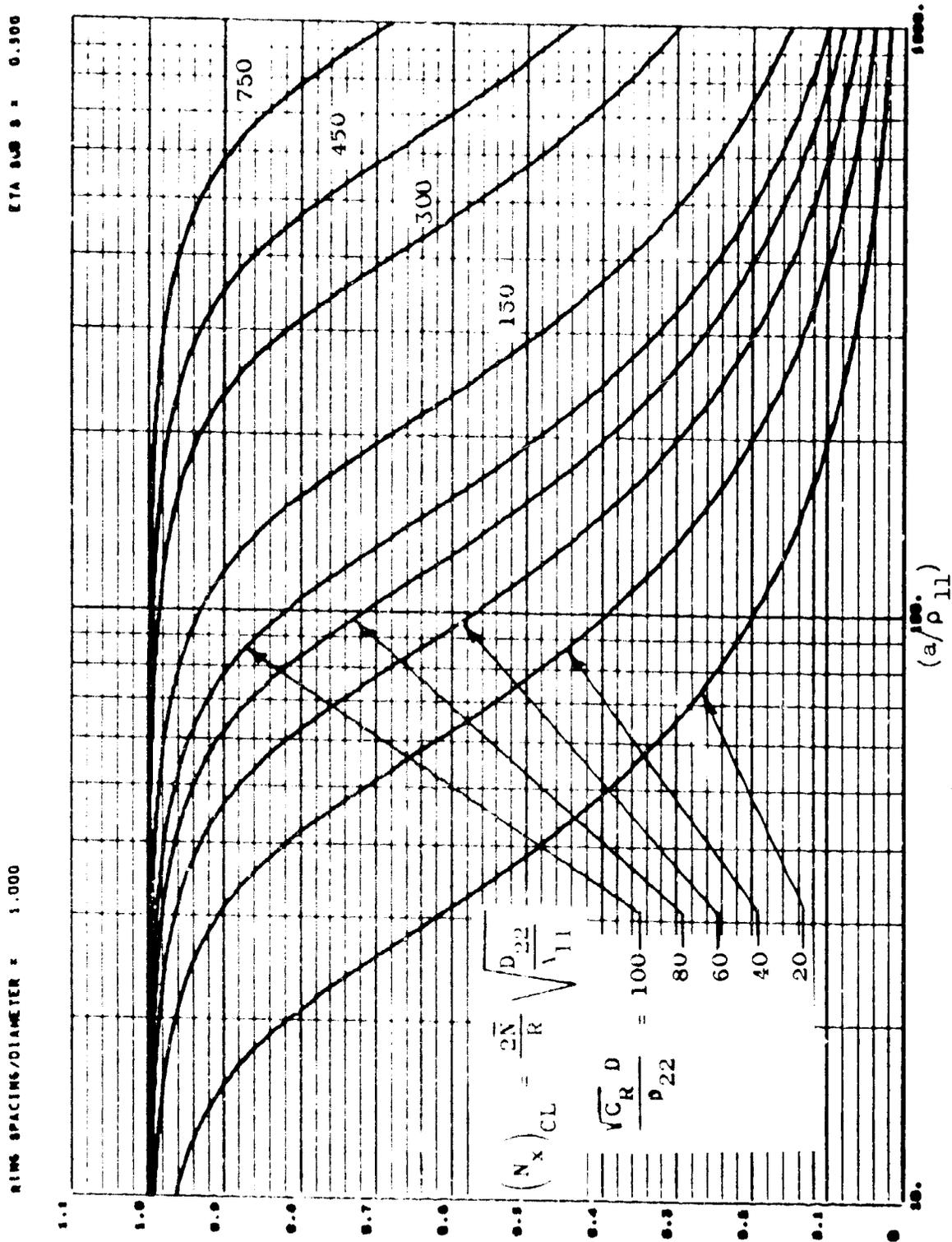


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS

Figure 36(t) - (See Table XIX)

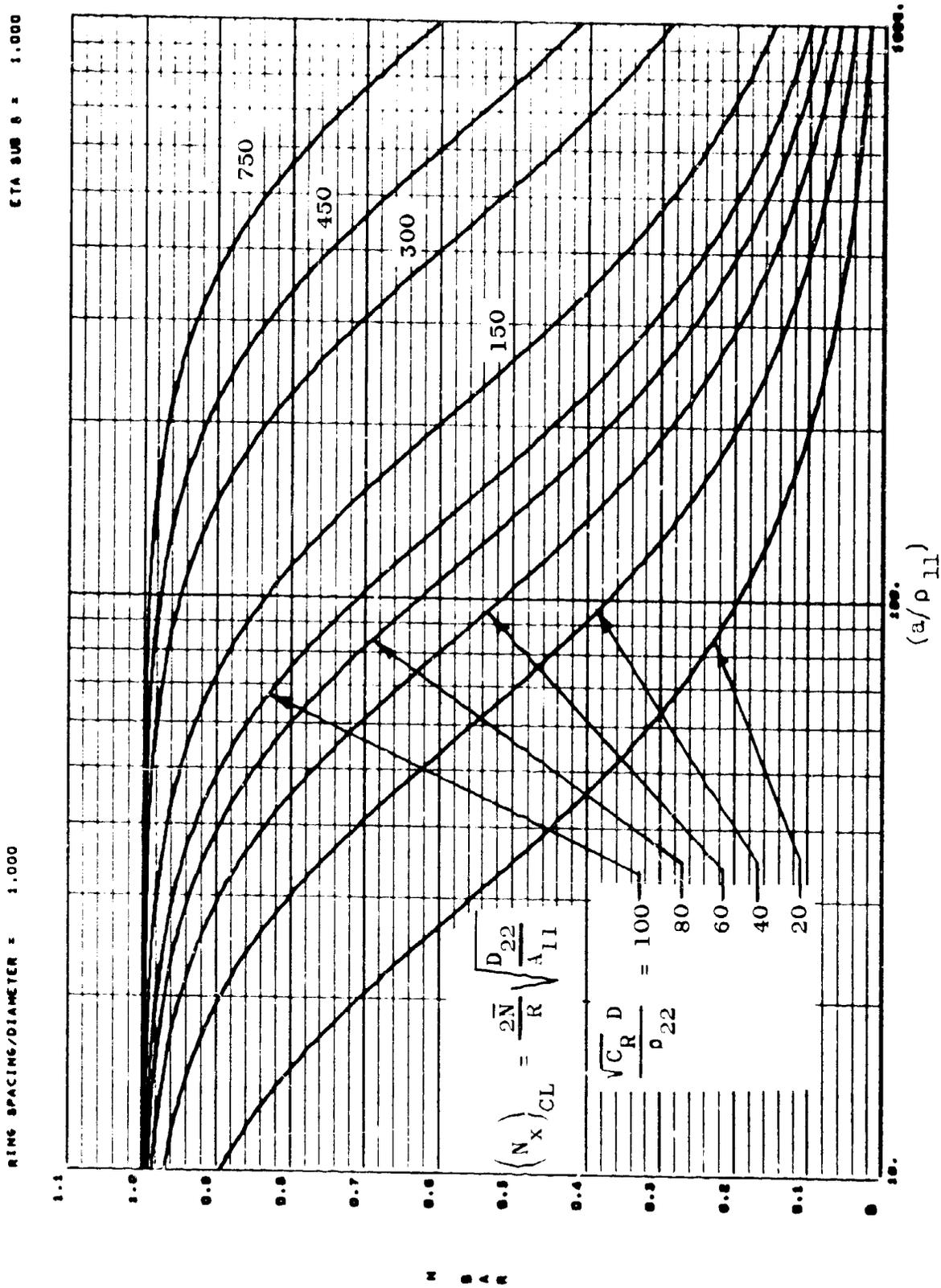


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
THIN-WALLED CIRCULAR CYLINDERS  
(See Table XIX)



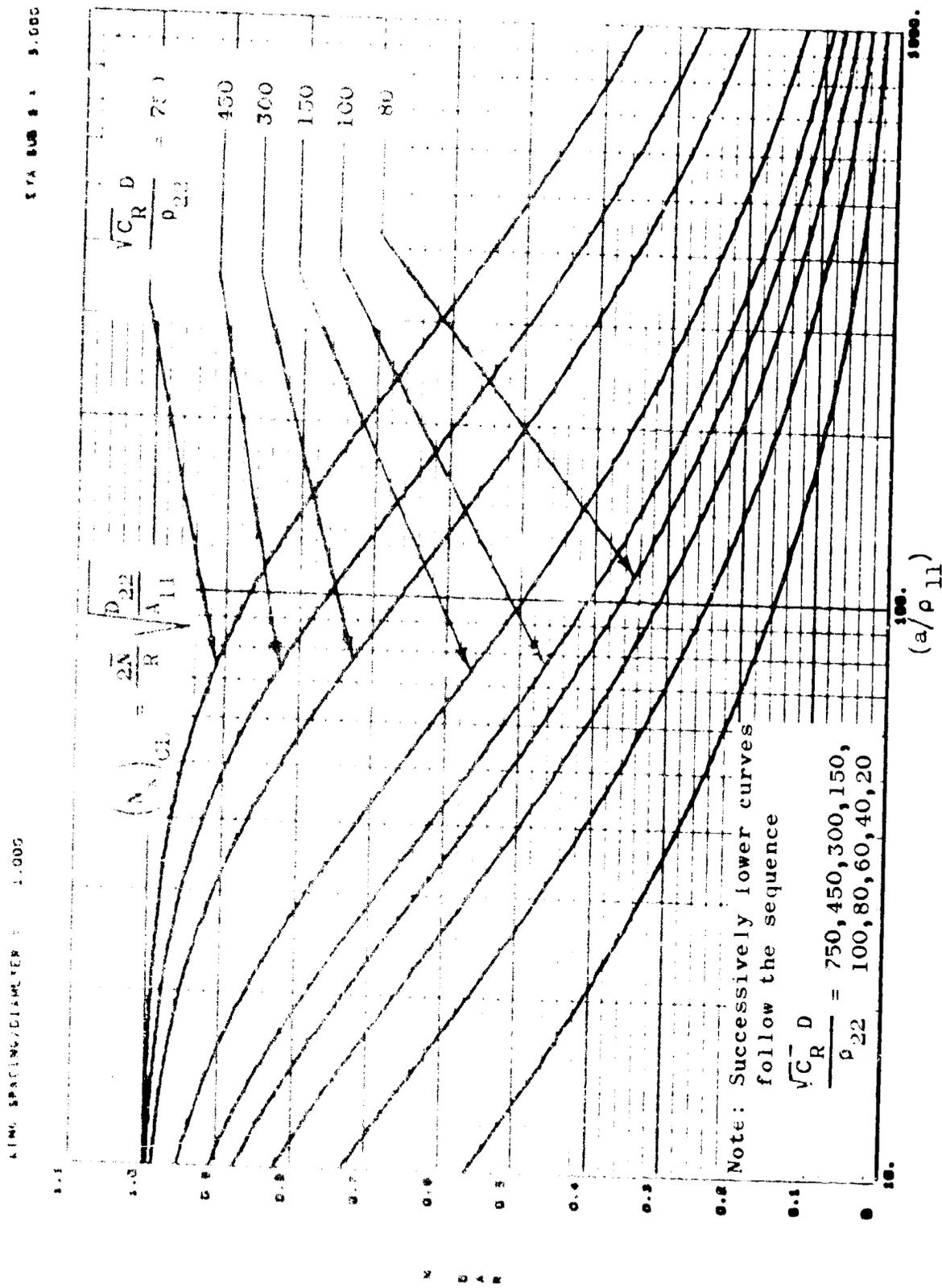
CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(v) - (See Table XIX)

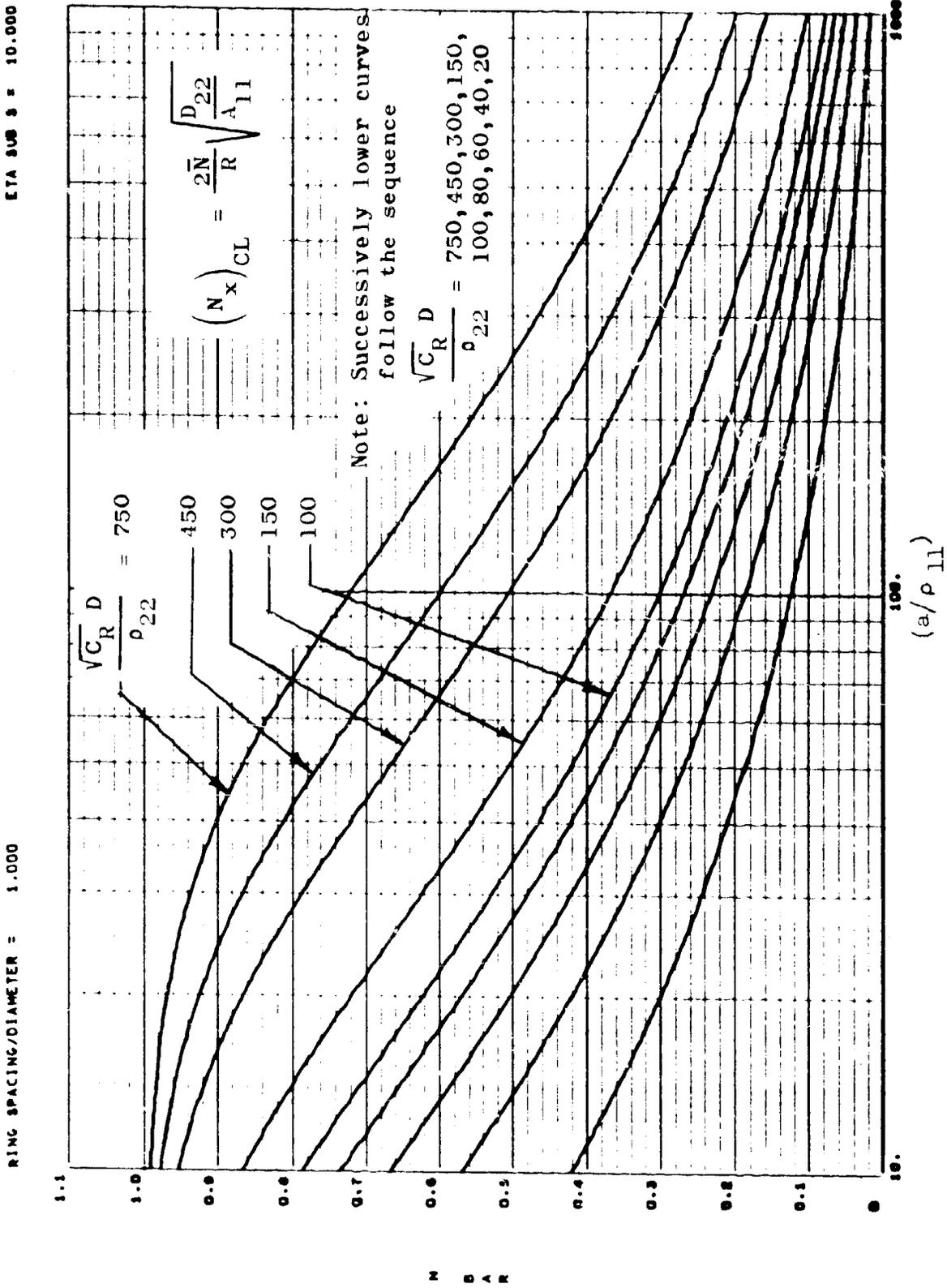


CRITICAL COMPRESSIVE LOADING COEFFICIENT  
FOR THE GENERAL INSTABILITY OF  
STIFFENED CIRCULAR CYLINDERS

Figure 36(w) - (See Table XIX)



**CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS**  
 Figure 36(x) - (See Table XIX)



CRITICAL COMPRESSIVE LOADING COEFFICIENT  
 FOR THE GENERAL INSTABILITY OF  
 STIFFENED CIRCULAR CYLINDERS  
 Figure 35(y) - (See Table XIX)

14.0 INTERACTION BEHAVIOR14.1 Procedures14.1.1 Axial Compression Plus Pure Bending

Step 1 - Using the procedures of Sections 12 and/or 13, find the critical longitudinal compressive running loads for each of the following cases:

- (a) Pure axial load acting along.
- (b) Pure bending moment acting alone.

Each of these load values may be identified by the symbol  $\left[ (N_x)_{cr} \right]_0$ . For the pure bending case, the classical ( $\Gamma = 1$ ) critical value for the peak running load is taken to be the same as that for pure axial loading. The respective  $\left[ (N_x)_{cr} \right]_0$  values will differ only because of differences in the actual  $\Gamma$ 's for the two cases.

Step 2 - Compute the ratio  $R_1$  as follows:

$$R_1 = \frac{N_x}{\left[ (N_x)_{cr} \right]_0} \quad (14-1)$$

where

$N_x$  = Applied uniformly distributed longitudinal compressive running load (lbs/in).

$\left[ \begin{matrix} (N_x) \\ \text{cr} \end{matrix} \right]_0$  = Critical value for uniformly distributed longitudinal compressive running load acting alone (lbs/in).

Step 3 - Compute the ratio  $R_2$  as follows:

$$R_2 = \frac{N_x}{\left[ \begin{matrix} (N_x) \\ \text{cr} \end{matrix} \right]_0} \quad (14-2)$$

where

$N_x$  = Peak longitudinal compressive running load (lbs/in) due to applied bending moment.

$\left[ \begin{matrix} (N_x) \\ \text{cr} \end{matrix} \right]_0$  = Critical value for peak longitudinal compressive running load (lbs/in) due to bending moment acting alone.

Step 4 - In Figure 38, locate the point B whose coordinates  $(R_1, R_2)$  come from Steps 2 and 3. Buckling can occur if this point lies on or above the straight-line interaction curve. The margin of safety may be computed as shown in Figure 38.

## 14.1.2

Axial Compression Plus External Pressure

THE PROCEDURES CITED HERE DO NOT ACCOUNT FOR DISCONTINUITY-TYPE DEFORMATIONS CREATED BY THE PRESSURE. SUCH DEFORMATIONS CAN BE VERY IMPORTANT TO ALL CONFIGURATIONS EXCEPT WHERE STIFFENER SPACINGS ARE VERY SMALL.

Step 1 - Using the procedures of Sections 12 and/or 13, find the critical longitudinal compressive running load  $\left[ (N_x)_{cr} \right]_0$  (lbs/in), for the case of pure axial load acting alone.

Step 2 - Using any suitable method such as the digital computer program of Section 18.3.2 [together with an appropriate correlation (knock-down) criterion], find the critical circumferential compressive running load  $\left[ (N_y)_{cr} \right]_0$  (lbs/in), for the case of external pressure acting alone.

Step 3 - Compute the ratio  $R_1$  as follows:

$$R_1 = \frac{N_x}{\left[ (N_x)_{cr} \right]_0} \quad (14-3)$$

where

$N_x$  = Applied uniformly distributed longitudinal compressive running load (lbs/in). If applicable, include axial loading due to external pressure.

Step 4 - Compute the ratio  $R_2$  as follows:

$$R_2 = \frac{N_y}{\left[ (N_y)_{cr} \right]_0} \quad (14-4)$$

where

$N_y$  = Circumferential compressive running load (lbs/in) due to applied external pressure.

Step 5 - In Figure 38, locate the point B whose coordinates  $(R_1, R_2)$  come from Steps 3 and 4. Buckling can occur if this point lies on or above the straight-line interaction curve. The margin of safety may be computed as shown in Figure 38.

NOTE: The straight-line interaction curve is suggested here in the interest of simplicity. More accurate analysis can be made (still neglecting discontinuity-type deformations) by using the digital computer program of Section 18.3.2. This program may be used to find either

(a):  $(N_x)_{cr}$  for given  $N_y$

or

(b):  $(N_y)_{cr}$  for given  $N_x$

Based on the limited interaction studies conducted under NASA Contract NAS8-11181, it appears that the digital computer program

results will indicate greater load-carrying capability than will Figure 38 (refer to Section 8.3, Part I).

14.1.3 Axial Compression Plus Internal Pressure

No interaction curve is presented for this case. In this report, the digital computer program of Section 18.3.2 is the only means provided for analysis of the axial compression - internal pressure loading combination. Note that THIS PROGRAM DOES NOT ACCOUNT FOR THE DISCONTINUITY-TYPE DEFORMATIONS CREATED BY THE PRESSURE. Within this framework, the program may be used to find either

(a):  $(N_x)_{cr}$  for given  $N_y$

or

(b):  $N_y$  required to obtain given  $(N_x)_{cr}$ .

14.1.4 Axial Compression Plus Shear

Step 1 - Using the procedures of Sections 12 and/or 13, find the critical longitudinal compressive running load  $\left[ (N_x)_{cr} \right]$  (lbs/in), for the case of pure axial load acting alone.

Step 2 - Find the critical in-plane running shear load  $\left[ (N_{xy})_{cr} \right]$  (lbs/in), for the case of shear load acting alone. (See Figure 37). No means are provided here for determination of this value.

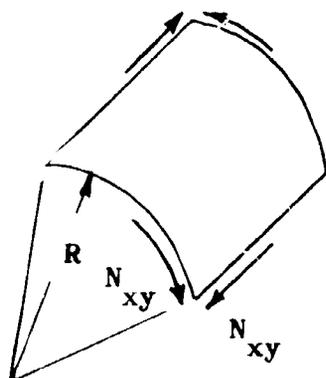


Figure 37 - In-plane Running Shear Load  $N_{xy}$

Step 3 - Compute the ratio  $R_1$  as follows:

$$R_1 = \frac{N_x}{\left[ \begin{matrix} (N_x) \\ x \\ cr \end{matrix} \right]_0} \quad (14-5)$$

where

$N_x$  = Applied uniformly distributed longitudinal compressive running load (lbs/in).

Step 4 - Compute the ratio  $R_2$  as follows:

$$R_2 = \frac{N_{xy}}{\left[ \begin{matrix} (N_{xy}) \\ xy \\ cr \end{matrix} \right]_0} \quad (14-6)$$

where

$N_{xy}$  = In-plane running shear load (lbs/in) due to applied transverse shear.

Step 5 - In Figure 38, locate the point B whose coordinates  $(R_1, R_2)$  come from Steps 3 and 4. Buckling can occur if this point lies on or above the straight-line interaction curve. The margin of safety may be computed as shown in Figure 38.

NOTE: The straight-line interaction curve is suggested here in the interest of simplicity. Based on the limited test data of reference [44], it appears that this is a conservative practice. See Section 8.5 for further clarification.

14.2            Design Curve

The recommended design interaction curve applicable to Sections 14.1.1, 14.1.2, and 14.1.4 is given in Figure 38.

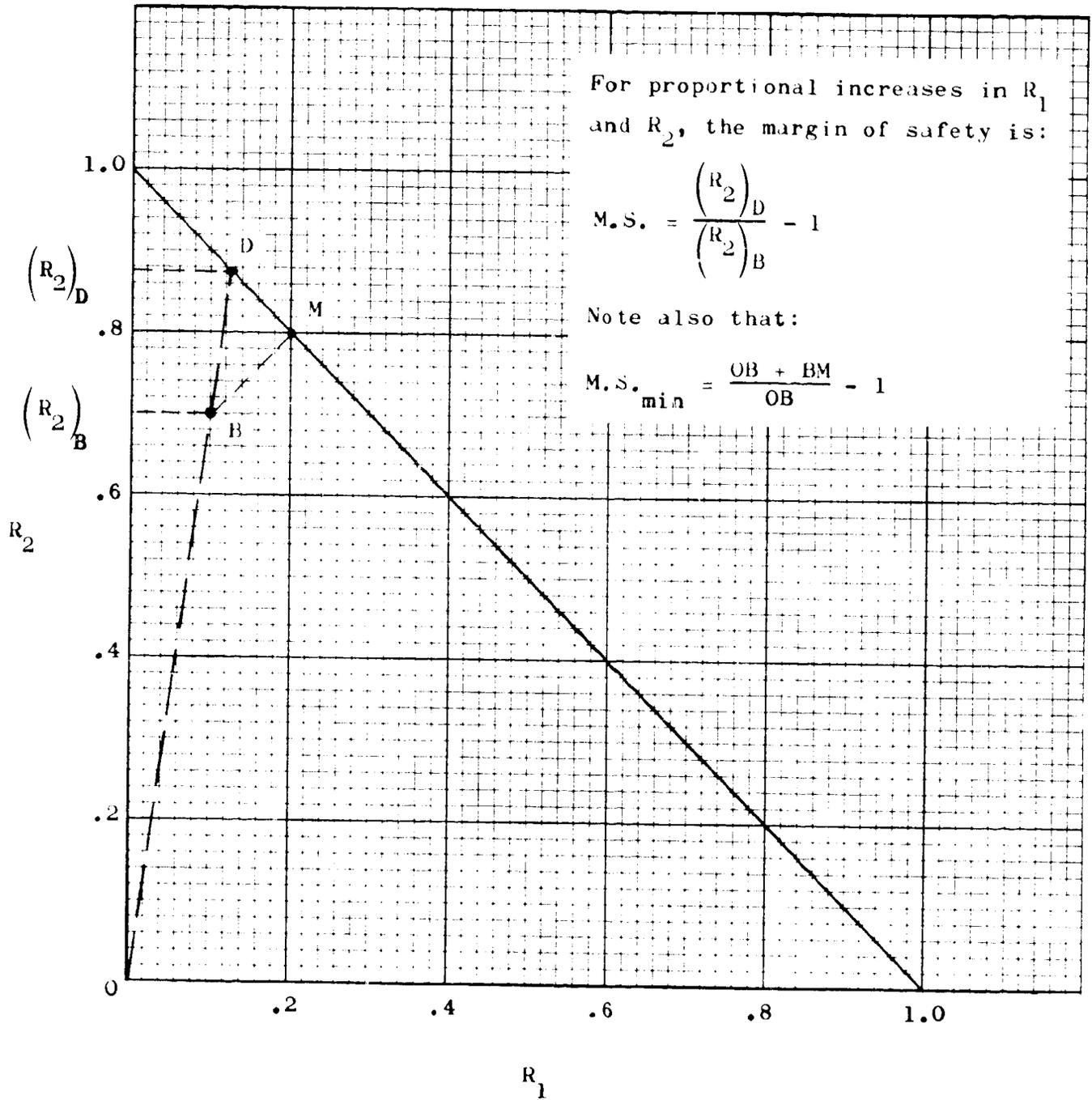


Figure 38 - Design Interaction Curve

GENERAL DYNAMICS  
Convair Division

15.0 INITIAL IMPERFECTIONS

## 15.1

Procedures

Step 1 - For longitudinally stiffened circular cylinders, compute the values  $A_{11}$ ,  $A_{22}$ ,  $D_{11}$ , and  $D_{22}$  using the value  $\nu = 0$  in the formulas of Table XVI. For the analysis of general instability in circular cylinders having both longitudinal and circumferential stiffeners, compute the values  $A_{11}$ ,  $A_{22}$ ,  $D_{11}$ , and  $D_{22}$  using  $\nu = 0$  in the formulas of Table XVIII.

Step 2 - Compute the effective thickness  $t_{\text{eff}}$  as follows:

$$t_{\text{eff}} = \left[ (144)(D_{11}A_{11})(D_{22}A_{22}) \right]^{1/4} \quad (15-1)$$

Step 3 - Compute the ratio  $\frac{R}{t_{\text{eff}}}$  where

$R$  = Radius to middle surface of basic cylindrical skin.

Step 4 - For the case of pure axial load, find the correlation (knock-down) factor  $\Gamma$  from Figure 39.

For the case of pure bending moment, find the correlation (knock-down) factor  $\Gamma$  from Figure 40.

## 15.2

Design Curves

Design curves for the correlation (knock-down) factor  $\Gamma$  are given in Figures 39 and 40 for the cases of pure axial load and pure bending moment, respectively.

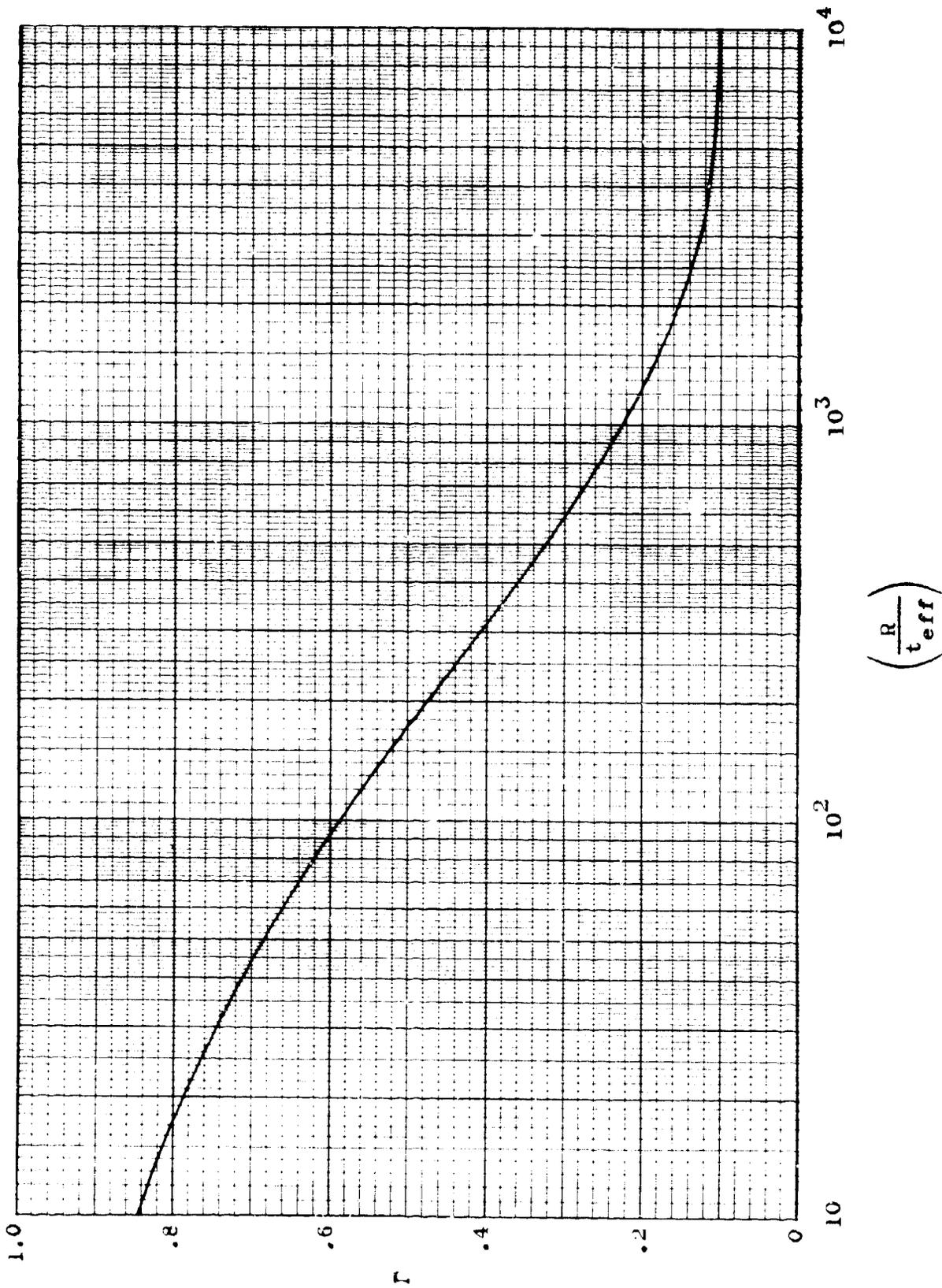


Figure 39 - Design Correlation (Knock-down)  
Factor for Pure Axial Load

GENERAL DYNAMICS  
Convair Division

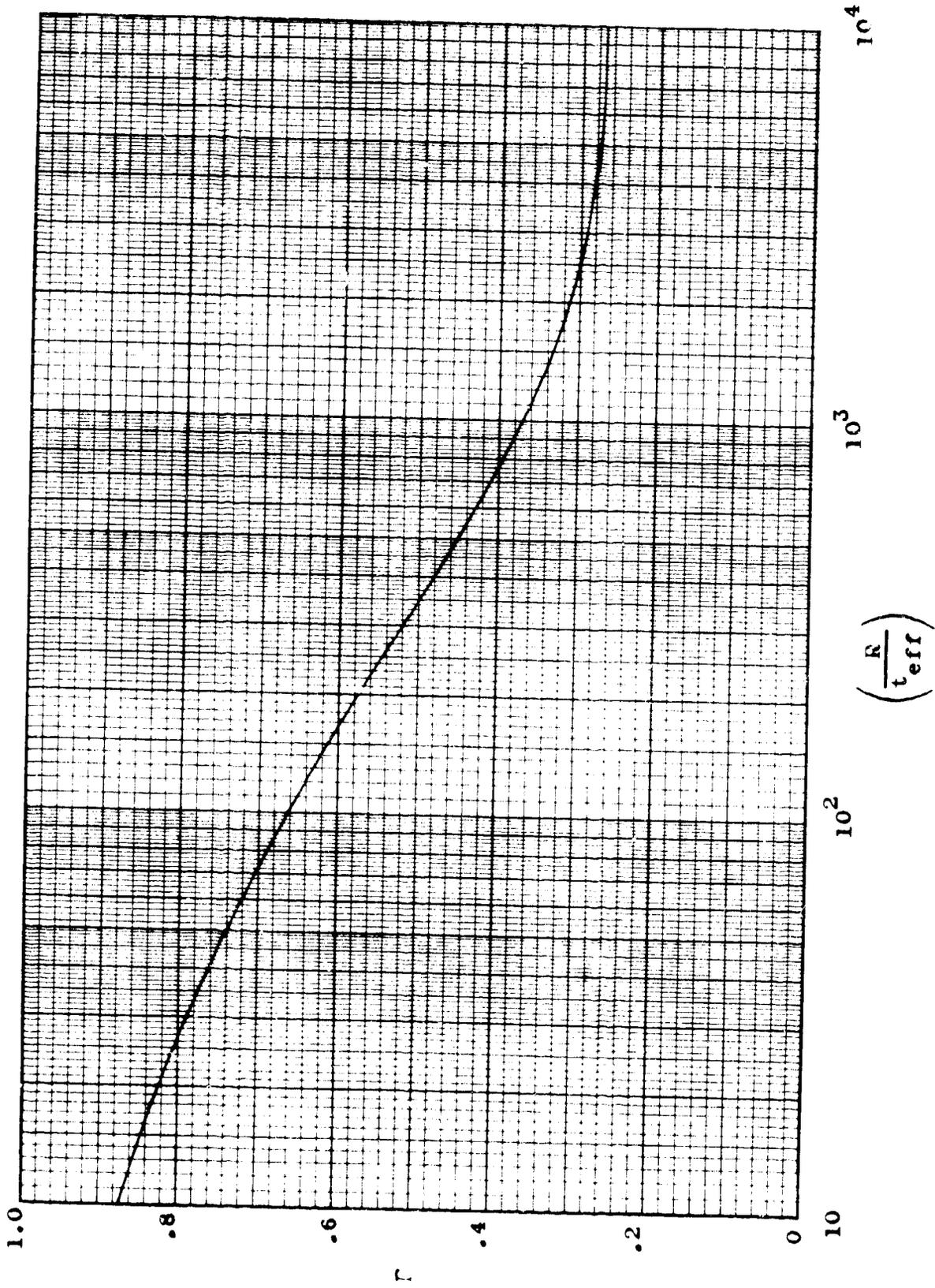


Figure 40 - Design Correlation (Knock-down) Factor for Pure Bending

GENERAL DYNAMICS  
Convair Division

16.0 BUCKLING OF MONOCOQUE CIRCULAR CYLINDERS

To provide a basis for comparison, curves are given here of  $\frac{\sigma_{cr}}{E}$  for monocoque circular cylinders subjected to each of the following loading conditions:

- (a) Unpressurized under pure axial load.
- (b) Unpressurized under pure bending moment.

These curves were developed by Convair through statistical analysis of test data [14]. Separate families are presented for each of the following statistical criteria:

- (a) Best fit
- (b) 90% probability; 95% confidence
- (c) 99% probability; 95% confidence

Each family consists of separate curves for  $\frac{L}{R} = 0.25, 1.0, \text{ and } 4.0$ . The criterion (a) was established using the conventional least squares technique and gives the mean expected or 50% probability level for buckling. Half of typical test values would fall below this level. The criterion (b) may be expressed: There is 95% confidence that at least 90% of the cylinders designed to this level would equal or exceed this buckling strength. Criterion (c) may be similarly expressed for 99% probability and 95% confidence. Criteria (b) and (c) statistically correspond respectively to the MIL-HDBK-5 "B" and "A" values.

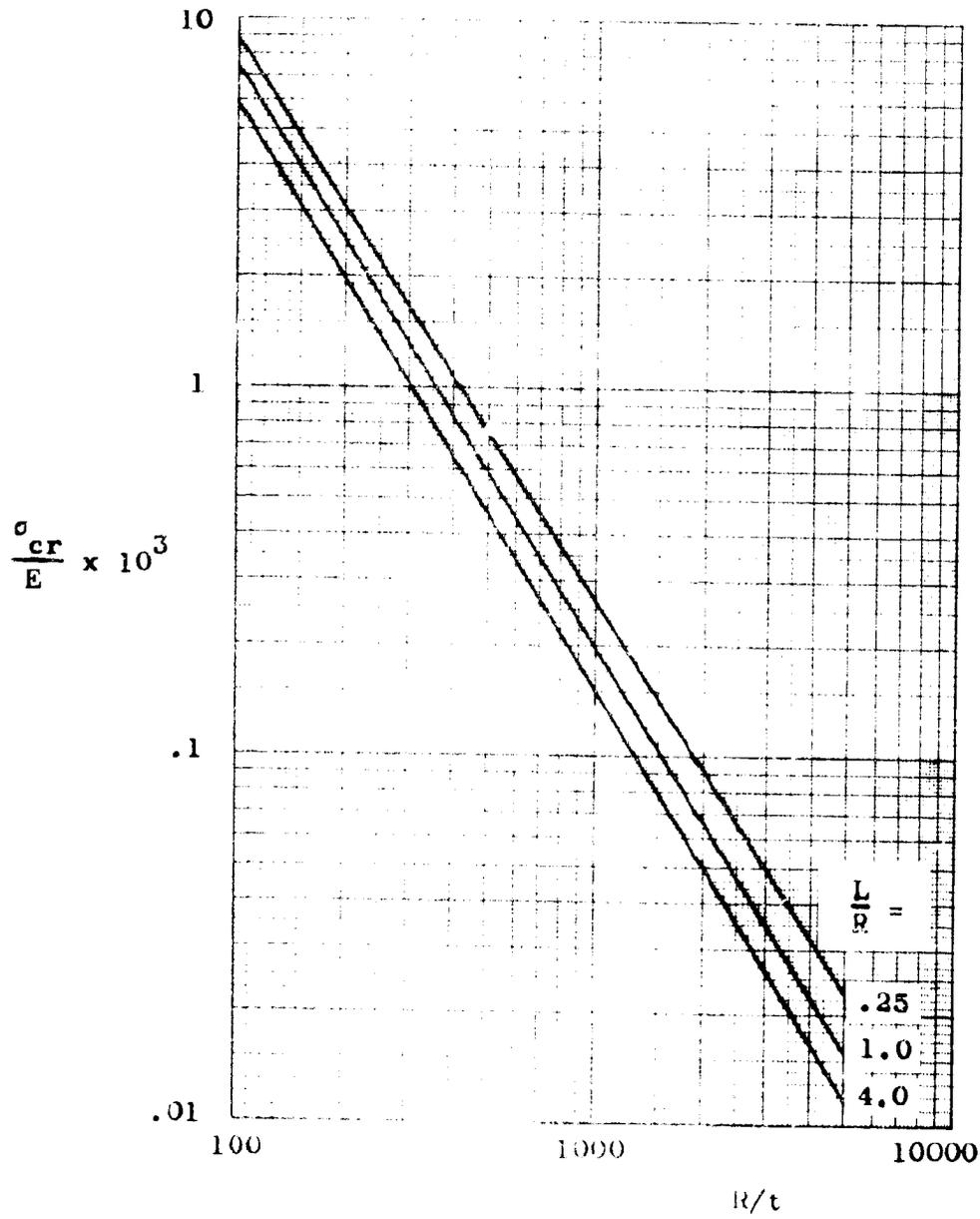


Figure 41 -  $\frac{\sigma_{cr}}{E}$  vs.  $R/t$  for Unpressurized Monocoque  
Circular Cylinders (Clamped Ends) Under  
Pure Axial Load; BEST FIT

GENERAL DYNAMICS  
 Convair Division

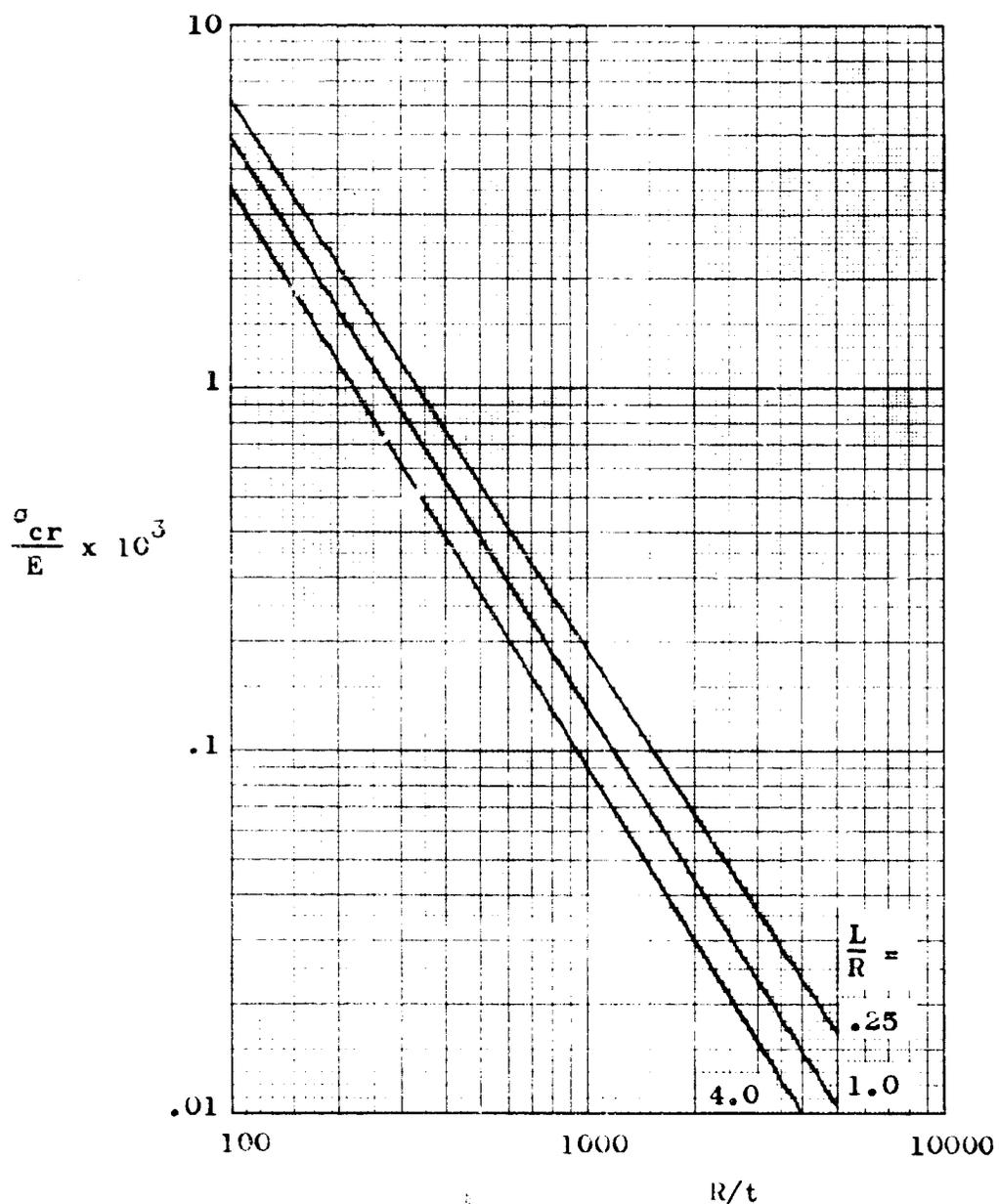


Figure 42 -  $\sigma_{cr}/E$  vs.  $R/t$  for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Axial Load; PROBABILITY = 90% CONFIDENCE = 95%

GENERAL DYNAMICS  
Convair Division

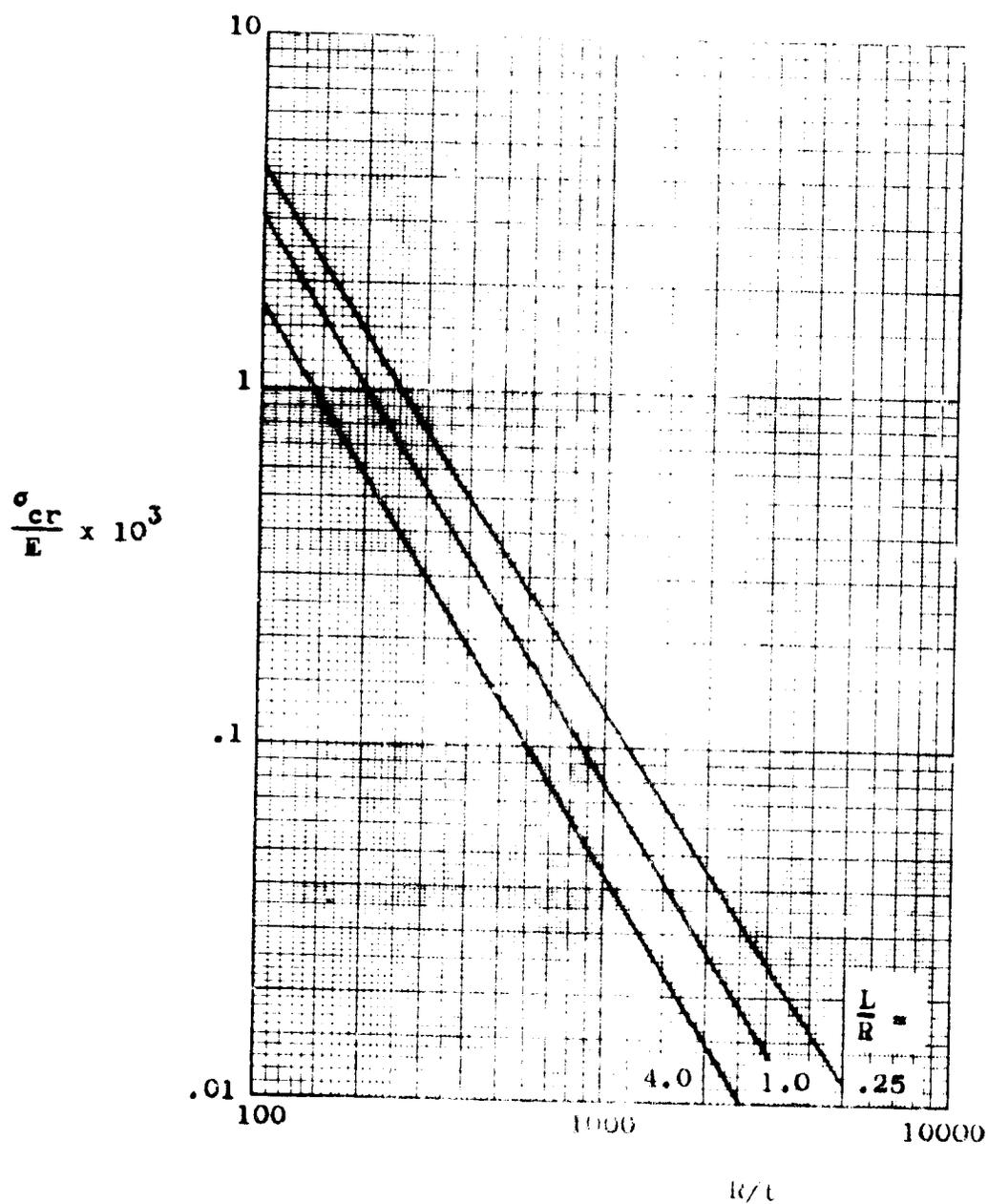


Figure 43 -  $\frac{\sigma_{cr}}{E}$  vs.  $R/t$  for Unpressurized Monocoque  
 Circular Cylinders (Clamped Ends) Under  
 Pure Axial Load; PROBABILITY = 99%  
 CONFIDENCE = 95%

GENERAL DYNAMICS  
 Convair Division

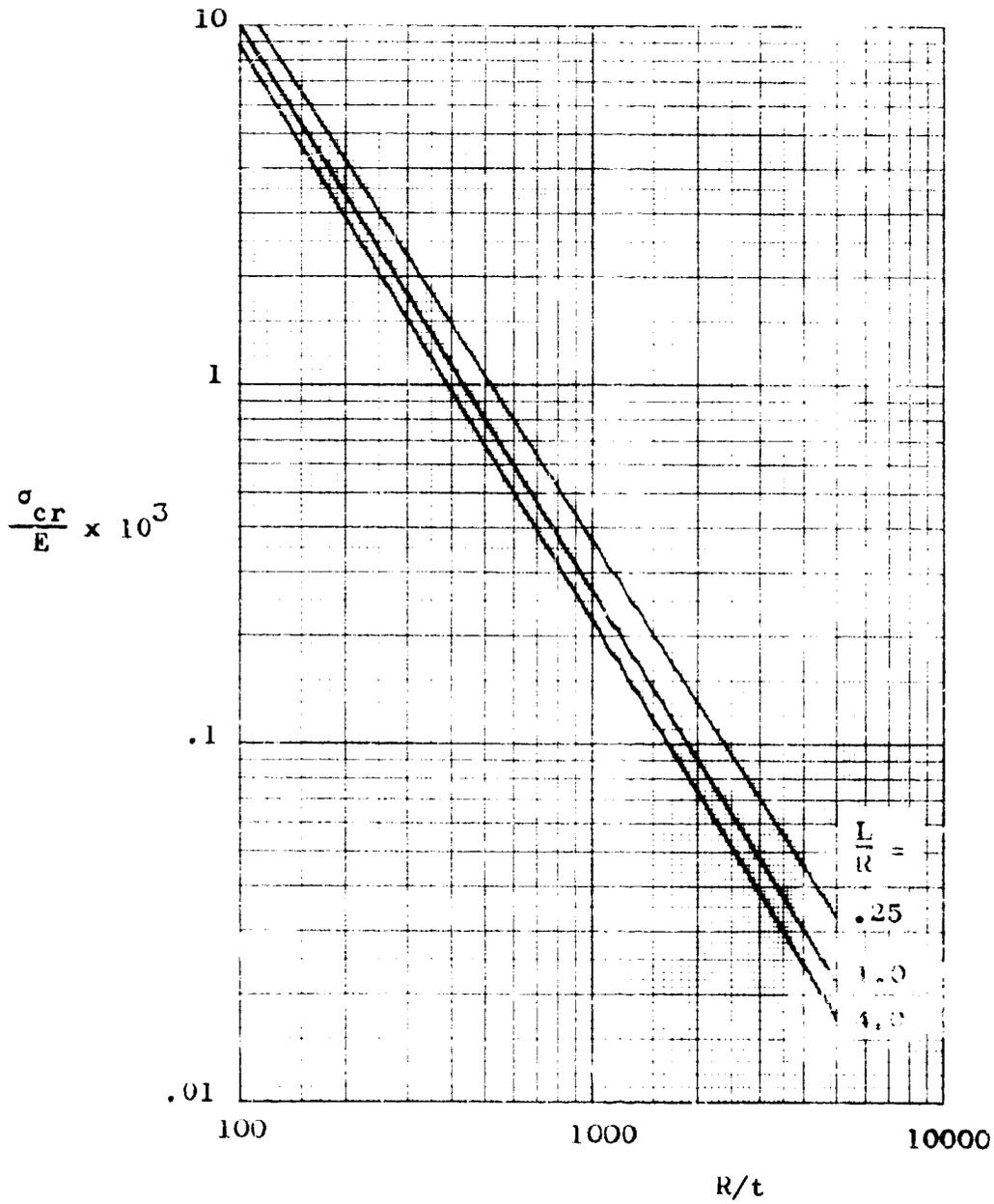


Figure 44 -  $\sigma_{cr}/E$  vs.  $R/t$  for Unpressurized Monocoque Circular Cylinders (Clamped Edges) Under Pure Bending Moment; BEST FIT

GENERAL DYNAMICS  
Convair Division

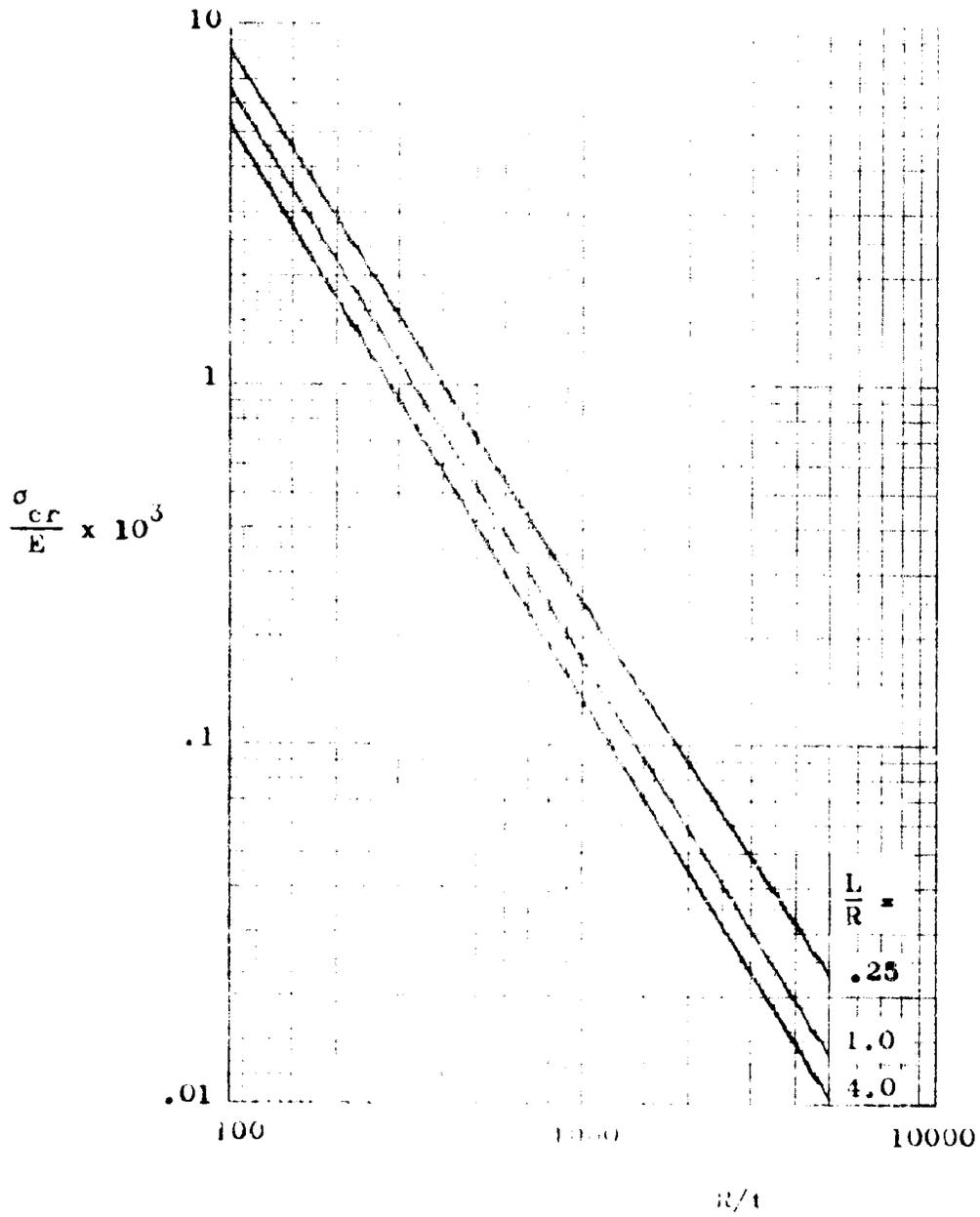


Figure 40 -  $\frac{\sigma_{cr}}{E}$  vs.  $R/t$  for Unpressurized Monocoque  
Circular Cylinders (Clamped Ends) Under  
Pure Bending Moment; PROBABILITY = 90%  
 (CRITICAL STRESS)

GENERAL DYNAMICS  
 Convair Division

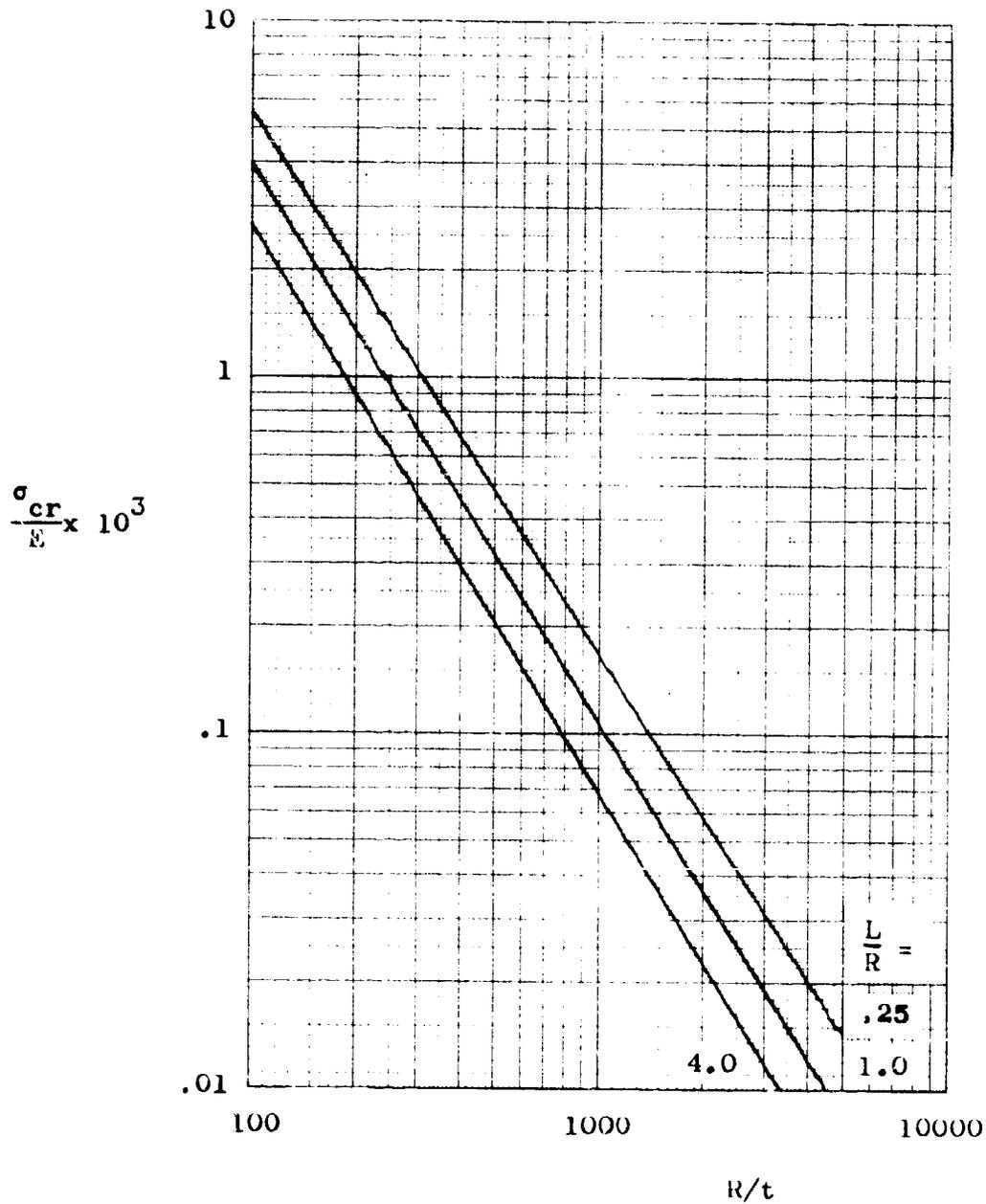


Figure 46 -  $\sigma_{cr}/E$  vs.  $R/t$  for Unpressurized Monocoque Circular Cylinders (Clamped Ends) Under Pure Bending Moment; PROBABILITY = 99% CONFIDENCE = 95%

GENERAL DYNAMICS  
Convair Division

17.0 SAMPLE PROBLEMS

Sample problems illustrating the use of digital computer programs developed in contract NAS8-11181 are given in Section 18. For each program, input data is shown both on a sample coding form and at the end of the applicable Fortran listing. Output listings are also provided for the sample problems.

The sample problem on buckling of isotropic skin panels displayed in Section 18.1 is to obtain listings and plots for  $\sigma_{cr}/E$  vs.  $R/t$  and  $\sigma_{cr}$  vs.  $R/t$  (for  $E = 10 \times 10^6$  psi and  $30 \times 10^6$  psi) for  $a/b = 0.4$  ( $K_s = K_c = 7.0$ ),  $0.6$  ( $K_s = 4.0$ ),  $0.6$  ( $K_c = 5.7$ ), and  $0.8$  ( $K_s = 3.29$ ). Calculations are desired for  $b/R = 0.01, 0.02, 0.05, 0.10, 0.20, 0.50$  and  $1.0$ . Plots are to be made for  $100 \text{ psi} \leq \sigma_{cr} \leq 100,000 \text{ psi}$  and  $10^{-6} \leq \sigma_{cr}/E \leq 10^{-1}$ . In addition, solutions for  $R/t = 100$  and  $1000$  are sought first. The input coding form for solution of this problem is shown in Figure 48 of Section 18.1. Sample pages from the output listing are shown in Figure 49 and the corresponding plots may be found among the design curves of Section 11.2.

Four sample problems on buckling of longitudinally stiffened cylinders are shown in Section 18.2. Buckling stresses are desired for four specimens from Reference 4 for which  $E = 10.5 \times 10^6$  psi,  $\sigma_{cy} = 38,000$  psi,  $\sigma_{pL} = 20,000$  psi,  $\sigma_{.7} = 37,000$  psi,  $\nu = 0.33$ ,  $n = 10$ ,  $\sigma_{cc} = 47,500$  psi, and  $C_F = 3.75$ . It was found for these specimens (see Section 6.4) that:

<u>Specimen</u>	$\Gamma \bar{N}$	$R/\bar{t}$	$L/\rho_{11}$
1	1.497	479	368
2	.526	495	368
3	1.826	501	430
4	.359	488	430

Using these values for input to Program 3896 as shown in Figure 52 of Section 18.2, solutions are obtained as listed in the output displayed in Figure 53.

An example problem using the Thielemann solution for general instability (Program 3942) is shown in the sample data coding form of Figure 56 in Section 18.3.1. The example is specimen I-1 of reference 29. Since the skin panels buckled very early, an iterative technique (refer to Section 7.3) was employed to obtain the effective skin width and corresponding stiffnesses. The required input information for program solution becomes:

$$a/D = 0.077702, \quad \eta_s = 1.8740, \quad \eta_p = 0,$$

$$a/\rho_{11} = 29.039, \quad \sqrt{C_R} D/\rho_{22} = 372.82.$$

Results of the computer run are shown in Figure 57 of Section 18.3.1 where it can be seen that  $\bar{N} = 0.58923$ . From equation (13-4),  $(N_x)_{CL} = 1038$  lbs/in. This result, along with other pertinent information regarding this specimen, is shown in Table VII of Section 7.3.

The evaluation of  $C_R$  for the specimens of reference 29 (including the above specimen I-1) forms sample problems demonstrating the use of Program 3942I. For specimen I-1,  $N_s = 98$  and  $R^2 A_r/I_r = 34,751$ .

The input coding form and output listing are shown in Figures 64 and 65 of Section 18.3.3 where the result is given for specimen I-1:

$$C_R = 0.99918 \text{ (or } C_R \approx 1.0).$$

An example problem demonstrating the general instability solution of reference 5 is shown on the coding form in Figure 60 of Section 18.3.2. This is the configuration described in Example 4 of Section 8.3. The corresponding input values are shown on the coding form and the upper portion of the output listing, Figure 61. The critical axial loading was found for the case of applied hoop compression equal to 799.1 lbs/in. This is one of the loading cases shown under Example 4 in Table IX of Section 8.3. Note also in Figure 61 that checks of critical loadings corresponding to buckle modes having configurations  $(m-1, n)$ ,  $(m+1, n)$ ,  $(m, n-1)$ , and  $(m, n+1)$  are also calculated to show that the value found for  $(m, n)$  is a relative minimum. Note in this example that the  $(m-1 = 0, 2n = 10)$  configuration is meaningless (see restrictions on  $m$  and  $n$  in discussion of MINM and MINN for CARD TYPE 3 in Section 18.3.2).

In addition to these sample problems demonstrating the use of the computer programs, various other solutions were obtained employing these programs during the course of this study. Included are the design curves of Sections 11.2, 12.2 and 13.2, the test data comparisons of Sections 5.3, 6.4, and 7.3, and the interaction studies of Sections 8.3, and 8.4.

The step-by-step procedures presented in Sections 11.1, 12.1, 13.1, and 15.1 are also of sufficient detail to guide solutions for a broad class of stability problems associated with the design of stiffened cylindrical shells. In particular, Tables XVI and XVIII of Sections 12.1 and 13.1 detail the appropriate means for obtaining required  $A_{ij}$  and  $D_{ij}$  values for specific types of stiffening.

## 18.0 DIGITAL COMPUTER PROGRAMS

Digital computer programs developed within contract NAS8-11181 for determination of buckling strength levels for isotropic skin panels and the panel and general instability modes (see Glossary) are discussed in this section. These programs were employed to obtain the design curves of Part II and the test comparisons of Part I. Detail descriptions of input and output, sample problems, program notation, flow diagrams, and Fortran listings are presented.

### 18.1 Program for Buckling of Isotropic Skin Panels Subjected to Edge Compression

The program developed for analyzing buckling of isotropic skin panels bounded by elements of a stiffened cylindrical shell under axial compression is designated as General Dynamics Convair Program No. 3875. The theoretical and empirical considerations leading to the analyses in the program are discussed in Section 5.2 of Part I. Because of the formulation employed, the program has the capability of analyzing buckling of flat or curved plates and also may be used for complete isotropic cylindrical shells.

The input format is shown in Figure 47. Symbols are shown in Table XX. Detailed discussion of input, card by card, follows. Runs may be stacked.

CARD TYPE 1: One card per run. Each entry must be integer(s) right adjusted in appropriate columns.

3875      80 COLUMN - GENERAL PURPOSE - WORKSHEET      L.S. FØSSUM

NBR	NBR	NBTRN	IBUG	K1	K2	K3
AB1	BR1	CKM1	CAPK1			
BR1	BR2	BR2	BR2	....	BRNBR-1	BRNBR
RT1	RT2	RT2	RT2	....	RTNBRN-1	RTNBRN
YBSIG	YBSIG	YBR	YBR	YBR	YBSIG	YBSIG
BT1	BT2	BT2	BT2	....	BTNAT-1	BTNAT

GENERAL DYNAMICS  
Convair Division

Figure 47 - Input Format-Program 3875

Enter NAB (the number of a/b ratios to be entered on cards TYPE 2) in columns 1-5 (30 maximum).

Enter NBR (the number of b/R ratios to be entered on cards TYPE 3) in columns 6-10 (20 maximum).

Enter NBT (the number of b/t ratios to be entered on cards TYPE 6) in columns 11-15 (20 maximum).

Enter NRTIN (the number of R/t ratios to be entered on cards TYPE 4) in columns 16-20 (99 maximum).

Enter NOP (determines which OPTIONS are used) in column 25:

OPTIONS	NOP
1	4
2	2
3	1
1 and 2	6
1 and 3	5
2 and 3	3
1, 2, and 3	7

As discussed in Section 5.2 of Part I, OPTION 1 uses the lower bound relation of Seide, et al. [13] for  $\sigma_R$  and would be generally employed for design. OPTION 2 uses mean expected results for  $\sigma_R$  and OPTION 3 uses 90% probability, 95% confidence values for  $\sigma_R$ . If more than one OPTION is called for (NOP = 3, 5, 6, or 7), complete sets of output for each desired OPTION are obtained.

Enter IBUG (debugging and printout option) in column 30. If IBUG = 0 (or is blank), only calculations for the R/t values input on cards TYPE 2 will be printed out. If IBUG = 1 (or  $\neq 0$ ), calculations will be run and printed for 101 values of R/t generated within the program whose logarithms are equally spaced.

Enter K1 (plotting option) in column 35. If K1 = 0 (or is blank), calculations will be run and logarithmic plots will be made using 101 values of R/t generated within the program whose logarithms are equally spaced. If K1 = 1, no plots will be made.

Enter K2 (header card option) in columns 36-40. This option allows the use of one TYPE 1 header card if applicable to a number of runs. If K2 is blank, 0, or 1, a TYPE 1 header card will be used as usual. If K2 > 1, say "n", then "n" runs may be stacked without repeating the header card TYPE 1. When "n" runs have been read, the program will then begin reading TYPE 1 cards, if additional runs are present.

Disregard K3 (not used).

CARD TYPE 2: There will be NAB cards per run.

Enter  $AB_i$  ( $i^{\text{th}}$  value of the panel length/width ratio a/b to be input) in columns 1-10 (E10.5).

Enter  $CKNM_i$  (heading for  $i^{th}$  output listing and/or  $i^{th}$  plot) right adjusted ending in column 20:

Hollerith characters  $KS=$  or  $KC=$  or  $KS=KC=$  (maximum of 6 characters).

Enter  $CAPK_i$  ( $i^{th}$  value of  $KS$  or  $KC$ ) in columns 21-30 ( $E10.5$ ). Note that these three entries must correspond; i.e., the  $i^{th}$  TYPE 2 card shows  $a/b$  and  $K_s$  or  $K_c$  values which are related since  $K_s$  and  $K_c$  are functions of  $a/b$  [see equation (5-12), Part I and reference 15].

**CARD TYPE 3:** There will be  $NBR/6$  (rounded to higher whole number) cards per run.

Enter  $BR$  values (panel width to radius ratio  $b/R$ , 6 to a card ( $6E10.5$ )).

**CARD TYPE 4:** There will be  $NRTIN/6$  (rounded to higher whole number) cards per run.

Enter  $RT$  values (panel radius to thickness ratio  $R/t$ ), 6 to a card ( $6E10.5$ ).

**CARD TYPE 5:** One card per run. (If  $K1 = 1$  omit this card)

Enter  $YBSIG$  (lower limit of  $\sigma_{cr}$  for plots of Buckling Stress, psi vs.  $R/t$ ) in columns 1-10 ( $E10.5$ ).

Enter  $YTSIG$  (upper limit of  $\sigma_{cr}$  for plots of Buckling Stress, psi vs.  $R/t$ ) in columns 11-20 ( $E10.5$ ).

Disregard  $YBBR$  (not used).

Disregard  $YTBR$  (not used).

Enter  $YBSIGE$  (lower limit of  $\sigma_{cr}/E$  for plots of Buckling Stress/ $E$  vs.  $R/t$ ) in columns 41-50 ( $E10.5$ ).

Enter YTSIGE (upper limit of  $\sigma_{cr}/E$  for plots of Buckling Stress/E vs. R/t) in columns 51-60 (E10.5).

CARD TYPE 6: There will be NBT/6 (rounded to higher whole number) cards per run.

Enter BT values (panel width to thickness ratio b/t for flat panels only), 6 to a card (6E10.5). Omit this card if NBT = 0. These values will be used only if b/R = 0 (flat plate).

A sample input coding form is shown in Figure 48.

Approximate execution times at the General Dynamics Convair 7094 DCS installation were 1.5 seconds per a/b and b/R combination for NOP = 7 and K1 = 0.

The program output consists of a listing of all input in addition to calculated results. The calculated results ( $\sigma_{cr}/E$ ,  $\sigma_{cr}$  for  $E = 10 \times 10^6$  psi and  $\sigma_{cr}$  for  $E = 30 \times 10^6$  psi) are tabulated (and/or plotted) for the input values of R/t and the 101 internally determined values of R/t, if desired. Separate tables (and/or plots) are formed for each a/b and  $K_{s[c]}$ , b/R, and OPTION combination selected on input. A column having the heading "BASIS" is also printed. The symbol I in this column indicates that the buckling stress was determined by equation (5-2) of Part I. The symbol II in this column indicates that the buckling stress was determined by equation (5-4) of Part I. The latter case indicates behavior of the panel to be similar to that of a

80 COLUMN-GENERAL PURPOSE-WORKSHEET  
 3785 BUCKLING OF ISOTROPIC PANELS L.S. FOSSUM

1	4	7	0	2	4	1
	.4		KS=	KC=	7.0	
2	.6		KS=		4.0	
	.6		KC=		5.7	
	.8		KS=		3.29	
3	.01		.C	.1	.2	.5
	1.					
4	100.				1000.	
5	100.				100000.	.1

CARD TYPE:

GENERAL DYNAMICS  
 Convair Division

Figure 48 - Sample Input Data-Program 3875

complete cylinder of the same radius, thickness and length.

A sample output listing is shown in Figure 49 where the first and two typical pages of the sample problem coded in Figure 48 are shown. Output Plots from this sample input are included in Section 11.2, Part II. Notation used in program 3875 vs. symbols employed in Parts I and II of this report appears in Table XX. A basic flow diagram for the program is given in Figure 50 and a Fortran listing of the program, including input data from the sample problem (Figure 48), is given in Table XXI.

LOCAL BUCKLING OF ISOTROPIC PANELS

MAR = 4    NPT = 7    NBT = 7    NPTN = 2    NBP = 4    IRUS = 1    K1 = -2    K2 = -0    K3 = -0

PANEL ASPECT RATIO = 7.490    KS=KC= 7.000  
 PANEL ASPECT RATIO = 2.500    KS= 4.000  
 PANEL ASPECT RATIO = 3.400    KC= 5.700  
 PANEL ASPECT RATIO = 0.900    KS= 3.200

PANEL WIDTH / RADIUS  
 0.200000    0.050000    0.100000    0.200000

RADIUS / THICKNESS  
 1.00000

YBSIG = 1.0E+02    YTSIG = 1.0E+05    YBR = 0.    YTR = 0.    YBSIGF = 10.00E-05    YTSIGF = 1.00E-01

PANEL WIDTH / THICKNESS

-0.

Figure 49 - Output Listing - Program 3875

CASE NO. 3875  
 CASE NAME: CASE 3875  
 CASE TYPE: 1  
 CASE NO.: 3875  
 PANEL WIDTH/RADIUS = 0.2000  
 RUCKLING STRESS, PSI  
 F = 1,000,000  
 RUCKLING STRESS, PSI  
 F = 30,000,000

NO. OF STRESS	RUCKLING STRESS, PSI	RUCKLING STRESS, PSI	BASIS
1	3.3777E-04	2.8130E-05	I
2	1.6546E-04	4.2161E-03	I
3	9.3773E-03	2.8130E-05	I
4	9.5763E-03	2.5729E-05	I
5	7.8434E-03	2.3529E-05	I
6	7.1733E-03	2.1529E-05	I
7	6.7413E-03	1.9686E-05	I
8	6.3337E-03	1.8011E-05	I
9	5.9221E-03	1.6482E-05	I
10	5.5286E-03	1.5085E-05	I
11	4.4032E-03	1.3910E-05	I
12	4.2148E-04	1.2644E-05	I
13	3.8533E-04	1.1580E-05	I
14	3.5357E-04	1.0607E-05	I
15	3.2394E-04	9.7183E-06	I
16	2.9686E-04	8.9059E-06	I
17	2.7211E-04	8.1632E-06	I
18	2.4948E-04	7.4843E-06	I
19	2.2878E-04	6.8634E-06	I
20	2.1085E-04	6.2956E-06	I
21	1.9254E-04	5.7762E-06	I
22	1.7670E-04	5.3010E-06	I
23	1.6220E-04	4.8661E-06	I
24	1.4893E-04	4.4680E-06	I
25	1.3679E-04	4.1036E-06	I
26	1.2566E-04	3.7699E-06	I
27	1.1548E-04	3.4643E-06	I
28	1.0614E-04	3.1843E-06	I
29	9.7589E-05	2.9277E-06	I
30	8.9749E-05	2.6925E-06	I
31	8.2561E-05	2.4758E-06	I
32	7.5968E-05	2.2791E-06	I
33	6.9921E-05	2.0976E-06	I
34	6.4372E-05	1.9312E-06	I
35	5.9280E-05	1.7784E-06	I
36	5.4606E-05	1.6381E-06	I
37	5.0310E-05	1.5093E-06	I
38	4.6366E-05	1.3910E-06	I
39	4.2742E-05	1.2923E-06	I
40	3.9411E-05	1.1823E-06	I
41	3.6349E-05	1.0905E-06	I
42	3.3533E-05	1.0060E-06	I
43	3.0944E-05	9.2831E-07	I
44	2.8561E-05	8.5683E-07	I
45	2.6369E-05	7.9106E-07	I
46	2.4351E-05	7.3052E-07	I
47	2.2493E-05	6.7479E-07	I
48	2.0783E-05	6.2348E-07	I
49	1.9207E-05	5.7622E-07	I
50	1.7756E-05	5.3269E-07	I
51	1.6420E-05	4.9260E-07	I

Figure 49 - Output Listing - Program 3875 (Cont'd)

1.51584E-06	1.61095E-03	6.55465E-02	I
1.47646E-06	1.47547E-03	4.21615E-03	I
1.37131E-06	1.30038E-03	3.90245E-03	I
1.27444E-06	1.27444E-03	3.61335E-03	I
1.17035E-06	1.11545E-03	3.24635E-03	I
1.07041E-06	1.09535E-03	3.10135E-03	I
9.77997E-07	9.88497E-03	2.87505E-03	I
8.94789E-07	8.84789E-03	2.66645E-03	I
8.24698E-07	8.24698E-03	2.47415E-03	I
7.65029E-07	7.65029E-03	2.29695E-03	I
7.11197E-07	7.11197E-03	2.13345E-03	I
6.61039E-07	6.61039E-03	1.98315E-03	I
6.14917E-07	6.14917E-03	1.84445E-03	I
5.72215E-07	5.72215E-03	1.71555E-03	I
5.32945E-07	5.32945E-03	1.59905E-03	I
4.96947E-07	4.96947E-03	1.49145E-03	I
4.63745E-07	4.63745E-03	1.39545E-03	I
4.32835E-07	4.32835E-03	1.29745E-03	I
4.03535E-07	4.03535E-03	1.21155E-03	I
3.75255E-07	3.75255E-03	1.13175E-03	I
3.47635E-07	3.47635E-03	1.05775E-03	I
3.21175E-07	3.21175E-03	9.89745E-04	I
2.95375E-07	2.95375E-03	9.26425E-04	I
2.70575E-07	2.70575E-03	8.66425E-04	I
2.47045E-07	2.47045E-03	8.09555E-04	I
2.24245E-07	2.24245E-03	7.60955E-04	I
2.02615E-07	2.02615E-03	7.13875E-04	I
1.81695E-07	1.81695E-03	6.70175E-04	I
1.61095E-07	1.61095E-03	6.29405E-04	I
1.41545E-07	1.41545E-03	5.91915E-04	I
1.23095E-07	1.23095E-03	5.55905E-04	I
1.05775E-07	1.05775E-03	5.24345E-04	I
8.94789E-08	8.94789E-03	4.94065E-04	I
7.65029E-08	7.65029E-03	4.65975E-04	I
6.51325E-08	6.51325E-03	4.39525E-04	I
5.52945E-08	5.52945E-03	4.15145E-04	I
4.69945E-08	4.69945E-03	3.92315E-04	I
3.97775E-08	3.97775E-03	3.70395E-04	I
3.34695E-08	3.34695E-03	3.51075E-04	I
2.79195E-08	2.79195E-03	3.32445E-04	I
2.30095E-08	2.30095E-03	3.14905E-04	I
1.87045E-08	1.87045E-03	2.98655E-04	I
1.49545E-08	1.49545E-03	2.83325E-04	I
1.17035E-08	1.17035E-03	2.68925E-04	I
8.94789E-09	8.94789E-03	2.55405E-04	I
6.51325E-09	6.51325E-03	2.42675E-04	I
4.69945E-09	4.69945E-03	2.30635E-04	I
3.34695E-09	3.34695E-03	2.19305E-04	I
2.47045E-09	2.47045E-03	2.08735E-04	I
1.87045E-09	1.87045E-03	1.98675E-04	I
1.49545E-09	1.49545E-03	1.89165E-04	I
1.17035E-09	1.17035E-03	1.80155E-04	I

Figure 49 - Output Listing - Program 3875 (Cont'd)

TABLE XX - Program 3875 Notation

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
AB	a/b	Panel length/width ratio.
NAB	-	Number of a/b values input.
BR	b/R	Panel width/radius ratio.
NBR	-	Number of b/R values input.
BT	b/t	Panel width/thickness ratio.
NBT	-	Number of b/t values input.
CAPK	$K_s$ or $K_c$	Buckling coefficient.
E1, E2	E	Elastic modulus, $10 \times 10^6$ and $30 \times 10^6$ psi respectively.
FMU	$\nu$	Poisson's Ratio, $\nu = 0.3$ .
NOP	-	Option No. (1, 2, or 3; or 4 meaning 1, 2, and 3).
RT	R/t	Radius/thickness ratio.
NRTIN	-	Number of R/t values input.
NRT	-	Total number of R/t values, both input and calculated.
SIGRE	$\sigma_R/E$	Nondimensional buckling parameter.
SIGPE	$\sigma_p/E$	Nondimensional buckling parameter
SIGCRE	$\sigma_{cr}/E$	Nondimensional buckling parameter
SIGCR1	$\sigma_{cr}$ (for $E = 10 \times 10^6$ psi)	Buckling stress (psi) for aluminum where $E = 10 \times 10^6$ psi.
SIGCR2	$\sigma_{cr}$ (for $E = 30 \times 10^6$ psi)	Buckling stress (psi) for steel where $E = 30 \times 10^6$ psi.

TABLE XX - Program 3875 Notation (Cont'd)

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
YBSIGE	-	Lower limit of $\sigma_{cr}/E$ for plots.
YTSIGE	-	Upper limit of $\sigma_{cr}/E$ for plots.
YBSIG	-	Lower limit of SIGCR1 and SIGCR2 for plots, psi.
YTSIG	-	Upper limit of SIGCR1 and SIGCR2 for plots, psi.
IB	-	Basis number (1 or 2); e.g., BASIS(IB) = BASIS II for IB = 2, and $\sigma_{cr} = \sigma_R$ .



TABLE XXI - Fortran Listing-Program 3875

```

$SETUP LB4      DISK,PLOT01,SAFE
$EXECUTE       IBJOB
$IBJOB
$IBFTC MAIN
C      MAIN PROGRAM 3875 - LOCAL BUCKLING OF ISOTROPIC PANELS
C
100 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
1     E1, E2, FMU, IBUG, K1, K2, K3, LAB, NBR, NOP, NRTIN, NBT,
2     RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
3     YBSIG, YTSIG, YBRK, YBR, YBSIGE, YTSIGE
COMMON /COMPUT/ C, IB, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
1     SIGCR2(200,20), SIGCR1(200,20), SIGCR2(200,20)
150 SKIP = 0.
200 WRITE (6,201)
201 FORMAT (1H1 /// 49X,34HLOCAL BUCKLING OF ISOTROPIC PANELS ////)
250 CALL DATAIN (SKIP)
300 OMIMU2 = 1.-FMU**2
325 IF (K1.NE.1) GO TO 350
326 NRT = NRTIN
327 NRTC = 0
340 GO TO 400
350 CALL RTC (RT,NRTIN,NRT,NRTC)
CALL SETMIV (288,0,24,24)
CALL SMXYV (1,1)
400 GO TO ( 650, 550, 550, 450, 450, 450 ), NOP
450 CALL CALC(1)
500 IF (NOP.EQ. 4) GO TO 700
501 IF (NOP.EQ. 5) GO TO 650
550 CALL CALC(2)
600 IF (NOP.EQ.2) GO TO 700
601 IF (NOP.EQ. 6) GO TO 700
650 CALL CALC(3)
700 CONTINUE
720 GO TO 200
750 END
$IBFTC BLKDAT
110 BLOCK DATA
111 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
1     E1, E2, FMU, IBUG, K1, K2, K3, LAB, NBR, NOP, NRTIN, NBT,
2     RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
3     YBSIG, YTSIG, YBRK, YBR, YBSIGE, YTSIGE
120 DATA TITLE(1), YLAB1(1), YLAB2(1), YLAB3(1), XLAB1(1)
1
2 / 36HLOCAL BUCKLING OF ISOTROPIC PANELS ,
3 36H BUCKLING STRESS, PSI FOR ALUMINUM ,
4 36HBUCKLING STRESS / ELASTIC MODULUS ,
5 36H BUCKLING STRESS, PSI FOR STEEL ,
6 36H RADIUS / THICKNESS /
121 DATA (BASIS(1E), IBS(1,2) /6H I 6H II /,
+ E1, E2, FMU /1.E7, 3.E7, 0.30 /
122 END
$IBFTC DATAIN
SUBROUTINE DATAIN (SKIP)
C      SUBROUTINE TO READ IN AND PRINT OUT INPUT DATA.

```

TABLE XXI - Fortran Listing - Program 3875 (Cont'd)

```

C
50 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
   1  E1, E2, FMU, IBUG, K1, K2, K3, NAB, NBR, NOP, NRTIN, NBT,
   2  RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
   3  YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
51 COMMON /COMPUT/ C, IB, N, NRT, NRTC, OM1MU2, SIGRE, SIGPE,
   1  SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
75 IF (SKIP.NE.0.) GO TO 299
100 READ (5,101) NAB, NBR, NBT, NRTIN, NOP, IBUG, K1, K2, K3
101 FORMAT (9I5)
200 WRITE (6,201) NAB, NBR, NBT, NRTIN, NOP, IBUG, K1, K2, K3
201 FORMAT (4X,5HNAB =I5,4X,5HNBR =I5,4X,5HNBT =I5,4X,7HNRTIN =I5,4X,
   * 5HNOP =I5,4X,6HIBUG =I5,4X,5H K1 =I5,4X,5H K2 =I5,4X,5H K3 =I5)
250 IF (K2.EQ.0) GO TO 300
251 SKIP = K2 - 1
252 GO TO 300
299 SKIP = SKIP - 1.
300 READ (5,301) (AB(K),CKNM(K),CAPK(K),K=1,NAB)
301 FORMAT (E10.5,A10,E10.5)
400 WRITE (6,401) (AB(K),CKNM(K),CAPK(K),K=1,NAB)
401 FORMAT ( /// (33X,20HPANEL ASPECT RATIO =F10.3,16X,A10,F10.5) )
500 READ (5,501) (BR(J),J=1,NBR)
501 FORMAT (6E10.5)
600 WRITE (6,601) (BR(J),J=1,NBR)
601 FORMAT ( /// 56X,20HPANEL WILTH / RADIUS //(5F23.6) )
700 READ (5,701) (RT(I),I=1,NRTIN)
701 FORMAT (6E10.5)
800 WRITE (6,801) (RT(I),I=1,NRTIN)
801 FORMAT ( /// 57X,18HRADIUS / THICKNESS // (5F23.2) )
850 IF (K1.EQ.1) GO TO 1050
900 READ (5,901) YBSIG,YTSIG,YBBR,YTBR,YBSIGE,YTSIGE
901 FORMAT ( 6E10.5 )
1000 WRITE (6,1001) YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
1001 FORMAT ( // 4X, 6HYBSIG=1PE12.2, 3X, 6HYTSIG=1PE12.2,
   1 3X, 5HYBBR=1PE12.2, 3X, 5HYTBR=1PE12.2,
   2 3X, 7HYBSIGE=1PE12.2, 3X, 7HYTSIGE=1PE12.2 )
1050 IF (NBT.EQ.0) GO TO 2000
1100 READ (5,1101) (BT(I),I=1,NBT)
1101 FORMAT (6E10.5)
1200 WRITE (6,1201) (BT(I),I=1,NBT)
1201 FORMAT ( /// 54X,23HPANEL WIDTH / THICKNESS // (6F23.5) )
2000 RETURN
3000 END
$IBFTC RTC
SUBROUTINE RTC (RT,NRTIN,NRT,NRTC)
C TO GENERATE RT(I)'S FOR I FROM N1 TO N2,
C EVENLY SPACED RELATIVE TO LOG10 SCALE.
DIMENSION RT(200)
100 N1 = NRTIN + 1
150 N2 = NRTIN + 101
200 RT(N1) = 100.
250 RT(N2) = 10000.
300 DLOG = .02
350 ONE = 2.0
400 NRTC = 101
450 DO 650 I=1,99
500 RI = N1 + I
550 RTLOG = ONE + FLOAT(I)*DLOG

```

TABLE XXI - Fortran Listing - Program 3875 (Cont'd)

```

600 RT(N1) = 10.**RTLOG
650 CONTINUE
700 RETURN
750 END
8IBFTC CALC
      SUBROUTINE CALC(NN)
C      SUBROUTINE TO CALCULATE SIGMA CRITICAL AND SIGMA CRITICAL / E .
C      SUBROUTINE NOVA IS CALLED TO MAKE PLOTS.
C
10 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BI(20),CAPK(30),CKNM(30),
   1  E1, E2, FMO, IBUG, K1, K2, K3, NAL, NBR, NOP, NRTIN, NET,
   2  RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
   3  YBSIG, YTSIG, YHBR, YTBR, YBSIGE, YTSIGE
11 COMMON /COMPUT/ C, Ib, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
   1  SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
      N = NN
50 DO 1090 K=1,NAB
70 DO 1030 J=1,NBR
71 IF (BR(J).NE.0.0) GO TO 90
72 WRITE (6,73) N, AB(K), CKNM(K), CAPK(K), BR(J)
73 FORMAT (1H1,62X,7HOPTION ,I1 /// 14X,19HPANEL ASPECT RATIO=F8.2,
   1  14X,A10,F8.3,15X,19HPANEL WIDTH/RADIUS=F10.4 ///
   2  11X,21HPANEL WIDTH/THICKNESS 9X,17HBUCKLING STRESS/E 13X,
   3  19HBUCKLING STRESS,PSI 11X,19HBUCKLING STRESS,PSI/
   4  73X,14HE = 10,000,000 16X,14HE = 30,000,000 /// )
74 DO 84 I=1,NBT
75 IF (BT(I).EQ.0.0) GO TO 82
76 SIGCRE(I,J) = CAPK(K) / OMIMU2 * (1./BT(I))**2
77 SIGCR1(I,J) = SIGCRE(I,J) * E1
78 SIGCR2(I,J) = SIGCRE(I,J) * E2
79 WRITE (6,80) BT(I), SIGCRE(I,J), SIGCR1(I,J), SIGCR2(I,J)
80 FORMAT (1H F25.4, 3(1PE30.4))
81 GO TO 84
82 WRITE (6,83) BT(I)
83 FORMAT (1H F25.4, 10X, 31H CALCULATIONS OMITTED, B/T = 0.)
84 CONTINUE
85 GO TO 1030
90 WRITE (6,91) N, AB(K), CKNM(K), CAPK(K), BR(J)
91 FORMAT (1H1,62X,7HOPTION ,I1 /// 14X,19HPANEL ASPECT RATIO=F8.2,
   1  14X,A10,F8.3,15X,19HPANEL WIDTH/RADIUS=F10.4 ///
   2  7X,16HRADIUS/THICKNESS,11X,17HBUCKLING STRESS/E,12X,
   3  19HBUCKLING STRESS,PSI,11X,19HBUCKLING STRESS,PSI,6X,5HBASIS /
   4  60X,14HE = 10,000,000 16X,14HE = 30,000,000 // )
92 NOP = NOP
100 DO 1020 I=1,NRT
120 C=0.005-(0.546*(1.-EXP(-1./16.*SQRT(RT(I))))))
130 GO TO (150,210,350),N
150 SIGRE = C*(1./RT(I))
190 GO TO 490
210 IF (AB(K)*BR(J).GE.1.0) GO TO 290
230 SIGRE = 11.28*RT(I)**(-1.6) + 0.109*(RT(I)*AB(K)*BR(J))**(-1.3)
270 GO TO 490
290 SIGRE = 11.28*RT(I)**(-1.6) + 0.109*(RT(I)*AB(K)*BR(J))**(-1.3)
   + - 1.418 * RT(I)**(-1.6) * ALOG (AB(K)*BR(J))
330 GO TO 490
350 IF (AB(K)*BR(J).GE.1.0) GO TO 430
370 SIGRE = 0.01*RT(I)**(-1.6) + 0.076*(RT(I)*AB(K)*BR(J))**(-1.3)
410 GO TO 490

```

TABLE XXI - Fortran Listing - Program 3875 (Cont'd)

```

430 SIGRE = 7.52 * RT(I)**(-1.6) + 0.072 * (RT(I)*AB(K)*BR(J))**(-1.
      * - 1.418 * RT(I)**(-1.6) * ALOG (AB(K)*BR(J))
490 SIGPE = CAPK(K) / OMIMU2 * (1./(RT(I)**2 * BR(J)**2))
630 IF (SIGRE .LE. 2.*SIGPE) GO TO 730
650 IB = 2
670 SIGCRE(I,J) = SIGRE
710 GO TO 770
730 IB = 1
750 SIGCRE(I,J) = SIGPE + SIGRE**2 / (4. * SIGPE)
770 CONTINUE
780 SIGCR1(I,J) = SIGCRE(I,J) * E1
790 SIGCR2(I,J) = SIGCRE(I,J) * E2
810 IF (IBUG.NE.0) GO TO 1010
830 IF (I.LE.NRTIN) GO TO 1010
850 GO TO 1020
1010 WRITE (6,1011) RT(I),SIGCRE(I,J),SIGCR1(I,J),SIGCR2(I,J),BASIS(I)
1011 FORMAT (1H ,F17.0,3(1PE30.4),A16)
1020 CONTINUE
1030 CONTINUE
1050 IF (K1.EQ.1) GO TO 1090
1070 CALL NOVA(K)
1090 CONTINUE
2010 RETURN
2030 END
$IBFTC NOVA
      SUBROUTINE NOVA (K)
      COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKIM(30),
1      E1, E2, FMU, IBUG, K1, K2, K3, NAB, NBR, NOP, NRTIN, NET,
2      RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
3      YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
      COMMON /COMPUT/ C, IB, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
1      SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
C
C      TO PLOT THREE FRAMES WITH NBR CURVES EACH FRAME
C      LOG-LOG SCALE FOR ALL PLOTS
C
C      PLOT 1 --- SIGCR1 VS R/T      (E=10,000,000)
C      PLOT 2 --- SIGCR2 VS R/T      (E=30,000,000)
C      PLOT 3 --- SIGCRE VS R/T
C
C      CALL SETMIV(288,0,24,60)
C
C
C      CALL SUBPLT (SIGCR1, YLAB1, YBSIG, YTSIG, K )
C      CALL SUBPLT (SIGCR2, YLAB3, YBSIG, YTSIG, K )
C      CALL SUBPLT (SIGCRE, YLAB2, YBSIGE, YTSIGE, K )
      RETURN
      END
$IBFTC SUBPLT
      SUBROUTINE SUBPLT (F, YNAME, YB, YT, K)
      COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKIM(30),
1      E1, E2, FMU, IBUG, K1, K2, K3, NAB, NBR, NOP, NRTIN, NET,
2      RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
3      YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
      COMMON /COMPUT/ C, IB, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
1      SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
      DIMENSION OPT (2,3)

```

TABLE XXI - Fortran Listing - Program 3875 (Cont'd)

```

DIMENSION F(200, 20) , YNAME(12)
DATA (OPT(1,I),I=1,3)/ 12H OPTION 1      , 12H OPTION 2      ,
* 12H OPTION 3      /

```

```

C
AA= AB(K)
CC=CAPK(K)
102 XR = 1.E4
103 XL =100.
    NEN
CALL GRIDIV(4,XL,XR,YB,YT, 1., 1., 0, 0, 0, 0, 5, 6)
C
PRINT TOP TITLE, VERT,  HORIZ
CALL APRINTV (0, -16, 36, YNAME, 200, 900)
CALL PRINTV (36, XLAB1, 600, 12)
CALL PRINTV(-6, 5HA/B.= ,300, 980 )
CALL PRINTV(-6, 5H K = ,600, 980 )
CALL PRINTV(12,OPT(1,N) ,900, 980 )
CALL LABLV ( AA,360,980, 6, 1,3)
CALL LABLV (CC, 660,980,6,1,2)
CALL RITE2V (380,1012,1023,90,1, 36, -1, 36HLOCAL BUCKLING OF ISOT
*ROPIC PANELS , NLAST )
DO 150 J=1,NBR
NB=NRTIN+1
IF (BR(J)) 150,150,101
101 DO 100 I=NB,NRT
IF (F(I,J) .LE. YT) GO TO 105
100 CONTINUE
GO TO 150
105 NB=I+1
DO 106 I=NB,NRT
IY = NRT+NB -I
IF (F(IY,J) .GE. YB) GO TO 110
106 CONTINUE
GO TO 150
110 NE=IY
RTI= RT(NB-1)
FF = F (NB-1,J)
IX1 = NXV(RTI)
IY1 = NYV( FF)
DO 140 I=NB,NE
RTI= RT(I)
FF= F(I,J)
IX2 = NXV(RTI)
IY2 = NYV( FF)
DO 130 NF= 1,5
CALL LINEV (IX1, IY1, IX2, IY2)
130 CONTINUE
IX1=IX2
IY1=IY2
140 CONTINUE
150 CONTINUE
200 RETURN
END

```

```

6      END OF FILE OR DATA CARD  DATA FOLLOWS

```

4	7	0	2	4	1
.4		KS=KC=		7.0	
.5		KS=		4.0	
.6		KC=		5.7	
.8		KS=		3.29	

TABLE XXI - Fortran Listing - Program 3875 (Cont'd)

.01	.02	.05	.1	.2	.5
1.					
100.	1000.				
100.	100000.			.0001	.1

18.2 Program for the Compressive Buckling of Longitudinally Stiffened Circular Cylindrical Shells

The program developed for analyzing the panel instability mode (see Glossary) of buckling is designated as General Dynamics Convair Program No. 3896. The program determines the buckling stress for cylinders employing only longitudinal stiffening including tangent modulus and crippling stress effects. The case of buckling between rings for cylinders stiffened both longitudinally and circumferentially, including tangent modulus and crippling stress effects, is also analyzed by this program. For the latter case, it is assumed that the intermediate ring(s) are sufficient to prevent the occurrence of the general instability mode (see Glossary). A separate check employing the procedures of Section 13 is required to verify this assumption.

The input format for Program 3896 is shown in Figure 51. Symbols used are shown in Table XXII. Detailed discussion of input, card by card, follows. Runs may be stacked.

CARD TYPE 1: One card per run.

Enter Problem Identification in columns 1-72.

Alphanumeric characters.

CARD TYPE 2: One card per run.

Enter E (Young's modulus, psi) in columns 1-10 (E10.5).

Enter SIGCY (Compressive yield stress, psi) in columns 11-20 (E10.5).

80 COLUMN-GENERAL PURPOSE-WORKSHEET

3896 BUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS

L. S. FØSSUM

PROBLEM IDENTIFICATION									
E	SIGCY	SIGPL	SIGPT7	FNU	VALUENINCREASES	INLRHØX	LRHØX <sub>2</sub>	LRHØX <sub>3</sub>	LRHØX <sub>4</sub>
LRHØX	LRHØX <sub>2</sub>	LRHØX <sub>3</sub>	LRHØX <sub>4</sub>	LRHØX <sub>5</sub>	LRHØX <sub>6</sub>	LRHØX <sub>7</sub>	LRHØX <sub>8</sub>	LRHØX <sub>9</sub>	LRHØX <sub>10</sub>
CASENO	OPTION	SIGCC	C	GAMMAN	RTBAR	LHRØX	LRHØX <sub>11</sub>	LRHØX <sub>12</sub>	LRHØX <sub>13</sub>

Figure 51 - Input Format - Program 3896

Enter SIGPL (Stress at assumed proportional limit, psi) in columns 21-30 (E10.5). PRESENT PROGRAM LIMITATION REQUIRES THAT THIS VALUE NOT BE LESS THAN 20,000 PSI.

Enter SIGPT7 (Ramberg-Osgood parameter,  $\sigma_{.7}$ , psi) in columns 31-40 (E10.5).

Enter FNU (Poisson's ratio,  $\nu$ ) in columns 41-45 (F5.3).

Enter VALUEN (Ramberg-Osgood parameter,  $n$ ) in columns 46-50 (F5.1).

Enter NCASES (number of cases) as right adjusted integer(s) in columns 51-55 (I5).

Enter NLRHOX (number of  $L/\rho_{11}$  values to be read in on CARD TYPE 3 and used in TABLE and PLOTS option if called for on CARD TYPE 4) as right adjusted integer(s) in columns 56-60 (I5).

CARD TYPE 3: There will be NLRHOX/8 (rounded to higher whole number) cards per run.

Enter LRHOX (slenderness ratio,  $L/\rho_{11}$ ) values, 8 to a card (8E10.5).

CARD TYPE 4: There will be NCASES cards per run.

Enter CASENO (case number) as right adjusted integer(s) in columns 1-5 (I5).

Enter OPTION (TABLE, PLOTS, or POINT) in columns 6-10(A5).

If TABLE, 301 values of  $R/\bar{t}$  are generated evenly spaced on a logarithmic scale and calculations are made for each combination of  $R/\bar{t}$  and  $L/\rho_{11}$ .

If PLOTS, plots are made in addition to calculations of TABLE. If POINT, one set of calculations only is run using the RTBAR and LRHOX values in columns 36-45 and 46-55, respectively, on the same card.

Enter SIGCC (crippling stress,  $\sigma_{cc}$ , psi) in columns 11-20 (E10.5).

Enter C (fixity factor,  $C_F$ ) in columns 21-25 (F5.1).

Enter GAMMAN (correction factor,  $\tilde{\Gamma}$ ) in columns 26-35 (E10.5).

Enter RTBAR ( $R/\bar{t}$  for use only in POINT Option) in columns 36-45 (E10.5). Not necessary in TABLE or PLOTS options.

Enter LRHOX ( $L/\rho_{11}$  for use only in POINT option) in columns 46-55 (E10.5). Not necessary in TABLE or PLOTS options.

A sample coding form is shown in Figure 52.

Approximate execution times at the General Dynamics Convair 7091 DCS installation were 2 seconds per  $L/\rho_{11}$  value for PLOTS option.

The program output consists of a listing or a listing plus plots depending on OPTION specified on CARD TYPE 4. If TABLE or PLOTS options are called for, the listing consists of all input plus a table of 301  $R/\bar{t}$  values with corresponding buckling stress values for each specified value of slenderness ratio  $L/\rho_{11}$ . A separate table is constructed for each input combination of crippling stress  $\sigma_{cc}$ , fixity

80 COLUMN-GENERAL PURPOSE-WORKSHEET

3896 BUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS L.S. F0SSUM

2024 T351 LONGITUDINALLY STIFFENED CYLINDERS OF REFERENCE 4

10.5	+0638.0	+0320.0	+0337.0	+03	.33	10.0	4
1	PRINT47.5	+03 3.75	1.497	.479	+03.368		+03
2			.5260	.495	+03.368		+03
3			1.826	.501	+03.230		+03
4	PRINT47.5	+03 3.75	3.590	.488	+03.230		+03

GENERAL DYNAMICS  
Convair Division

Figure 52 - Sample Input Data - Program 3896

factor  $C_F$ , and correction factor  $\tilde{\Gamma}^N$ . For the POINT option, the same quantities are listed but only for the specified input  $R/\bar{t}$  and  $L/\rho_{11}$  combinations. A sample output listing of the POINT option is shown in Figure 53 for the sample input data of Figure 51. Sample S.C. 4020 plots from the PLCTS option appear in Section 12.2. It should be noted that although the program can accommodate a wide variety of materials through the input material properties, the plots will always bear the title "COMPRESSIVE BUCKLING STRESS FOR LONGITUDINALLY STIFFENED 7075-T6 AL ALLOY CIRCULAR CYLINDERS". Changes must therefore be made to the titles of plots when run for other materials.

Notation used in program 3896 vs. symbols employed in Parts I and II of this report appears in Table XXII. A basic flow diagram for the program is given in Figure 54 and a Fortran listing of the program, including input data from the sample problem (Figure 52), is given in Table XXIII.

BUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS

2024 T351 LONGITUDINALLY STIFFENED CYLINDERS OF REFERENCE 4

INITIAL E, PSI	COMPRESSIVE YIELD STRESS, PSI	ELASTIC LIMIT STRESS, PSI	SIGMA PT SEVEN, PSI	POISSONS RATIO	N	NUMBER OF CASES	NO. SLENDERNESS RATIOS
1.0500E 07	3.8000E 04	2.0000E 04	3.7000E 04	0.330	10.0	4	-1
CASE NO.	CRIPPLING STRESS, PSI	FIXITY FACTOR	CORRECTION FACTOR	RADIUS/T BAR	SLENDERNESS RATIO		BUCKLING STRESS, PSI
1	4.750E 04	3.750E 00	1.49700E 00	4.750E 02	3.6800E 02		2.1347E 04
CASE NO.	CRIPPLING STRESS, PSI	FIXITY FACTOR	CORRECTION FACTOR	RADIUS/T BAR	SLENDERNESS RATIO		BUCKLING STRESS, PSI
2	4.750E 04	3.750E 00	5.26000E-01	4.950E 02	3.6800E 02		9.6937E 03
CASE NO.	CRIPPLING STRESS, PSI	FIXITY FACTOR	CORRECTION FACTOR	RADIUS/T BAR	SLENDERNESS RATIO		BUCKLING STRESS, PSI
3	4.750E 04	3.750E 00	1.82600E 00	5.010E 02	2.3000E 02		2.4552E 04
CASE NO.	CRIPPLING STRESS, PSI	FIXITY FACTOR	CORRECTION FACTOR	RADIUS/T BAR	SLENDERNESS RATIO		BUCKLING STRESS, PSI
4	4.750E 04	3.750E 00	3.59000E-01	4.880E 02	2.3000E 02		1.2071E 04

Figure 53 - Sample Output Listing - Program 3896

TABLE XXII - Program 3896 Notation

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
C	$C_F$	Fixity Factor.
CASENO	-	Case number.
CONST1	$\sqrt{3(1-\nu^2)}$	
E	E	Young's Modulus, psi.
ETANCY	$(E_{\tan})_{cy}$	Tangent modulus at compressive yield stress, psi.
FNU	$\nu$	Poisson's Ratio.
GAMMAN	$\tilde{\Gamma}_N$	Correction Factor.
ISTOP	-	Indicates R/t value at which no further calculations are made for that $L/\rho_{11}$ .
LRHOX	$L/\rho_{11}$	Slenderness ratio.
RTBAR	$R/\bar{t}$	Radius/effective thickness ratio.
SIGCY	$\sigma_{cy}$	Compressive yield stress, psi.
SIGPL	$\sigma_{PL}$	Stress at assumed proportional limit.
SIGPT7	$\sigma_{.7}$	Ramberg-Osgood parameter.
SIGCC	$\sigma_{cc}$	Crippling stress.
SIGCR	$\sigma_{cr}$	Buckling stress.
SCRCY	$(\sigma_{cr})_{cy}$	Buckling stress using $E_{\tan} = (E_{\tan})_{cy}$
SWCCY	$(\sigma_{wc})_{cy}$	Wide column buckling stress using $E_{\tan} = (E_{\tan})_{cy}$
VALUEN	n	Ramberg-Osgood parameter.
YT	-	Upper limit of plotting grid.

GENERAL DYNAMICS  
Convair Division

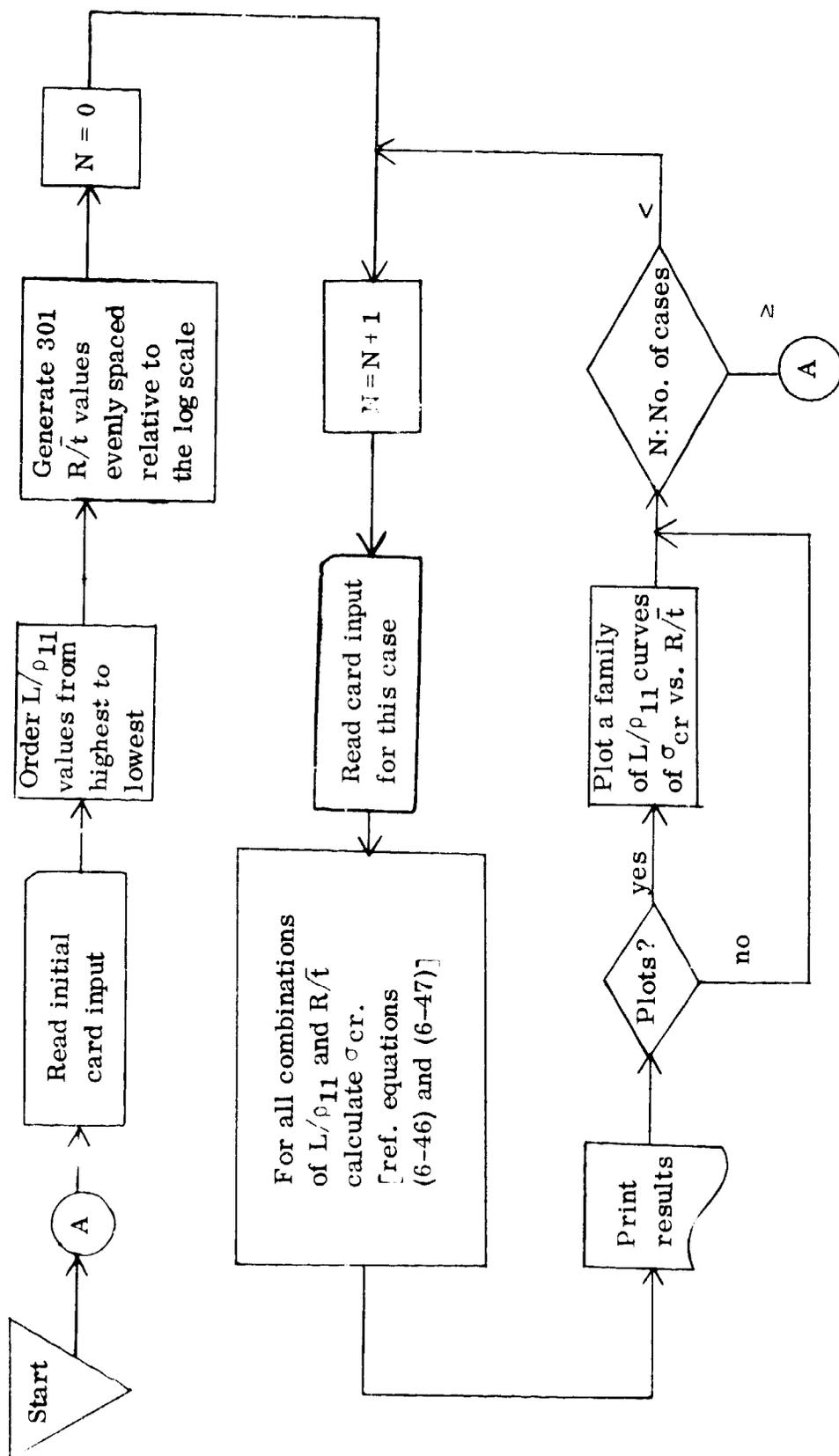


Figure 54 - Flow Diagram - Program 3896

TABLE XXIII - Fortran Listing - Program 3896

```

$SETUP LB4      DISK,SIGPLT,SAVE
$EXECUTE       IBJOB
$IBJOB
$IBFTC MAIN
COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
1      NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
2      SIGPT7, SIGCC, SIGCR(302,25), SRCY, VALUEN, YT,
3      PROPID(12), PI,CONST1
REAL LRHOX, LRHOXD
DATA PLOTS, TABLE, POINT /5HPLOTS,5HTABLE,5HPOINT/
50 WRITE (6,51)
51 FORMAT (1H1,42X,46HBUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS)
60 READ (5,61) (PROPID(K),K=1,12)
61 FORMAT (12A6)
70 WRITE (6,71) (PROPID(K),K=1,12)
71 FORMAT (//// 29X,12A6 ////)
80 WRITE (6,81)
81 FORMAT (7X, 10HINITIAL E, 5X, 11HCOMPRESSIVE 5X, 13HELASTIC LIMIT
1      5X, 8HSIGMA PT 10X, 8HPOISSONS 21X, 6HNUMBER 3X,
2      15HNO. SLENDERNESS /10X, 3HPSI 7X, 16HYIELD STRESS,PSI
3      3X, 10HSTRESS,PSI 6X, 10HSEVEN, PSI 11X, 5HRATIO 13X, 1HN
4      7X, 8HOF CASES 7X, 6HRATIO5 )
90 READ (5,91) E, SIGCY, SIGPL, SIGPT7, FNU, VALUEN, NCASES, NLRHOX
91 FORMAT (4E10.5,F5.3,F5.1,2I5)
100 WRITE (6,101)E, SIGCY, SIGPL, SIGPT7, FNU, VALUEN, NCASES, NLRHOX
101 FORMAT (1H0, 1P4E16.4, 0PF16.3, F16.1, I10, I14 )
105 IF (NLRHOX.EQ.0) GO TO 140
110 WRITE (6,111)
111 FORMAT (//// 47X, 37HSLENDERNESS RATIOS FOR AUTOMATIC SEQ. // )
120 READ (5,121) (LRHOX(K),K=1,NLRHOX)
121 FORMAT ( 8E10.5)
130 WRITE (6,131) (LRHOX(K),K=1,NLRHOX)
131 FORMAT ( 1P6E20.5 )
140 CONST1 = SQRT ( 3. *(1. - FNU**2 ) )
141 PI=3.14159
142 CALL SETMIV(125,0,160,176)
143 CALL SMXYV(1,0)
C
C      CALCULATE UPPER LIMIT OF GRID
XX=SIGCY+15000.
YT=XX-AMOD(XX,10000.)
150 CALL SEARCH (LRHOX,NLRHOX)
151 CALL RTBARC (RTBAR,NRTBAR)
160 DO 255 NREAD=1,NCASES
C      STORE R/TBAR FOR POINT OPTION IN RTBAR(302) AND L/RX IN LRHOX(25).
170 READ (5,171) CASENO,OPTION,SIGCC,C,GAMMAN,RTBAR(302),LRHOX(25)
171 FORMAT (15,A5,E10.5,F5.1,3E10.5)
175 NOP=0
180 IF(PLOTS.EQ.OPTION) NOP=1
190 IF(TABLE.EQ.OPTION) NOP=2
200 IF(POINT.EQ.OPTION) NOP=3
210 IF(NOP.NE.0) GO TO 250
220 WRITE (6,221) CASENO
221 FORMAT (//// 49X, 28HILLEGAL OPTION FOR CASE NO. ,I5)

```

TABLE XXIII - Fortran Listing - Program 3896 (Cont'd)

```

230 GO TO 255
250 CALL COMPUT
255 CONTINUE
260 GO TO 50
270 END
$IBFTC SEARCH
  SUBROUTINE SEARCH (LRHOX,NLRHOX)
  DIMENSION LRHOX(25)
C   TO PLACE INPUT L/RX IN ORDER FROM HIGHEST TO LOWEST.
  REAL LRHOX, LRHOXD
  100 NLRX1=NLRHOX-1
  150 DO 650 K=1,NLRX1
  200 TEMP=LRHOX(K)
  250 K1=K+1
  275 ITRANS=0
  300 DO 500 I=K1,NLRHOX
  350 IF (TEMP.GE.LRHOX(I)) GO TO 500
  375 ITRANS=1
  400 NBLANK=I
  450 TEMP=LRHOX(I)
  500 CONTINUE
  525 IF (ITRANS.EQ.0) GO TO 650
  550 LRHOX(NBLANK)=LRHOX(K)
  600 LRHOX(K)=TEMP
  650 CONTINUE
  700 RETURN
  750 END
$IBFTC RTBARC
  SUBROUTINE RTBARC ( RTBAR, NRTBAR )
C   TO GENERATE 301 VALUES OF RTBAR FROM 10 TO 10,000
C   EVENLY SPACED RELATIVE TO LOG SCALE.
  DIMENSION RTBAR(301)
  100 NRTBAR = 301
  200 RTBAR(1) = 10000.
  300 RTBAR(301) = 10.
  400 DO 800 I = 1,299
  500 I1 = 301-I
  600 EXP = 1.0 + FLOAT(I) * .01
  700 RTBAR(I1) = 10. **EXP
  800 CONTINUE
  900 RETURN
  1000 END
$IBFTC COMPUT
  SUBROUTINE COMPUT
C
  COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
  1   NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
  2   SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
  3   PROPID(12), PI,CONST1
  REAL LRHOX, LRHOXD
C
C   THIS SUBROUTINE WILL DO THE FOLLOWING ACCORDING TO THE OPTION
C
C   POINT OPTION
C   THE BUCKLING STRESS VALUE IS COMPUTED FOR THE INPUT L/RX AND R/T.
C
C   TABLE OPTION
C   301 R/TBARS ARE GENERATED FROM 10 TO 10,000 (IN SUB. RTBARC).

```

TABLE XXIII - Fortran Listing - Program 3896 (Cont'd)

```

C      BUCKLING STRESS VALUES ARE CALCULATED FOR EACH L/RX AND R/TBAR.
C
C      PLOTS OPTION
C      SAME AS TABLE OPTION. IN ADDITION A PLOT OF BUCKLING STRESS VS.
C      R/TBAR IS MADE FOR EACH L/RX VALUE.
C
C      STEP NOS. REFER TO STEPS IN OUTLINE OF PROBLEM WRITTEN
C      BY GEORGE SMITH. SEE DOCUMENTATION.
C
C
100 GO TO (125,125,135),NOP
C
C      LOOP FOR TABLE AND PLOTS OPTIONS.
C
125 J1=1
126 J2=NLRHOX
127 I1=1
128 I2=301
129 GO TO 150
C
C      POINT OPTION. CALCULATE ONLY FOR ONE POINT USING SPECIAL VALUES
C      STORED IN RTBAR(302) AND LRHOX(25).
C
135 J1=25
136 J2=25
137 I1=302
138 I2=302
C
C      STEPS 1 - 3
150 LRHOXD = SQRT(2.*C) * PI * SQRT(E/SIGCC)
160 ETANCY = E * (1./(VALUEN * (SIGCY/SIGPT7)**(VALUEN-1.) + 1.))
170 DO 2080 J=J1,J2
C
C      STEPS 5 AND 6
190 IF (LRHOX(J).GE.LRHOXD) GO TO 300
C
C      PROCEED AS IN STEPS 22 - 41 (IG0=2)
200 IG0 = 2
C
C      STEPS 22, 23, AND 24
210 STEP22 = (C * PI**2 * ETANCY)/(LRHOX(J)**2)
220 STEP23 = SIGCC - ( (SIGCC**2 * LRHOX(J)**2)/(4.*C*PI**2 * E) )
230 SWCCY = AMINI(STEP22,STEP23)
240 AA = SWCCY
250 GO TO 325
C
C      PROCEED AS IN STEPS 7 - 21 (IG0=1)
300 IG0 = 1
310 AA = (C * PI**2 * ETANCY) / (LRHOX(J)**2)
C
C      STEP 4
325 DO 2060 I=I1,I2
C
C      STEPS (7 AND 8) AND (25 AND 26)
350 SCRCY = AA + GAMMAN * (ETANCY/CONST1) * (1./RTBAR(I))
360 IF (SCRCY - SIGCY) 450,370,400
C
C      STEPS 10 AND 28

```

TABLE XXIII - Fortran Listing - Program 3896 (Cont'd)

```

370 SIGCR(I,J) = SCPLY
   IF (SIGCR(I,J).LE.SIGCC) GO TO 2060
   ISTOP(J)=1
   GO TO 2080
C
C   STEPS 9 AND 27
400 IF (1.E0.302) GO TO 425
420 ISTOP(J)=1
421 GO TO 2080
425 ISTOP(J)=303
430 GO TO 2080
C
C   STEPS (11 - 16) AND (29 - 34)
450 GO TO (460,500),160
460 BB = (C * PI**2 * E) / LRHOX(J)**2
470 GO TO 550
500 BB = SIGCC - (SIGCC**2 * LRHOX(J)**2) / (4. * C * PI**2 * E)
550 SCRBB = LB + GAMMAN * (E / CONST1) * (1./RTBAR(I))
560 IF (SCRBB.GT.SIGPL) GO TO 600
570 SIGCR(I,J) = SCRBB
   IF (SIGCR(I,J).LE.SIGCC) GO TO 2060
   ISTOP(J)=1
   GO TO 2080
600 IF (SCRBB.GE.SIGCY) GO TO 660
620 SCRNEW = SCRBB
640 GO TO 680
660 SCRNEW = SIGCY
C
C   STEPS (17 - 21) AND (35 -41)
C
C   INITIALIZE
680 X = 12500.
700 SCRNEW = SCRNEW + X
720 NCNT = 0
C
C   ENTER SCHEME TO CLOSE IN ON OUTPUT VALUE
740 NCNT = NCNT + 1
760 SCRNEW = SCRNEW - X
780 ETANNU = E*(1./(VALUEN * (SCRNEW/SIGHT7)**(VALUEN-1.) + 1.))
800 GO TO (920,820),160
820 STEP36 = (C * PI**2 * ETANNU) / LRHOX(J)**2
840 STEP37 = SIGCC - (SIGCC**2 * LRHOX(J)**2) / (4. * C * PI**2 * E)
860 SWC36 = AMIN1(STEP36,STEP37)
880 CC = SWC36
900 GO TO 940
920 CC = (C * PI**2 * ETANNU) / LRHOX(J)**2
C
C   STEPS 18 AND 39 (EFN 940)
940 SCRNU = CC + GAMMAN * (ETANNU/CONST1) * (1./RTBAR(I))
960 IF (SCRNU.LT.SCRNEW) GO TO 740
1000 IF (NCNT.EQ.1) GO TO 1020
1010 IF (X.GT.100.) GO TO 1080
1020 SIGCR(I,J) = SCRNEW
   IF (SIGCR(I,J).LE.SIGCC) GO TO 2060
   ISTOP(J)=1
   GO TO 2080
1080 SCRNEW = SCRNEW + X
2010 X = X/5.

```

TABLE XXIII - Fortran Listing - Program 3896 (Cont'd)

```

2020 GO TO 760
2060 CONTINUE
2070 ISTOP(J)=302
2080 CONTINUE
3000 CALL OUTPUT (J1, J2, I1, I2)
3020 GO TO (4000,5000,5000),NOP
4000 CALL SIGPLT
5000 RETURN
6000 END
$IBFTC OUTPUT
SUBROUTINE OUTPUT(J1, J2, I1, I2)
COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
1 NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
2 SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
3 PROPID(12), PI,CONST1
REAL LRHOX, LRHOXD
C
C SUBROUTINE TO PRINT OUT OUTPUT.
C
100 WRITE (6,101)
101 FORMAT ( // 20X,9HCRIPLING 12X, 6HFIXITY 9X, 10HCORRECTION 28X,
1 11HSLENDERNES 9X, 8HBUCKLING // 5X, 8HCASE NO. 6X,
2 11HSTRESS, PSI 11X, 6HFACTOR 11X, 6HFACTOR 11X,
3 12HRADIUS/T BAR 10X, 5HRATIO 11X, 11HSTRESS, PSI // )
150 IF (ISTOP(J1).EQ.1) GO TO 200
175 IF (ISTOP(J1).NE.303) GO TO 300
200 WRITE (6,201) CASENO,SIGCC,C,GAMMAN,LRHOX(J1)
201 FORMAT (1H ,I9,1P2E19.3,1PE19.5,19X,
* 27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
250 GO TO (500,500, 900),NOP
300 WRITE (6,301) CASENO,SIGCC,C,GAMMAN,RTBAR(I1),LRHOX(J1),
* SIGCR(I1,J1)
301 FORMAT (1H ,I9,1P2E19.3,1PE19.5,1PE19.3,1P2E19.4)
IF (NOP.EQ.3) GO TO 900
399 II=ISTOP(I)-1
400 WRITE (6,401) (RTBAR(I),LRHOX(I), SIGCR(I,1), I=2,II)
401 FORMAT (67X, 1PE19.3, 1P2E19.4)
425 WRITE (6,426) LRHOX(1)
426 FORMAT (1H ,74X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
500 DO 800 J=2,J2
550 IF (ISTOP(J).NE.1) GO TO 675
600 WRITE (6,601) LRHOX(J)
601 FORMAT(/// 75X, 27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
650 GO TO 800
675 II=ISTOP(J)-1
700 WRITE (6,701) (RTBAR(I),LRHOX(J),SIGCR(I,J),I=1,II)
701 FORMAT (///(66X,1PE19.3,1P2E19.4))
750 WRITE (6,751) LRHOX(J)
751 FORMAT (75X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
800 CONTINUE
900 RETURN
1000 END
$IBFTC OUTPUT
SUBROUTINE OUTPUT(J1, J2, I1, I2)
COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
1 NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
2 SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
3 PROPID(12), PI,CONST1

```

TABLE XXIII - Fortran Listing - Program 3896 (Cont'd)

REAL LRHOX, LRHOXD

SUBROUTINE TO PRINT OUT OUTPUT.

```

C
C
C
100 WRITE (6,101)
101 FORMAT ( /// 20X,9HCRIPPLING 12X, 6HFIXITY 9X, 10HCORRECTION 28X,
1      11HSLENDERNES 9X, 8HBUCKLING // 5X, 8HCASE NO. 6X,
2      11HSTRESS, PSI 11X, 6HFACTOR 11X, 6HFACTOR 11X,
3      12HRADIUS/T BAR 10X, 5HRATIO 11X, 11HSTRESS, PSI ///)
150 IF (ISTOP(J1).EQ.1) GO TO 200
175 IF (ISTOP(J1).NE.303) GO TO 300
200 WRITE (6,201) CASENO,SIGCC,C,GAMMAN,LRHOX(J1)
201 FORMAT (1H I9,1P2E19.3,1PE19.5,19X,
*      27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
250 GO TO (500,500, 900),NOP
300 WRITE (6,301) CASENO,SIGCC,C,GAMMAN,RTBAR(I1),LRHOX(J1),
*      SIGCR(I1,J1)
301 FORMAT (1H ,I9,1P2E19.3,1PE19.5,1PE19.3,1P2E19.4)
IF (NOP.EQ.3) GO TO 900
399 II=ISTOP(1)-1
400 WRITE (6,401) (RTBAR(I),LRHOX(1), SIGCR(I,1), I=2,II)
401 FORMAT (67X, 1PE19.3, 1P2E19.4)
425 WRITE (6,426) LRHOX(1)
426 FORMAT (1H ,75X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
475 IF(NLRHOX.EQ.1) GO TO 900
500 DO 800 J=2,J2
550 IF (ISTOP(J).NE.1) GO TO 675
600 WRITE (6,601) LRHOX(J)
601 FORMAT(/// 76X, 27HNO CALCULATIONS FOR L/RX = ,1PE12.4)
650 GO TO 800
675 II=ISTOP(J)-1
700 WRITE (6,701) (RTBAR(I),LRHOX(J),SIGCR(I,J),I=1,II)
701 FORMAT (///(66X,1PE19.3,1P2E19.4))
750 WRITE (6,751) LRHOX(J)
751 FORMAT (76X,34HNO FURTHER CALCULATIONS FOR L/RX= ,1PE12.4)
800 CONTINUE
900 RETURN
1000 END
SIBFTC SIGPLT
SUBROUTINE SIGPLT
COMMON C, CASENO, E, FNU, GAMMAN, ISTOP(25), LRHOX(25), NLAST,
1      NCASES, NLRHOX, NOP, OPTION, RTBAR(302), SIGCY, SIGPL,
2      SIGPT7, SIGCC, SIGCR(302,25), SCRCY, VALUEN, YT,
3      PROPID(12), PI,CONST1
REAL LRHOX, LRHOXD
USED FOR PLOTS OPTION. WILL PLOT SIGCR VS R/TBAR FOR VARIOUS L/RX.
C
C
C
SET GRID
100 CALL GRIDIV (4,10.,10000.,0.,YT,1.,2000.,0, 5,-10,- 5,6,6)
C
C
PRINT SIGCC, C, AND GAMMA N AT TOP
150 CALLPRINTV (-19,19HCRIPPLING STRESS = ,192,876)
151 CALL LABELV (SIGCC,344,876,-4,1,1)
160 CALL PRINTV(-16,16HFIXITY FACTOR = ,524,876)
161 CALL LABELV (C,652,876,5,1,3)
170 CALL PRINTV(-20,20HCORRECTION FACTOR = ,784,876)
171 CALL LABELV (GAMMAN,944,876,7,1,3)
C

```

TABLE XXIII - Fortran Listing - Program 38'S (Cont'd)

```

C   PRINT SIGCR TITLE DOWN SIDE
C
180 CALL APRNTV(0,-14,-19,19HBUCKLING STRESS PSI,76,637)
C   PRINT R/T BAR AT BOTTOM
190 CALL PRINTV(-14,14HRADIUS / T BAR,551,120)
C
C   PRINT MAIN TITLE AT BOTTOM
200 CALL RITE2V(328,77,1023,90,1,31,-1,31HCOMPRESSIVE BUCKLING STRESS
*FOR,NLAST)
201 CALL RITE2V(319,45,1023,90,1,32,-1,32HLONGITUDINALLY STIFFENED 707
*5-T6,NLAST)
202 CALL RITE2V(364,13,1023,90,1,27,-1,27HAL ALLOY CIRCULAR CYLINDERS,
*NLAST)
C
C   PLOT CURVES
300 DO 700 J=1,NLRHOX
310 IF (ISTOP(J).EQ.1) GO TO 700
320 RTB = RTBAR(1)
330 SIG = SIGCR(1,J)
340 IX1 = NXV(RTB)
350 IY1 = NYV(SIG)
    II=ISTOP(J)-1
400 DO 600 I=2,II
410 RTB = RTBAR(I)
420 SIG = SIGCR(I,J)
440 IX2 = NXV(RTB)
450 IY2 = NYV(SIG)
500 DO 501 NN=1,3
501 CALL LINEV(IX1,IY1,IX2,IY2)
550 IX1 = IX2
560 IY1 = IY2
600 CONTINUE
700 CONTINUE
C
C   PLOT CUT OFF LINE AT SIGMA CC FROM X=10 TO X OF LAST POINT PLOTTED
750 NSCC=NYV(SIGCC)
775 NTEN=NXV(10.)
800 DO 801 MM=1,3
801 CALL LINEV(NTEN, NSCC, IX1, NSCC)
900 RETURN
1000 END
C
C   END OF FILE OR DATA CARD DATA FOLLOWS
2024 T351 LONGITUDINALLY STIFFENED CYLINDERS OF REFERENCE 4
10.5  +0638.0  +0320.0  +0337.0  +03  .33 10.0  4
    1POINT47.5  +03 3.751.497  .479  +03.368  +03
    2POINT47.5  +03 3.75.5260  .495  +03.368  +03
    3POINT47.5  +03 3.751.826  .501  +03.230  +03
    4POINT47.5  +03 3.75.3590  .488  +03.230  +03

```

18.3 Programs for General Instability of Orthotropically Stiffened Circular Cylindrical Shells Subjected to Axial Compression

18.3.1 Thielemann Solution - The Thielemann solution for general instability of stiffened circular cylindrical shells under axial compression, as discussed in Section 7, was programmed to obtain the design curves shown in Section 13.2. The program also provides the capability of solution for particular configurations and was so applied in the test data comparisons of Section 7.3. The program is designated as General Dynamics Convair Program No. 3942.

The governing stability relationship for the programmed solution is given by equation (7-25) which relates the buckling load parameter  $\bar{N} = \frac{1}{2} N_x R \sqrt{A_{11}/D_{22}}$  to  $D/a_{22}$ ,  $a/\rho_{11}$ ,  $a/D$ ,  $C_R$ ,  $\eta_p$ ,  $\eta_s$ , and  $\beta$ . To find the critical buckling stress value, the minimum value of  $N_x$  must be found. To do this, the expression for  $\bar{N}$  [equation (7-25)] was minimized with respect to the wave length parameter  $\beta^2$  resulting in the following expression which may be recognized to be a quadratic in  $\beta^2$ :

$$\left[ (C_R \gamma) \eta_s - \frac{\eta_p}{\sqrt{C_R}} \sqrt{C_R \gamma} \right] \beta^4 + \left[ C_R \gamma - 1 \right] \beta^2 + \left[ \frac{\eta_p}{\sqrt{C_R}} \sqrt{C_R \gamma} - \eta_s \right] = 0 \quad (18-1)$$

The roots of this equation give values of  $\beta^2$  which are employed in minimizing equation (7-25). In the programmed solution procedure, the discriminant of the quadratic equation (18-1) is denoted  $B$  and the roots of the equation are designated  $X_1 (= \beta_1^2)$  and  $X_2 (= \beta_2^2)$ .  $\bar{N}_1$  and  $\bar{N}_2$  values are obtained using the  $X_1$  and  $X_2$  solutions respectively in equation (7-25). The values  $\bar{N}_3 = \sqrt{C_R \gamma}$  and  $\bar{N}_4 = 1$  are also calculated.

To establish the minimum applicable  $\bar{N}$  value, the logic shown in the flow diagram, Figure 58, is followed. The resulting modes are identified as follows:

- If  $\bar{N}_1$  is lowest, the checkerboard mode predominates.
- If  $\bar{N}_2$  is lowest, the checkerboard mode predominates.
- If  $\bar{N}_3$  is lowest, the axisymmetric mode predominates.
- If  $\bar{N}_4$  is lowest, the "Zero Beta" mode predominates.

The checkerboard and axisymmetric modes are as conventionally defined and the "Zero Beta" mode is a bounding condition characterized by nonaxisymmetric modes which approach infinite axial half-wave lengths.

TO DATE THE PROGRAM HAS BEEN DEBUGGED ONLY FOR THE FOLLOWING SITUATIONS:

SIGN OF B	SIGN OF $X_1$	SIGN OF $X_2$	CRITICAL $\bar{N}$
+	-	-	$\bar{N}_4$
+	-	+	$\bar{N}_4$
+	+	-	$\bar{N}_1$
-	Not Applicable	Not Applicable	$\bar{N}_3$
-	Not Applicable	Not Applicable	$\bar{N}_4$

The minimization logic presently built into the program restricts its use to cases where  $\eta_p = 0$ . This rather special condition will always result in the third tabulated combination (positive B, positive  $X_1$ , negative  $X_2$ , critical  $\bar{N} = \bar{N}_1$ ).

Since only the general instability mode is evaluated using this program, other possible modes of failure must be checked. For example, the panel instability mode must be examined using the procedures of Section 12.

The input format for program 3942 is shown in Figure 55. Symbols used are shown in Table XXIV. Detailed discussion of input, card by card, follows. Runs may be stacked.

CARD TYPE 1: One card per run.

Enter PROPID (problem identification) in columns 1-72.

Alphanumeric characters.

CARD TYPE 2: One card per run.

Enter NCASES (number of cases) as right adjusted integer(s) in columns 1-5 (I5).

Enter NRING (number of  $\sqrt{C_R} D / \rho_{22}$  values) as right adjusted integer(s) in columns 6-10 (I5).  $0 \leq \text{NRING} \leq 21$ .

Enter IRITPT (printout option) as right adjusted integer in columns 11-15. If  $\text{IRITPT} \neq 0$ , values of  $\bar{N}_1$ ,  $\bar{N}_2$ ,  $\bar{N}_3$ , and  $\bar{N}_4$  will be printed out for all POINT cases in the run.



If IRITPT = 0 (or is blank), these values will not be printed.

Enter IR1OR2 (printout option) as right adjusted integer in columns 16-20. Same as IRITPT except applies to TABLE and PLOTS cases.

CARD TYPE 3: There will be NRING/7 (rounded to higher integer) cards per run. When NRING = 0, there must be one blank CARD TYPE 3 per run.

Enter RINGRA  $\left(\sqrt{C_R} D/\rho_{22}\right)$  values, 7 to a card (7E10.5). These are values used in automatic sequencing for TABLE and PLOTS options.

CARD TYPE 4: There will be NCASES cards per run.

Enter CASENO (case number) in columns 1-4. Alphanumeric characters. Do not use column 5.

Enter OPTION (option for PLOTS, TABLE or POINT solutions) in columns 6-10. If PLOTS, output values will be calculated and listed for all combinations of the input  $\sqrt{C_R} D/\rho_{22}$  values on CARD TYPE 3 and 301 internally generated  $a/\rho_{11}$  values. In addition, plots of  $\bar{N}$  vs.  $a/\rho_{11}$  for each  $\sqrt{C_R} D/\rho_{22}$  value will be made.

Enter RINGSP (a/D) value in columns 11-20 (E10.5).

Enter ETAS ( $\eta_s$ ) value in columns 21-30 (E10.5).

Enter ETAP ( $\eta_p$ ) value in columns 31-40 (E10.5).

PRESENT PROGRAM LOGIC REQUIRES THAT THIS VALUE BE ZERO (OR BLANK).

Enter LONGRA ( $a/\rho_{11}$  value to be used in POINT option) in columns 41-50 (E10.5). May be blank if OPTION is not POINT.

Enter RINGRA ( $\sqrt{C_R} D/\rho_{22}$  value to be used in POINT option) in columns 51-60 (E10.5). May be blank if OPTION is not POINT.

Enter CR ( $C_R$ ) value in columns 61-70 (E10.5). If  $\eta_p = 0$ , CR may be left blank even though  $C_R \neq 0$ .  $C_R$  values may be obtained from Figure 35 of Section 13.1 or from the program of Section 18.3.3.

A sample input coding form is shown in Figure 56.

Approximate execution times at the General Dynamics Convair 7094 DCS installation were 2.75 seconds per  $a/\rho_{11}$  value for PLOTS option.

The output consists of a listing of all input and calculated data or listing plus plots depending on OPTION specified on CARD TYPE 4. If TABLE or PLOTS options are called for, the listing includes a table of 301  $a/\rho_{11}$  values with corresponding values of  $B$ ,  $X_1$ ,  $X_2$ ,  $\beta$ ,  $\bar{N}$ , and the mode of buckling. Double precision is used in the evaluation of  $B$ ,  $X_1$ , and  $X_2$ . A separate table is constructed for each value of  $\sqrt{C_R} D/\rho_{22}$  specified. For the POINT option, the same quantities are listed but only for the input  $a/\rho_{11}$  and  $\sqrt{C_R} D/\rho_{22}$  values specified on CARD TYPE 4. A sample output listing of the POINT option is shown in Figure 57 for the



sample input data of Figure 55. Sample S.C. 4020 plots from the PLOTS option appear in Section 13.2. Notation used in program 3942 vs. symbols employed in Parts I and II of this report appears in Table XXIV. A basic flow diagram for the program is given in Figure 58 and a Fortran listing of the program, including input data from the sample problem (Figure 56), is given in Table XXV.

CASE NO.	NON-DIM. RING SPACING	FTA SUR S	FTA SUR P	C SUR R
1	7.7721E-02	1.8740E 00	-C.	
	CORRECTED HOOP STRENGTH RATIO	B	BETA	MODE
LONGITUDINAL STRENGTH RATIO	3.7732E 02	1.59860 01	1.00170 00	5.80230-01
2.90390 01	1.00340 00	-1.00100 00		CHKBOARD
	N BAR 1	N BAR 2	N BAR 3	N BAR 4
5.8922546554481360-01			5.0781570222378300-01	10.00000000000000-01

GENERAL DYNAMICS  
Convair Division

Figure 57 - Sample Output Listing - Program 3942

TABLE XXIV - Program 3942 Notation

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
B	B	Discriminant of the quadratic equation whose roots are X1 and X2.
BETA	$\beta$	Wave length parameter.
CASENO	-	Case number.
CR	$C_R$	Correction factor
CRGAM	$C_R \gamma$	
ETAP	$\eta_p$	
ETAS	$\eta_s$	
FMODE	-	Alphanumeric words for the 5 modes (see block data).
IFLAG	-	1, 2, 3, 4, or 5 denotes mode to be printed out.
IPRNTX	-	IPRNTX(I) = 0: print X1 and X2. IPRNTX(I) = 1: print imaginary for X1 and X2.
ISTOP	-	ISTOP = I at which stop calculations.
LONGRA	$a/\rho_{11}$	Longitudinal slenderness ratio. 301 generated for NOP = 1 and 2.
NCASES	-	Number of cases.
NRING	-	Number of $\sqrt{C_R} D/\rho_{22}$ values input for use with TABLE or PLOTS option.
NBAR	$\bar{N}$	Buckling load parameter.
NOP	-	Options: PLOTS NOP = 1 TABLE NOP = 2 POINT NOP = 3

GENERAL DYNAMICS  
Convair Division

TABLE XXIV - Program 3942 Notation (Cont'd)

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
OPTION	-	PLOTS, TABLE, or POINT
PROBID	-	Problem identification
RINGRA	$\sqrt{C_R} D / p_{22}$	Ring ratios, input, up to 21.
RINGSP	a/D	Ring spacing/diameter ratio.
SCRGAM	$\sqrt{C_R} \gamma$	
SQRTCR	$\sqrt{C_R}$	
X1	$X_1$	$\beta_1^2$
X2	$X_2$	$\beta_2^2$

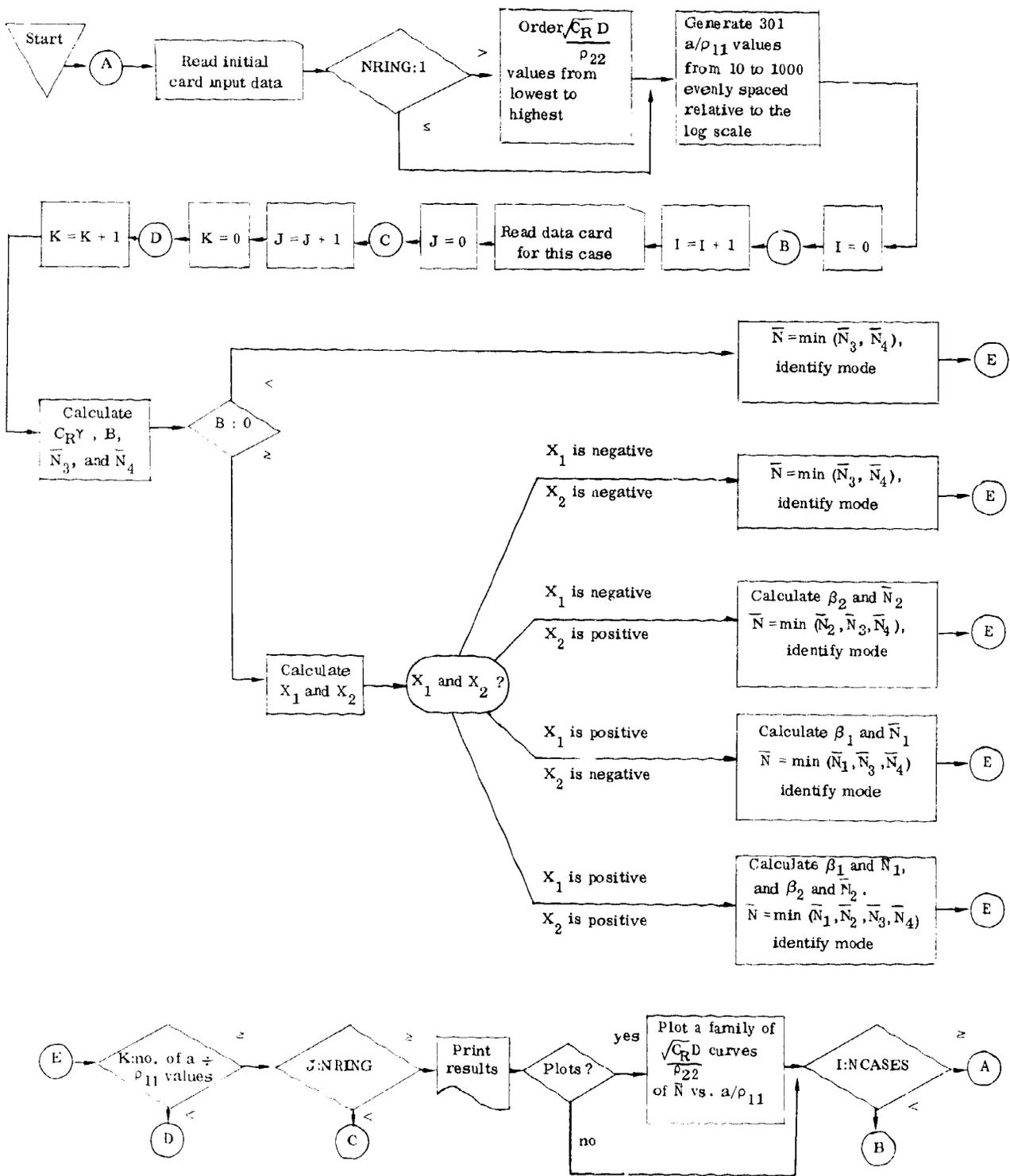


Figure 58 - Flow Diagram - Program 3942

TABLE XXV - Fortran Listing - Program 3942 (Cont'd)

```

$SETUP LB4      LISK,SCPLOT,SAVE
$EXECUTE       1BJOB
$1BJOB
$IBFIC MAIN    LIST
COMMON /INPUT/  CASENO,CR,CRGAM,ETAP,ETAS,LONGRA(302),NRING,
1              RINGRA(22),RINGSP,IRITPT,IR1OR2
COMMON /OPTION/ PLOTS, PLOT,POINT,TABLE
COMMON /MISC/   IFLAG(302),ISTOP,NOP,J
DIMENSION PROID(12)
DOUBLE PRECISION CRGAM, LONGRA
C
C
50 CALL SETMIV (50,0,160,176)
51 CALL SMXYV (1,0)
100 READ (5,101) (PROID(I),I=1,12)
101 FORMAT (12A6)
150 WRITE (6,151) (PROID(I),I=1,12)
151 FORMAT (1H1,20(/),19X 84HTHIELEMANN SMALL DEFLECTION SOLUTION FOR
*STABILITY OF ORTHOTROPIC CIRCULAR CYLINDERS 10(/), 30X,12A6 )
200 READ (5,201) NCASES,NRING,IRITPT,IR1OR2
201 FORMAT (4I5)
300 READ (5,301) (RINGRA(J),J=1,NRING)
301 FORMAT (7E10.5)
    IF (NRING.GT.1) CALL ORDER (NRING,RINGRA)
    CALL GNRATE (LONGRA)
    DO 2000 NREAD=1,NCASES
400 READ (5,401) CASENO,OPTION,RINGSP,ETAS,ETAP,LONGRA(302),RINGRA(22)
1      ,CR
401 FORMAT (A4,1X,A5,6E10.5)
500 NOP=0
    IF (PLOTS.EQ.OPTION) NOP=1
    IF (TABLE.EQ.OPTION) NOP=2
    IF (POINT.EQ.OPTION) NOP=3
    IF (PLOT.EQ.OPTION) NOP=1
    IF (NOP.EQ.0) GO TO 600
    GO TO 700
600 WRITE (6,601) CASENO
601 FORMAT (///87X,31HOPTION GIVEN INCORRECTLY, CASE A3, 6H DELETED)
GO TO 2000
700 CALL CALC
2000 CONTINUE
GO TO 100
END
$IBFIC BLKDAT
BLOCK DATA
COMMON /OPTION/ PLOTS, PLOT,POINT,TABLE
COMMON /MODE/   FMODE(5,2)
DATA          PLOTS,TABLE,POINT,PLOT
1            / 5HPLOTS, 5HTABLE, 5HPPOINT, 4HPLOT /
DATA          ((FMODE(I,J),J=1,2),I=1,5)
1            / 6H  CHK, 6HRBOARD, 6H  CHK, 6HRBOARD,
2            6H  AX, 6HISYM , 6H  ZER, 6HO BETA,
3            6H  INDE, 6HFINITE /
END

```

TABLE XXV - Fortran Listing - Program 3942 (Cont'd)

```

$IBFTC NBARI
  DOUBLE PRECISION FUNCTION NBARI (ETAP, SQRTCR, SCRGAM, BETA, CRGAM,
  *   ETAS)
  DOUBLE PRECISION SQRTCR, SCRGAM, BETA, CRGAM
  NBARI = DSQRT( ( 1. + (2. * ETAP * (1./SQRTCR) * SCRGAM * BETA**2)
  1   + CRGAM * BETA**4 ) /
  2   ( 1. + 2. * ETAS * BETA**2 + BETA**4 ) )
  RETURN
  END

$IBFTC ORDER LIST
  SUBROUTINE ORDER (NRING, RINGRA)
  DIMENSION RINGRA(22)
  C
  C   SUBROUTINE TO ORDER RING RATIOS FROM LOWEST TO HIGHEST.
  NRING1=NRING-1
  DO 650 N=1, NRING1
  TEMP=RINGRA(N)
  NI=N+1
  ITRANS=0
  DO 500 NN=NI, NRING
  IF (TEMP.LE.RINGRA(NN)) GO TO 500
  ITRANS=1
  NBLANK=NN
  TEMP=RINGRA(NN)
  500 CONTINUE
  IF (ITRANS.EQ.0) GO TO 650
  RINGRA(NBLANK)=RINGRA(N)
  RINGRA(N)=TEMP
  650 CONTINUE
  RETURN
  END

$IBFTC GNRATE
  SUBROUTINE GNRATE(LONGRA)
  DOUBLE PRECISION LONGRA, TOTHRD, EXP
  DIMENSION LONGRA(302)
  C
  C   SUBROUTINE TO GENERATE 301 VALUES OF LONGRA FROM 10 TO 1000,
  C   EVENLY SPACED RELATIVE TO THE LOG SCALE.
  TOTHRD = 2./300.
  LONGRA(1)=1000.
  LONGRA(301)=10.
  DO 200 I=1, 299
  I1=301-I
  EXP = 1.0 + FLOAT(I) * TOTHRD
  LONGRA(I1) = 10. ** EXP
  200 CONTINUE
  LONGRA(151)=100.
  RETURN
  END

$IBFTC CALC LIST
  SUBROUTINE CALC
  COMMON /INPUT/ CASENO, CR, CRGAM, ETAP, ETAS, LONGRA(302), NRING,
  1 RINGRA(22), RINGSP, IRITPT, IR1OR2
  COMMON /OUTFT/ BETA(302), NBAR(302), B(302), X1(302), X2(302),
  * IPRINTX(302)
  COMMON /MODE/ FMODE(5,2)
  COMMON /MISC/ IFLAG(302), ISTOP, NOP, J
  COMMON /FTOPT/ NBAR1, NBAR2, NBAR3, NBAR4, IRIT

```

## TABLE XXV - Fortran Listing - Program 3942 (Cont'd)

```

DOUBLE PRECISION LONGRA,CRGAM,SQRTCR,SCRGAM,NBAR1,NBAR2,NBAR3,
* NBAR4,NBARI,B,DIV,X1,X2,BETA1,BETA2,BETA,NBAR
C
C SUBROUTINE TO COMPUTE THE OUTPUT VALUES.
C
      GO TO (100,100,125), NOP
100  I1=1
      I2=301
      J1=1
      J2=NRKING
      GO TO 200
125  I1=302
      I2=302
      J1=22
      J2=22
200  DO 4000 J=J1,J2
      IF (IR1OR2.NE.0 .AND. NOP.NE.3) GO TO 204
      GO TO 207
204  WRITE (6,205) CASENO,RINGRA(J)
205  FORMAT (1H1,26X,28HINTERMEDIATE VALUES FOR CASE A4,36H, CORRECTED
1 HOOP SLENDERNESS RATIO =,1PD14.4 /// 4X,12HLONGITUDINAL / 5X,
2 1HSLENDERNESS / 7X,5HRATIO,17X,7HN BAR 1,21X,7HN BAR 2,21X,
3 7HN BAR 3,21X,7HN BAR 4 // )
C
C FORMATS
210  FORMAT (1X,1PD14.4,56X,1P2D28.15)
211  FORMAT (1X,1PD14.4,28X,1P3D28.15)
212  FORMAT (1X,1PD14.4,1PD28.15,28X,1P2D28.15)
213  FORMAT (1X,1PD14.4,1P4D28.15)
220  CONTINUE
207  DO 3000 I=I1,I2
      IPRINTX(I)=0
C
C CALCULATE B
      CRGAM = RINGRA(J)**2 / LONGRA(I)**2 * RINGSP**2
      SQRTCR = SQRT(CRGAM)
      SCRGAM = DSQRT(CRGAM)
      NBAR3 = SCRGAM
      NBAR4 = 1.
      B(I) = (CRGAM-1.)**2 - ((4.*ETAP*ETAS) / SQRTCR * (CRGAM**3)**.5 )
1 + (4.*ETAP**2) / CR * CRGAM + 4. * ETAS**2 * CRGAM
2 - (4.*ETAP*ETAS) / SQRTCR * SCRGAM
      IF (B(I).GE.0.) GO TO 400
C
C B IS NEGATIVE. NO BETA VALUES CALCULATED.
      IPRINTX(I)=1
      NBAR(I) = DMIN1(NBAR3,NBAR4)
      IF (NBAR(I).EQ.NBAR3) IFLAG(I)=3
      IF (NBAR(I).EQ.NBAR4) IFLAG(I)=4
      IF (NBAR3.EQ.NBAR4) IFLAG(I)=5
      IF (IR1OR2.NE.0 .AND. NOP.NE.3) WRITE (6,210) LONGRA(I),NBAR3,
* NBAR4
      IRIT=1
      GO TO 2000
C
C B IS POSITIVE. CALCULATE X1 AND X2
400  DIV = 2.*( CRGAM*ETAS - (ETAP / SQRTCR) * SCRGAM )
      X1(I) = ( (1.-CRGAM) + DSQRT(B(I)) ) / DIV

```

## TABLE XXV - Fortran Listing - Program 3942 (Cont'd)

```

X2(I) = ( (1.-CRGAM) - DSQRT(B(I)) ) / DIV
C
C   IS X1 OR X2 NEGATIVE
NX1=1
NX2=1
IF (X1(I).GE.0.) NX1=2
IF (X2(I).GE.0.) NX2=2
IF (NX1.EQ.2 .AND. NX2.EQ.2) GO TO 1000
IF (NX1.EQ.1 .AND. NX2.EQ.2) GO TO 600
IF (NX1.EQ.2 .AND. NX2.EQ.1) GO TO 700
C
C
C   BOTH X1 AND X2 ARE NEGATIVE
500 NBAR(I) = DMIN1(NBAR3,NBAR4)
IF (NBAR(I).EQ.NBAR3) IFLAG(I)=3
IF (NBAR(I).EQ.NBAR4) IFLAG(I)=4
IF (NBAR3.EQ.NBAR4) IFLAG(I)=5
IF (IR1OR2.NE.0 .AND. NOP.NE.3) WRITE (6,210) LONGRA(I),NBAR3,
*                                     NBAR4
IR1I=1
GO TO 2000
C
C
C   X1 IS NEGATIVE AND X2 IS POSITIVE
600 BETA2=DSQRT(X2(I))
NBAR2 = NBAR1(ETAP,SQRTCR,SCRGAM,BETA2,CRGAM,ETAS)
IF (IR1OR2.NE.0 .AND. NOP.NE.3) WRITE (6,211) LONGRA(I),NBAR2,
*                                     NBAR3,NBAR4
IR1I=2
BETA(I) = BETA2
NBAR(I) = DMIN1(NBAR2,NBAR3,NBAR4)
IF (NBAR(I).EQ.NBAR2) IFLAG(I)=2
IF (NBAR(I).EQ.NBAR3) IFLAG(I)=3
IF (NBAR(I).EQ.NBAR4) IFLAG(I)=4
IF=IFLAG(I)
GO TO (650,650,660,670), IF
650 IF (NBAR2.EQ.NBAR3 .OR. NBAR2.EQ.NBAR4) IFLAG(I)=5
GO TO 2000
660 IF (NBAR3.EQ.NBAR2 .OR. NBAR3.EQ.NBAR4) IFLAG(I)=5
GO TO 2000
670 IF (NBAR4.EQ.NBAR2 .OR. NBAR4.EQ.NBAR3) IFLAG(I)=5
GO TO 2000
C
C
C   X1 IS POSITIVE AND X2 IS NEGATIVE
700 BETA1=DSQRT(X1(I))
NBAR1 = NBAR1(ETAP,SQRTCR,SCRGAM,BETA1,CRGAM,ETAS)
IF (IR1OR2.NE.0 .AND. NOP.NE.3) WRITE (6,212) LONGRA(I),NBAR1,
*                                     NBAR3,NBAR4
IR1I=3
BETA(I) = BETA1
NBAR(I) = DMIN1(NBAR1,NBAR3,NBAR4)
IF (NBAR(I).EQ.NBAR1) IFLAG(I)=1
IF (NBAR(I).EQ.NBAR3) IFLAG(I)=3
IF (NBAR(I).EQ.NBAR4) IFLAG(I)=4
IF=IFLAG(I)
GO TO (750,750,760,770), IF
750 IF (NBAR1.EQ.NBAR3 .OR. NBAR1.EQ.NBAR4) IFLAG(I)=5

```

TABLE XXV - Fortran Listing - Program 3942 (Cont'd)

```

      GO TO 2000
760  IF (NBAR3.EQ.NBAR1 .OR. NBAR3.EQ.NBAR4) IFLAG(I)=5
      GO TO 2000
770  IF (NBAR4.EQ.NBAR1 .OR. NBAR4.EQ.NBAR3) IFLAG(I)=5
      GO TO 2000
C
C
C
      X1 AND X2 ARE BOTH POSITIVE
1000 BETA1=DSQRT(X1(I))
      BETA2=DSQRT(X2(I))
      NBAR1 = NBARI(ETAP, SQRTCR, SCRGAM, BETA1, CRGAM, ETAS)
      NBAR2 = NBARI(ETAP, SQRTCR, SCRGAM, BETA2, CRGAM, ETAS)
      IF (IR1OR2.NE.0 .AND. NOP.NE.3) WRITE (6,213) LONGRA(I),NBAR1,
*                                     NBAR2,NBAR3,NBAR4
      IRIT=4
      NBAR(I) = DMIN1(NBAR1,NBAR2,NBAR3,NBAR4)
      IF (NBAR(I).EQ.NBAR1) GO TO 1100
      IF (NBAR(I).EQ.NBAR2) GO TO 1200
      IF (NBAR(I).EQ.NBAR3) GO TO 1300
      IF (NBAR(I).EQ.NBAR4) GO TO 1400
1100 IFLAG(I)=1
      BETA(I)=BETA1
      IF (NBAR1.EQ.NBAR2 .OR. NBAR1.EQ.NBAR3 .OR. NBAR1.EQ.NBAR4)
*   IFLAG(I)=5
      GO TO 2000
1200 IFLAG(I)=2
      BETA(I)=BETA2
      IF (NBAR2.EQ.NBAR1 .OR. NBAR2.EQ.NBAR3 .OR. NBAR2.EQ.NBAR4 )
*   IFLAG(I)=5
      GO TO 2000
1300 IFLAG(I)=3
      IF (NBAR3.EQ.NBAR1 .OR. NBAR3.EQ.NBAR2 .OR. NBAR3.EQ.NBAR4)
*   IFLAG(I)=5
      GO TO 2000
1400 IFLAG(I)=4
      IF (NBAR4.EQ.NBAR1 .OR. NBAR4.EQ.NBAR2 .OR. NBAR4.EQ.NBAR3)
*   IFLAG(I)=5
C
C
C
      IS NBAR EQUAL TO ONE YET
2000 IF (NBAR(I).EQ.1.) GO TO 3500
3000 CONTINUE
3500 ISTOP=1
      J=J
      CALL OUTPUT
      IF (NOP.EQ.1) CALL SCPLLOT
4000 CONTINUE
      RETURN
      END
$IBFTC SCPLLOT LIST
      SUBROUTINE SCPLLOT
      COMMON /INPUT/  CASENO,CR,CRGAM,ETAP,ETAS, LONGRA(302),NKING,
1  RINGRA(22),RINGSP,IRITPT,IR1OR2
      COMMON /OUTPT/  BETA(302),NBAR(302),B(302),X1(302),X2(302),
*  IPRNTX(302)
      COMMON /MISC/  IFLAG(302),ISTOP,NOP,J
      DIMENSION LNGRA(302),NBR(302)
      DOUBLE PRECISION CRGAM, LONGRA,B,X1,X2,BETA,NBAR

```

TABLE XXV - Fortran Listing - Program 3942 (Cont'd)

```

REAL NB,LR,LNGRA,NBR
C
C SUBROUTINE TO PLOT NBAR VS. A/RHO11 FOR VARIOUS RING RATIOS.
IF (J.NE.1) GO TO 90
C
C SET GRID
CALL GRIDIV (4,10.,1000.,0.,1.1,1.,.02,0.5,-10,-5,5,3)
C
C PRINT A/D AND ETA S AT TOP
CALL PRINTV (-24,24HRING SPACING/DIAMETER = ,91,876)
CALL LABLV (RINGSP,299,876,6,1,2)
CALL PRINTV (-12, 12HETA SUB S = ,855,876)
CALL LABLV (ETAS,951,876,7,1,3)
C
C PRINT N BAR DOWN SIDE
CALL APRNTV (0,-14,-5,5HN BAR,5,538)
C
C PRINT MAIN TITLE AT BOTTOM
CALL RITE2V (189,77,1023,90,1,40,-1,40HCRITICAL COMPRESSIVE LOADING
*G COEFFICIENT,NLAST)
CALL RITE2V (279,45,1023,90,1,30,-1,30HFOR THE GENERAL INSTABILITY
*OF,NLAST)
CALL RITE2V (297,13,1023,90,1,28,-1,28HSTIFFENED CIRCULAR CYLINDER
*S,NLAST)
C
C PLOT CURVES
90 DO 100 II=1,ISTOP
LNGRA(II)=LNGRA(II)
NBR(II)=NBAR(II)
100 CONTINUE
300 IF (ISTOP.EQ.1) GO TO 700
LR=LNGRA(1)
IX1=NXV(LR)
NB=NBR(1)
IY1=NYV(NB)
DO 600 I=2,ISTOP
LR=LNGRA(I)
IX2=NXV(LR)
NB=NBR(I)
IY2=NYV(NB)
DO 500 NN=1,3
500 CALL LINEV (IX1,IY1,IX2,IY2)
IX1=IX2
IY1=IY2
600 CONTINUE
700 CONTINUE
IF (J.NE.NRING) RETURN
C
C PLOT CUT OFF LINE AT 1.0 FROM X=10 TO X OF LAST POINT PLOTTED.
IX1=NXV(10.)
IY=NYV(1.0)
DO 800 LL=1,5
800 CALL LINEV (IX1,IY,IX2,IY)
RETURN
END
$IBFTC OUTPUT LIST
SUBROUTINE OUTPUT
COMMON /INPUT/ CASLNO,CR,CRGAM,ETAP,ETAS,LNGRA(302),NRING,

```

TABLE XXV - Fortran Listing - Program 3942 (Cont'd)

```

1      RINGRA(22), RINGSP, IRITPT, IR1OR2
COMMON /OUTPT/ BETA(302), NBAR(302), B(302), X1(302), X2(302),
*           IPRINTX(302)
COMMON /MODE/  FMODE(5,2)
COMMON /MISC/  IFLAG(302), ISTOP, NOP, J
COMMON /PTOPT/ NBAR1, NBAR2, NBAR3, NBAR4, IRIT
DOUBLE PRECISION NBAR1, NBAR2, NBAR3, NBAR4
DOUBLE PRECISION CRGAM, LONGRA, B, X1, X2, NBAR, BETA

C
C      SUBROUTINE TO PRINT OUTPUT FOR EACH J VALUE
C
C
      IF (J.EQ.1 .OR. J.EQ.22) GO TO 75
      GO TO 250
75     IF (CR) 125,125,100
100    WRITE (6,101) CASENO, RINGSP, ETAS, ETAP, CR
101    FORMAT (1H1, 9X, 4HCASE, 18X, 8HNON-DIM. / 10X, 3HNO., 16X,
1       12HRING SPACING, 18X, 9HETA SUB S, 19X, 9HETA SUB P, 20X,
2       7HC SUB R // 8X, A4, 1P4E20.4 /// 22X, 9HCORRECTED / 4X,
3       12HLONGITUDINAL, 8X, 4HHOOP / 5X, 11HSLENDERNESS, 5X,
4       11HSLENDERNESS / 8X, 5HRATIO, 11X, 5HRATIO, 12X, 1HB, 15X, 2HX1,
5       14X, 2HX2, 13X, 4HBETA, 11X, 5HN BAR, 12X, 4HMODE //)
      GO TO 140
125    WRITE (6,126) CASENO, RINGSP, ETAS, ETAP
126    FORMAT (1H1, 9X, 4HCASE, 18X, 8HNON-DIM. / 10X, 3HNO., 16X,
1       12HRING SPACING, 18X, 9HETA SUB S, 19X, 9HETA SUB P, 20X,
2       7HC SUB R // 8X, A4, 1P3E20.4 /// 22X, 9HCORRECTED / 4X,
3       12HLONGITUDINAL, 8X, 4HHOOP / 5X, 11HSLENDERNESS, 5X,
4       11HSLENDERNESS / 8X, 5HRATIO, 11X, 5HRATIO, 12X, 1HB, 15X, 2HX1,
5       14X, 2HX2, 13X, 4HBETA, 11X, 5HN BAR, 12X, 4HMODE //)

C
C      FORMAT STATEMENTS
C
1001   FORMAT (1X, 1PD14.4, 1PE16.4, 1P5D16.4, 4X, 2A6)
1003   FORMAT (1X, 1PD14.4, 1PE16.4, 1P3D16.4, 16X, 1PD16.4, 4X, 2A6)
1004   FORMAT (1X, 1PD14.4, 1PE16.4, 1P3D16.4, 10X, 1H0, 5X, 1PD16.4, 4X, 2A6)
2003   FORMAT (1X, 1PD14.4, 1PE16.4, 1PD16.4, 6X, 9HIMAGINARY, 7X, 9HIMAGINARY,
*       17X, 1PD16.4, 4X, 2A6)
2004   FORMAT (1X, 1PD14.4, 1PE16.4, 1PD16.4, 6X, 9HIMAGINARY, 7X, 9HIMAGINARY,
*       11X, 1H0, 5X, 1PD16.4, 4X, 2A6)
3001   FORMAT (60X, 1P2D33.15)
3002   FORMAT (27X, 1P3D33.15)
3003   FORMAT (1X, 1PD26.15, 33X, 1P2D33.15)
3004   FORMAT (1X, 1PD26.15, 1P3D33.15)
140   IF (NOP.NE.3) GO TO 250

C
C      POINT OPTION
C
      IF=IFLAG(302)
      IF (IPRINTX(302).EQ.1) GO TO 200
145   GO TO (150,150,160,170,160), IF
150   WRITE (6,1001) LONGRA(302), RINGRA(22), B(302), X1(302), X2(302),
*       BETA(302), NBAR(302), FMOVL(1,1), FMODE(1,2)
      IF (IRITPT.NE.0) GO TO 235
      RETURN
160   WRITE (6,1003) LONGRA(302), RINGRA(22), B(302), X1(302), X2(302),
*       NBAR(302), FMODE(IF,1), FMODE(IF,2)
      IF (IRITPT.NE.0) GO TO 235
      RETURN
170   WRITE (6,1004) LONGRA(302), RINGRA(22), B(302), X1(302), X2(302),

```

TABLE XIV - Fortran Listing - Program 3942 (Cont'd)

```

*          NBAR(302),FMODE(4,1),FMODE(4,2)
  IF (IRITPT.NE.0) GO TO 235
  RETURN
200 GO TO (220,220,220,230,220), IF
220 WRITE (6,2003) LONGRA(302),RINGRA(22),B(302),NBAR(302),
*          FMODE(IF,1),FMODE(IF,2)
  IF (IRITPT.NE.0) GO TO 235
  RETURN
230 WRITE (6,2004) LONGRA(302),RINGRA(22),B(302),NBAR(302),
*          FMODE(4,1),FMODE(4,2)
  IF (IRITPT.NE.0) GO TO 235
  RETURN
235 WRITE (6,236)
236 FORMAT (/// 13X,7HN BAR 1,26X,7HN BAR 2,26X,7HN BAR 3,26X,
*          7HN BAR 4 // )
  GO TO (240,242,244,246),IRIT
240 WRITE (6,3001) NBAR3,NBAR4
  RETURN
242 WRITE (6,3002) NBAR2,NBAR3,NBAR4
  RETURN
244 WRITE (6,3003) NBAR1,NBAR3,NBAR4
  RETURN
246 WRITE (6,3004) NBAR1,NBAR2,NBAR3,NBAR4
  RETURN
C
C  TABLE OR PLOTS OPTION
250 DO 400 I=1,ISTOP
  IF=IFLAG(I)
  IF (IPRNTX(I).EQ.1) GO TO 350
  GO TO (300,300,310,320,310), IF
300 WRITE (6,1001) LONGRA(I),RINGRA(J),B(I),X1(I),X2(I),BETA(I),
*          NBAR(I),FMODE(1,1),FMODE(1,2)
  GO TO 400
310 WRITE (6,1003) LONGRA(I),RINGRA(J),B(I),X1(I),X2(I),NBAR(I),
*          FMODE (IF,1),FMODE(IF,2)
  GO TO 400
320 WRITE (6,1004) LONGRA(I),RINGRA(J),B(I),X1(I),X2(I),NBAR(I),
*          FMODE(4,1),FMODE(4,2)
  GO TO 400
350 GO TO (360,360,360,370,360), IF
360 WRITE (6,2003) LONGRA(I),RINGRA(J),B(I),NBAR(I),FMODE(IF,1),
*          FMODE(IF,2)
  GO TO 400
370 WRITE (6,2004) LONGRA(I),RINGRA(J),B(I),NBAR(I),FMODE(4,1),
*          FMODE(4,2)
400 CONTINUE
500 WRITE (6,501) RINGRA(J)
501 FORMAT (34X,63HNO FURTHER CALCULATIONS FOR CORRECTED HOOP SLENDERNESS
*          RATIO = ,1PE11.4 /// )
  RETURN
  END
Q  END OF FILE OR DATA CARD  DATA FOLLOWS
SPECIMEN 1-1 OF REFERENCE 29 USING SECOND ITERATION STIFFNESS FACTORS
  1      U      U      0
  1 POINT7.7702 -021.8740 +00          2.9039 +013.7282 +02

```

18.3.2            Langley Solution - The solution of reference 5 for general instability of stiffened circular cylindrical shells was programmed to give a method for evaluating the effects of stiffener eccentricities and to provide a means for investigating interaction between circumferential and longitudinal loadings. The program is identified as General Dynamics Convair Program No. 3962. The analysis approach is discussed in Section 7.2.2 and the program is applied in Section 7.3 to study test data comparisons. The program was also used to obtain the interaction results of Sections 8.3 and 8.4.

The stability relationship programmed is equation (15) of reference 5 which relates the applied longitudinal and circumferential running loads ( $\bar{N}_x$  and  $\bar{N}_y$ ) to  $m$  (the number of half waves in the buckle pattern in the longitudinal direction) and  $n$  (the number of full waves in the circumferential direction) as follows. The program simply allows for input of  $\bar{N}_y$  (or  $\bar{N}_x$ ) (+ compression, -tension) and  $m$  and  $n$  values for screening. The minimum  $\bar{N}_x$  (or  $\bar{N}_y$ ) value for the combinations of input  $m$  and  $n$  values is found. Integral or non-integral values may be used for  $m$  and  $n$ . Care must be exercised to ascertain that the minimum strength buckle pattern does not lie outside of the range of  $m$  and  $n$  values screened.

The governing stability equation is (refer to Table XXVI for definition of symbols):

$$\left(\frac{m\pi}{a}\right)^2 \bar{N}_x + \left(\frac{n}{R}\right)^2 \bar{N}_y = A_{33} + \left(\frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}^2}\right) A_{13} + \left(\frac{A_{12}A_{13} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}^2}\right) A_{23} \quad (18-2)$$

in which

$$A_{11} = \left( \frac{E_x}{1 - \mu_x' \mu_y'} + \frac{E_s A_s}{d} \right) \left( \frac{m\pi}{a} \right)^2 + G_{xy} \left( \frac{n}{R} \right)^2$$

$$A_{12} = \left( \frac{\mu_y' E_x}{1 - \mu_x' \mu_y'} + G_{xy} \right) \left( \frac{m\pi}{a} \right) \left( \frac{n}{R} \right)$$

$$A_{13} = \frac{1}{R} \left( \frac{\mu_y' E_x}{1 - \mu_x' \mu_y'} \right) \left( \frac{m\pi}{a} \right) + \frac{E_s A_s}{d} \bar{z}_s \left( \frac{m\pi}{a} \right)^3$$

$$A_{22} = G_{xy} \left( \frac{m\pi}{a} \right)^2 + \left( \frac{E_y}{1 - \mu_x' \mu_y'} + \frac{E_r A_r}{\ell} \right) \left( \frac{n}{R} \right)^2$$

$$A_{23} = \frac{1}{R} \left( \frac{E_y}{1 - \mu_x' \mu_y'} + \frac{E_r A_r}{\ell} \right) \left( \frac{n}{R} \right) + \frac{E_r A_r}{\ell} \bar{z}_r \left( \frac{n}{R} \right)^3$$

$$A_{33} = \left( \frac{D_x}{1 - \mu_x \mu_y} + \frac{E_s I_o}{d} \right) \left( \frac{m\pi}{a} \right)^4 + \left( \frac{2\mu_y D_x}{1 - \mu_x \mu_y} + 2D_{xy} + \frac{G_s J_s}{d} + \frac{G_r J_r}{\ell} \right) \left( \frac{m\pi}{a} \right)^2 \left( \frac{n}{R} \right)^2$$

$$+ \left( \frac{D_y}{1 - \mu_x \mu_y} + \frac{E_r I_c}{\ell} \right) \left( \frac{n}{R} \right)^4 + \frac{1}{R^2} \left( \frac{E_y}{1 - \mu_x' \mu_y'} + \frac{E_r A_r}{\ell} \right) + 2 \frac{E_r A_r}{\ell} \bar{z}_r \frac{n^2}{R^3}$$

The input format for Program No. 3962 is shown in Figure 59. Symbols used appear in Table XXVI. Detailed discussion of input, card by card, follows. Runs may be stacked.

CARD TYPE 1: One card per run.

Enter the problem identification (any alphanumeric characters) in columns 1-72.

CARD TYPE 2: One card per run.

Enter NCASES (number of cases) as right adjusted integer(s) in columns 1-5.

Enter PRNTOP (printout option) as right adjusted integer in columns 6-10. If PRNTOP = 0 (or blank), only output corresponding to minimum  $\bar{N}$  are printed. If PRNTOP = 1 (or  $\neq 0$ ), all output associated with each  $m$  and  $n$  combination screened are printed including the  $A_{ij}$  values used in equation (18-2).

CARD TYPE 3: One card per case.

Enter CASENO (case number) as right adjusted alphanumeric character(s) in columns 1-5.

Enter LOADIN (loading condition) in columns 6-10 with the symbols:

COMBY if  $\bar{N}_y$  value input on CARD TYPE 3

COMBX if  $\bar{N}_x$  value input on CARD TYPE 3.

Enter MINM (minimum  $m$  value for screening) in columns 11-20 (E10.5). MINM > 0.

80 COLUMN-GENERAL PURPOSE-WORKSHEET

3962 LANGLEY SOLUTION L.S. FØSØM

PROBLEM IDENTIFICATION

MINM LENGTH	MAXM STRSP	DELIAM FRAMSP	MINN	MAXN	DELTAN	NY ØR NX
EX	OX	PY	GXY	DXY		
PRIMMX	MX	MY				
ES	AS	IQS	JS	ZBARS		
ER	AR	IQR	JR	ZBARR		

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

GENERAL DYNAMICS  
Convair Division

Figure 59 - Input Format - Program 3962

Enter MAXM (maximum  $n$  value for screening) in columns 21-30 (E10.5).

Enter DELTAM ( $\Delta_m$ ) in columns 31-40 (E10.5).

Enter MINN (minimum  $n$  value for screening) in columns 41-50 (E10.5). MINN > 0 for COMBX.

MINN  $\geq$  0 for COMBY.

Enter MAXN (maximum  $n$  value for screening) in columns 51-60 (E10.5).

Enter DELTAN ( $\Delta_n$ ) in columns 61-70 (E10.5).

CARD TYPE 4: One card per case.

Enter RADIUS (R, cylinder radius measured to mid-surface of skin, in.) in columns 1-10 (E10.5).

Enter LENGTH (a, overall length of cylinder, in) in columns 11-20 (E10.5).

Enter STRSP (d, stringer spacing, in.) in columns 21-30 (E10.5).

Enter FRAMSP ( $\ell$ , frame spacing, in) in columns 31-40 (E10.5).

CARD TYPE 5: One card per case.

Enter EX ( $E_x$ , lbs/in) in columns 1-10 (E10.5).

Enter EY ( $E_y$ , lbs/in) in columns 11-20 (E10.5).

Enter DX ( $D_x$ , lb-in) in columns 21-30 (E10.5).

Enter DY ( $D_y$ , lb-in) in columns 31-40 (E10.5).

Enter GXY ( $G_{xy}$ , lbs/in) in columns 41-50 (E10.5)

Enter DXY ( $D_{xy}$ , lb-in) in columns 51-60 (E10.5).

CARD TYPE 6: One card per case.

Enter PRIMMX ( $\mu_x$ ) in columns 1-10 (E10.5).

Enter PRIMMY ( $\mu_y$ ) in columns 11-20 (E10.5).

Enter MX ( $\mu_x$ ) in columns 21-30 (E10.5).

Enter MY ( $\mu_y$ ) in columns 31-40 (E10.5).

CARD TYPE 7: One card per case.

Enter ES ( $E_s$ , psi) in columns 1-10 (E10.5).

Enter GS ( $G_s$ , psi) in columns 11-20 (E10.5).

Enter AS ( $A_s$ , in<sup>2</sup>) in columns 21-30 (E10.5).

Enter IOS ( $I_{os}$ , in<sup>4</sup>) in columns 31-40 (E10.5).

Enter JS ( $J_s$ , in<sup>4</sup>) in columns 41-50 (E10.5).

Enter ZBARS ( $\bar{z}_s$ , in) in columns 51-60 (E10.5).

CARD TYPE 8: One card per case.

Enter ER ( $E_r$ , psi) in columns 1-10 (E10.5).

Enter GR ( $G_r$ , psi) in columns 11-20 (E10.5).

Enter AR ( $A_r$ , in<sup>2</sup>) in columns 21-30 (E10.5).

Enter IOR ( $I_{or}$ , in<sup>4</sup>) in columns 31-40 (E10.5).

Enter JR ( $J_r$ , in<sup>4</sup>) in columns 41-50 (E10.5).

Enter ZBARR ( $\bar{z}_r$ , in) in columns 51-60 (E10.5).

An input coding form for sample data is shown in Figure 60.

Approximate execution times at the General Dynamics Convair 7094 DCS installation were .002 seconds per combination of m and n screened if PRNTOP = 0 and .02 seconds per case if PRNTOP  $\neq$  0.

3962 LANGLEY SOLUTION L.S. :  $\Phi$ SSUM

80 COLUMN - GENERAL PURPOSE - WORKSHEET

EXAMPLE 4 OF SECTION 8.3, HOPP COMPRESSIVE RUNNING LOAD = 799.1 LBS/IN.

ICPMBY	1.0	12.0	1.0	0.0	12.0	1.0	799.1
38.6	7.2	2.48	6.0				
1.5 +06	2.0 +06	2.5 +02	3.0 +02	2.0 +05	2.0 +02		
2.5 -01	3.5 -01	3.0 -01	4.0 -01				
3.0 +07	1.2 +07	2.0 -02	5.0 -03	4.0 -03	5.0 -01		
2.5 +07	1.0 +07	4.0 -02	1.0 -02	6.0 -03	7.5 -01		

Figure 60 - Sample Input Data - Program 3962

The program output consists of a listing of all input in addition to calculated results. The calculated results for  $PRNTOP \neq 0$  consist of an  $\bar{N}_x$  (or  $\bar{N}_y$ ) value and the associated  $m$ ,  $n$ , and  $A_{ij}$  values for each possible combination of input  $m$  and  $n$ . In addition, a short table is established showing the input  $\bar{N}_y$  (or  $\bar{N}_x$ ), the minimum calculated  $\bar{N}_x$  (or  $\bar{N}_y$ ), and the number of longitudinal and circumferential half waves ( $m$  and  $2n$ ). An indication of both sensitivity of the critical loading to buckle size and whether the selected minimum value is a relative minimum value is obtained by evaluating  $\bar{N}_x$  (or  $\bar{N}_y$ ) for  $(m, n-1)$ ,  $(m, n+1)$ ,  $(m-1, n)$ , and  $(m+1, n)$  where  $m$  and  $n$  correspond to the minimum  $\bar{N}_x$  (or  $\bar{N}_y$ ) value found. Note the limitations of meaningful values of  $m$  and  $n$  in the CARD TYPE 3 discussion. A sample page of output listing is shown in Figure 61 for the sample input coded in Figure 60. Notation used in program 3962 vs. symbols employed in Section 7.2.2 of this report and in reference 5 is given in Table XXVI. A basic flow diagram for the program is shown in Figure 62 and a Fortran listing of the program, including input data from the sample problem (Figure 60), is given in Table XXVII.

CASE NO.	MIN M	MAX M	DELTA M	MIN N	MAX N	DELTA N
1	1.0000E 00	1.2000E 01	1.0000E 00	0.	1.2000E 01	1.0000E 00
	CYLINDER RADIUS	OVERALL LENGTH	STRINGER SPACING	FRAME SPACING		N BAR Y
	3.8600E 01	7.2000E 01	7.4800E 00	6.0000E 00		7.9910E 02
	F SUR X	F SUR Y	D SUR X	D SUR Y	G SUR XY	D SUR XY
	1.5000E 04	2.0000E 06	2.5000E 02	3.0000E 02	2.0000E 05	2.0000E 02
	MU PRIME SUR X	MU PRIME SUR Y	MU SUR X	MU SUR Y		
	2.5000E-01	3.5000E-01	3.0000E-01	4.0000E-01		
	F SUR S	G SUR S	A SUR S	T SUR OS	J SUR S	Z BAR SUB S
	3.0000E 07	1.2000E 07	2.0000E-02	5.0000E-03	4.0000E-03	5.0000E-01
	F SUR R	G SUR R	A SUR R	T SUR OR	J SUR R	Z BAR SUR R
	2.5000E 07	1.0000E 07	4.0000E-02	10.0000E-03	6.0000E-03	7.5000E-01

CRITICAL COMBINED LOAD VALUES			
NUMBER OF LONGITUDINAL HALF WAVES	NUMBER OF CIRCUMFERENTIAL HALF WAVES	AXIAL LOAD BAR SUR X LBS PER IN	Hoop Load BAR SUB Y LBS PER IN
1.0000E 00	1.0000E 01	2.7015E 03	7.9910E 02
0.	1.0000E 01	-0.	7.9910E 02
2.0000E 00	1.0000E 01	7.6512E 03	7.9910E 02
1.0000E 00	8.0000E 00	6.0120E 03	7.9910E 02
1.0000E 00	1.2000E 01	3.3791E 03	7.9910E 02

Figure 61 - Sample Output Listing - Program 3962

TABLE XXVI - Program 3962 Notation

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
AR	$A_r$	Cross-sectional area of circumferential stiffener, in <sup>2</sup> .
AS	$A_s$	Cross-sectional area of longitudinal stiffener, in <sup>2</sup> .
CASENO	-	Case number.
COMBX	-	Indicates $\bar{N}_x$ value input, solve for $\bar{N}_y$ .
COMBY	-	Indicates $\bar{N}_y$ value input, solve for $\bar{N}_x$ .
D	-	See STRSP.
DX	$D_x$	Bending stiffness of skin in longitudinal direction, lb-in.
DY	$D_y$	Bending stiffness of skin in circumferential direction, lb-in.
DXY	$D_{xy}$	Twisting stiffness of skin, lb-in.
DELTAM	$\Delta m$	Value by which $m$ is incremented.
DELTAN	$\Delta n$	Value by which $n$ is incremented.
ER	$E_r$	Young's modulus for circumferential stiffener, psi.
ES	$E_s$	Young's modulus for longitudinal stiffener, psi.
EX	$E_x$	Extensional stiffness of skin in longitudinal direction, lbs/in.
EY	$E_y$	Extensional stiffness of skin in circumferential direction, lbs/in.

TABLE XXVI - Program 3962 Notation (Cont'd)

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
ERARL	$E_{rR}/l$	
FRAMSP	$l$	Frame spacing, in.
FMPIA	$m\pi/a$	
FMXMY	$1-\mu_x, \mu_y$	
GR	$G_r$	Shear modulus for circumferential stiffener, psi.
GS	$G_s$	Shear modulus for longitudinal stiffener, psi.
GXY	$G_{xy}$	In-plane shear stiffness of skin panel, lbs/in.
IOR	$I_{oR}$	Moment of inertia of circumferential stiffener cross-section about middle surface of skin, in <sup>4</sup> .
IOS	$I_{oS}$	Moment of inertia of longitudinal stiffener cross-section about middle surface of skin, in <sup>4</sup> .
JR	$J_r$	Torsional constant for circumferential stiffener, in <sup>4</sup> .
JS	$J_s$	Torsional constant for longitudinal stiffener, in <sup>4</sup> .
LENGTH	$a$	Length of stiffened cylinder, in.
LOADIN	-	Option describing input loading: COMBY or COMBX.
MAXM	-	Maximum value of $m$ used.

TABLE XXVI - Program 3962 Notation (Cont'd)

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
MINM	-	Minimum value of m used.
MAXN	-	Maximum value of n used.
MINN	-	Minimum value of n used.
MX	$\mu_x$	Poisson's ratio for bending of skin in longitudinal direction.
MY	$\mu_y$	Poisson's ratio for bending of skin in circumferential direction.
NBAR	$\bar{N}$	$\bar{N}_x$ or $\bar{N}_y$ (+ in compression) lbs/in.
NCASES	-	Number of cases.
NOP	-	NOP = 1 used for COMBY NOP = 2 used for COMBX
NXORNY	$\bar{N}_x$ or $\bar{N}_y$	$\bar{N}_x$ or $\bar{N}_y$ input, lbs/in.
PRIMMX	$\mu_x'$	Poisson's ratio for extension of skin in longitudinal direction.
PRIMMY	$\mu_y'$	Poisson's ratio for extension of skin in circumferential direction.
PRNTOP	-	Printout option. If PRNTOP = 0 (or blank), prints only output associated with minimum value of $\bar{N}$ calculated. If PRNTOP $\neq$ 0, prints output for all combinations of m and n values screened.
PROBID	-	Problem identification.
RADIUS	R	Radius of cylinder, measured to mid-surface of skin, in.

TABLE XXVI - Program 3962 Notation (Cont'd)

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
STOREM	-	m for minimum $\bar{N}$ .
STOREN	-	n for minimum $\bar{N}$ .
STRSP	d	Stringer spacing, in.
TEMP	-	Relative minimum $\bar{N}$ .
ZBARR	$\bar{z}_r$	Distance from middle surface of skin to centroid of circumferential stiffener, in., positive if stiffener is outside.
ZBARS	$\bar{z}_s$	Distance from middle surface of skin to centroid of longitudinal stiffener in., positive if stiffener is outside.

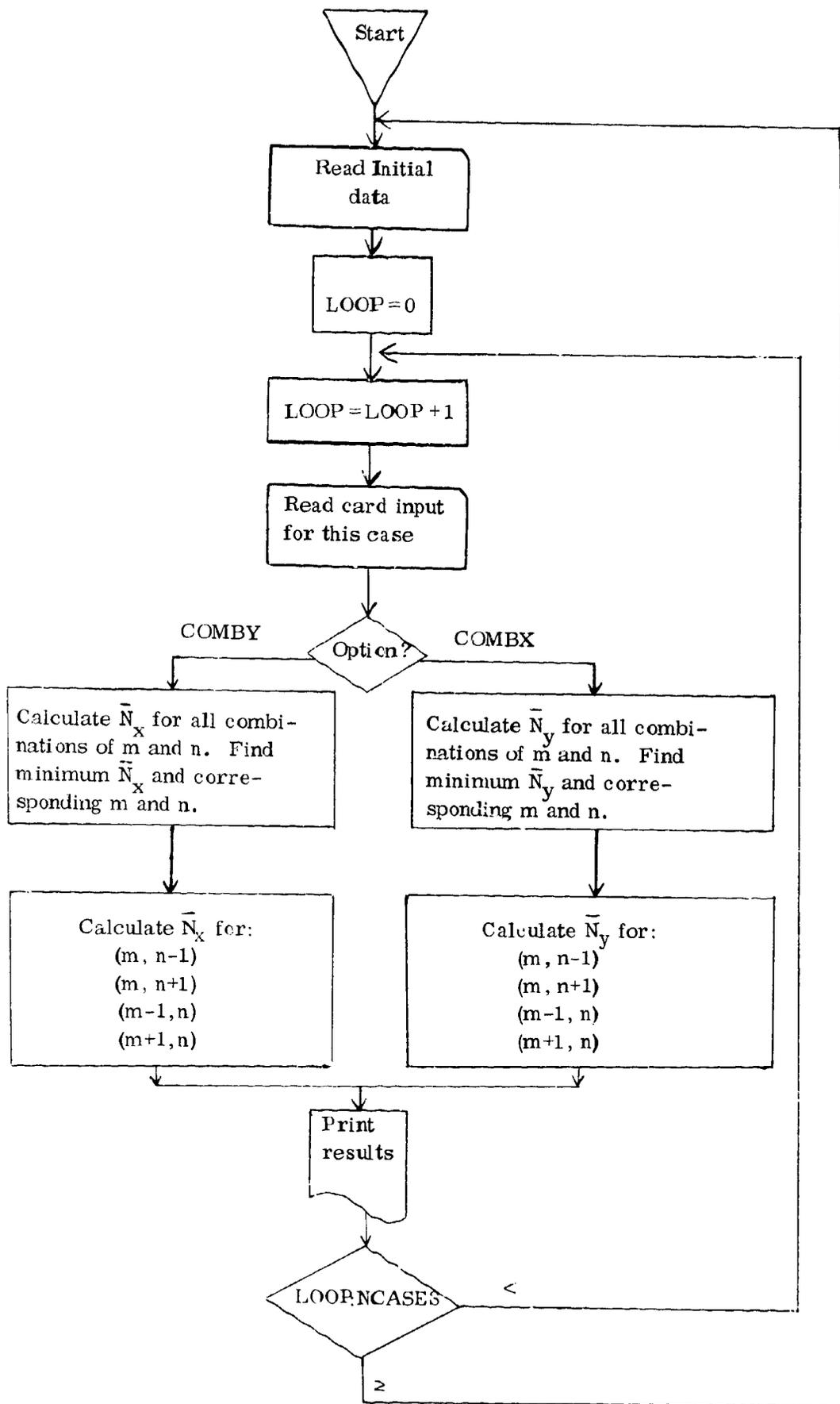


Figure 62 - Flow Diagram - Program 3962

TABLE XXVII - Fortran Listing - Program 3962

```

$EXECUTE      1BJOB
$1BJOB
$IBFTC MAIN   LIST
COMMON /INPT/ CASENO,LOADIN,MINM,MAXM,DELTAM,MINN,MAXN,DELTAN,
1             RADIUS,LENGTH,STRSP,FRAMSP,EX,EY,DX,DY,GXY,DXY,
2             PRIMMX,PRIMMY,MX,MY,ES,GS,AS,IOS,JS,ZBARS,ER,GR,
3             AR,IOR,JR,ZBARR,NOP,PI,PRNTOP,NXORNY
DIMENSION PROBID(12)
REAL MINM,MAXM,MINN,MAXN,LENGTH,MX,MY,IOS,JS,IOR,JR,NBAR,NXORNY
INTEGER COMBX,COMBY
DATA COMBX,COMBY /5HCOMBX,5HCOMBY/
DATA PI /3.1415926/
100 READ (5,101) PROBID
101 FORMAT (12A6)
WRITE (6,201) PROBID
201 FORMAT (1H1,20(/),58X,16HLANGLEY SOLUTION 10(/),30X,12A6)
READ (5,301) NCASES,PRNTOP
301 FORMAT (15,F5.0)
DO 1000 N=1,NCASES
CALL INPUT
IF (PRNTOP.EQ.0.) GO TO 310
WRITE (6,305) CASENO
305 FORMAT (1H1,50X,28HINTERMEDIATE VALUES FOR CASE,A5)
310 NOP=0
IF (LOADIN.EQ.COMBY) NOP=1
IF (LOADIN.EQ.COMBX) NOP=2
IF (NOP.NE.0) GO TO 400
WRITE (6,350) CASENO
350 FORMAT (/// 87X,31HOPTION GIVEN INCORRECTLY: CASE A3, 8H DELETED)
GO TO 1000
400 CALL EQUATN
500 CALL FINAL
600 CALL OUTPUT
1000 CONTINUE
GO TO 100
END
$IBFTC NBAR   LIST
REAL FUNCTION NBAR(EX,FMXMY,ES,AS,D,FMPIA,GXY,FNR,PRIMMY,R,ZBARS,
1             EY,ERARL,ZBARR,DX,IOS,DXY,GS,JS,GR,JR,L,DY,ER,
2             IOR,N,NOP,M,PRNTOP,NXORNY)
REAL IOS,JS,JR,L,IOR,N,M,NXORNY
A11 = (EX / FMXMY + ES*AS/D) * FMPIA**2 + GXY*FNR**2
A12 = (PRIMMY*EX/FMXMY + GXY) * FMPIA * FNR
A13 = (1./R)*(PRIMMY*EX/FMXMY)*FMPIA + ES*AS/D*ZBARS*FMPIA**3
A22 = GXY*FMPIA**2 + (EY/FMXMY + ERARL) * FNR**2
A23 = (1./R) * (EY/FMXMY + ERARL) * FNR + ERARL*ZBARR*FNR**3
A33 = (DX/FMXMY + ES*IOS/D)*FMPIA**4 + (2.*PRIMMY*DX/FMXMY
1      + 2.*DXY + GS*JS/D + GR*JR/L) * FMPIA**2 * FNR**2
2      + (DY/FMXMY + ER*IOR/L)*FNR**4 + (1./R**2)*(EY/FMXMY
3      + ERARL) + (2.*ERARL*ZBARR*(N**2/R**3) )
GO TO (1000,2000), NOP
C
C      NOP=1
1000 DIV = FMPIA**2

```

TABLE XXVII - Fortran Listing - Program 3962 (Cont'd)

```

SUB = FNR**2 * NXORNY
GO TO 4000
C
C   NOP=2
2000 DIV = FNR**2
    SUB = FMPIA**2 * NXORNY
C
4000 NBAR = (A33 + ( ((A12*A23)-(A13*A22))/((A11*A22)-A12**2))*A13
1      + (((A12*A13)-(A11*A23))/((A11*A22)-A12**2) ) * A23 ) - SUB)
2      / DIV
    IF (PRNTOP.EQ.0.) RETURN
    WRITE (6,6000) M,N,NBAR,A11,A12,A13,A22,A23,A33
6000 FORMAT (/// 11X,5HM  =,1PE11.3,28X,5HN  =,1PE11.3,28X,6HN BAR=,
1      E16.9 // 11X,5HA11 =,1PE17.9,22X,5HA12 =,1PE17.9,,22X,
2      5HA13 =,1PE17.9, / 11X,5HA22 =,1PE17.9,22X,5HA23 =,1PE17.9,
3      22X,5HA33 =,1PE17 9)
    RETURN
    END
$IBFTC INPUT LIST
SUBROUTINE INPUT
COMMON /INPT/  CASENO,LOADIN,MINM,MAXM,DELTAM,MINN,MAXN,DELTAN,
1              RADIUS,LENGTH,STRSP,FRAMSP,EX,EY,DX,DY,GXY,DXY,
2              PRIMMX,PRIMMY,MX,MY,ES,GS,AS,IOS,JS,ZBARS,ER,GR,
3              AR,IOR,JR,ZBARR,NOP,PI,PRNTOP,NXORNY
REAL MINM,MAXM,MINN,MAXN,LENGTH,MX,MY,IOS,JS,IOR,JR,NBAR,NXORNY
C
C   SUBROUTINE TO READ CARD INPUT FOR EACH CASE.
READ (5,101) CASENO,LOADIN,MINM,MAXM,DELTAM,MINN,MAXN,DELTAN
*
101 FORMAT (2A5,7E10.5)
READ (5,201) RADIUS,LENGTH,STRSP,FRAMSP
201 FORMAT (4E10.5)
READ (5,301) EX,EY,DX,DY,GXY,DXY
301 FORMAT (6E10.5)
READ (5,201) PRIMMX,PRIMMY,MX,MY
READ (5,301) ES,GS,AS,IOS,JS,ZBARS
READ (5,301) ER,GR,AR,IOR,JR,ZBARR
RETURN
END
$IBFTC EQUATN LIST
SUBROUTINE EQUATN
COMMON /INPT/  CASENO,LOADIN,MINM,MAXM,DELTAM,MINN,MAXN,DELTAN,
1              RADIUS,LENGTH,STRSP,FRAMSP,EX,EY,DX,DY,GXY,DXY,
2              PRIMMX,PRIMMY,MX,MY,ES,GS,AS,IOS,JS,ZBARS,ER,GR,
3              AR,IOR,JR,ZBARR,NOP,PI,PRNTOP,NXORNY
COMMON /INTER/ FMXMY,ERARL,TEMP,STOREM,STOREN
COMMON /RENAME/ A,D,L,R
REAL MINM,MAXM,MINN,MAXN,LENGTH,MX,MY,IOS,JS,IOR,JR,NBAR
REAL L,M,N,NBARX,NBARY,NXORNY
R=RADIUS
A=LENGTH
D=STRSP
L=FRAMSP
FMXMY = 1.-PRIMMX*PRIMMY
ERARL = ER*AR/L
ICNT=U
N=MINN-DELTAN
100 N=N+DELTAN

```

TABLE XXVII - Fortran Listing - Program 3962 (Cont'd)

```

      IF (M.GT.MAXM) GO TO 6000
      FNR = N/R
      MEMINM=DELTAM
200  MEM+DELTAM
      ICNT=ICNT+1
      IF (M.GT.MAXM) GO TO 5000
      FMPIA = M*PI/A
      GO TO (1000,2000), NOP
C
C      NOP=1  SOLVE FOR N BAR X
1000  NBARX = NBAR(EX,FMXMY,ES,AS,D,FMPIA,GXY,FNR,PRIMMY,R,ZBARS,EY,
1      ERARL,ZBARR,DX,IOS,UXY,GS,JS,GR,JR,L,DY,ER,IOR,N,NOP,
2      M,PRNTOP,NXORNY)
      IF (ICNT.NE.1) GO TO 1100
      TEMP=NBARX
      STOREM=M
      STOREN=N
      GO TO 4000
1100  IF (NBARX.GE.TEMP) GO TO 4000
      TEMP=NBARX
      STOREM=M
      STOREN=N
      GO TO 4000
C
C      NOP=2  SOLVE FOR N BAR Y
2000  NBARY = NBAR(EX,FMXMY,ES,AS,D,FMPIA,GXY,FNR,PRIMMY,R,ZBARS,EY,
1      ERARL,ZBARR,DX,IOS,UXY,GS,JS,GR,JR,L,DY,ER,IOR,N,NOP,
2      M,PRNTOP,NXORNY)
      IF (ICNT.NE.1) GO TO 2100
      TEMP=NBARY
      STOREM=M
      STOREN=N
      GO TO 4000
2100  IF (NBARY.GE.TEMP) GO TO 4000
      TEMP=NBARY
      STOREM=M
      STOREN=N
C
4000  GO TO 200
5000  GO TO 100
6000  RETURN
      END
$IBFTC FINAL  LIST
      SUBROUTINE FINAL
      COMMON /INPT/  CASENO,LOADIN,MINM,MAXM,DELTAM,MINN,MAXN,DELTAN,
1      RADIUS,LENGTH,STRSP,FRAMSP,EX,EY,DX,DY,GXY,UXY,
2      PRIMMX,PRIMMY,MX,MY,ES,GS,AS,IOS,JS,ZBARS,ER,GR,
3      AR,IOR,JR,ZBARR,NOP,PI,PRNTOP,NXORNY
      COMMON /OUT/   X(5),Y(5),VALUE(5)
      COMMON /INTER/ FMXMY,ERARL,TEMP,STOREM,STOREN
      COMMON /RENAME/ A,D,L,R
      DIMENSION FN(5)
      REAL MINM,MAXM,MINN,MAXN,LENGTH,MX,MY,IOS,JS,IOR,JR,NBAR,M,N
1      ,NXORNY
      VALUE(1)=TEMP
      X(1)=STOREM
      Y(1)=2.0*STOREN
      FI(1)=STOREN

```

TABLE XXVII - Fortran Listing - Program 3962 (Cont'd)

```

X(2)=X(1)-1.0
X(3)=X(1)+1.
X(4)=X(1)
X(5)=X(1)
Y(2)=Y(1)
FN(2)=FN(1)
Y(3)=Y(1)
FN(3)=FN(1)
Y(4)=Y(1)-2.0
FN(4)=FN(1)-1.0
Y(5)=Y(1)+2.0
FN(5)=FN(1)+1.0
DO 500 I=2,5
  FMPIA = X(I)*PI/A
  FNR = FN(I)/R
  N = FN(I)
  M=X(I)
  VALUE(I) = NBAR(EX,FMXY,ES,AS,D,FMPIA,GXY,FNR,PRIMMY,R,ZBARS,EY,
1          ERARL,ZBARR,DX,IOS,DX,Y,GS,JS,GR,JR,L,DY,ER,IOR,N,
2          NOP,M,PRNTOP,NXORNY)
500 CONTINUE
  RETURN
  END
$IBFTC OUTPUT LIST
  SUBROUTINE OUTPUT
  COMMON /INPT/  CASENO,LOADIN,MINM,MAXM,DELTAM,MINN,MAXN,DELTAN,
1          RADIUS,LENGTH,STRSP,FRAMSP,EX,EY,DX,DY,GXY,DX,Y,
2          PRIMMX,PRIMMY,MX,MY,ES,GS,AS,IOS,JS,ZBARS,ER,GR,
3          AR,IOR,JR,ZBARR,NOP,PI,PRNTOP,IOXORNY
  COMMON /OUT/   X(5),Y(5),VALUE(5)
  REAL MINM,MAXM,MINN,MAXN,LENGTH,MX,MY,IOS,JS,IOR,JR,NBAR,NXORNY
  WRITE (6,100) CASENO,MINM,MAXM,DELTAM,MINN,MAXN,DELTAN
100 FORMAT (1H1,1X,8HCASE NO.,9X,5HMIN M,16X,5HMAX M,14X,7HDELTA M,
1          14X,5HMIN N,15X,5HMAX N,14X,7HDELTA N // 1X,A6,1P6E20.4 )
  GO TO (200,250), NOP
200 WRITE (6,201) RADIUS,LENGTH,STRSP,FRAMSP,NXORNY
201 FORMAT (/// 18X,8HCYLINDER,12X,7HOVERALL,13X,8HSTRINGER,13X,
1          5HFRAME / 19X,6HRADIUS,14X,6HLENGTH,14X,7HSPACING,12X,
2          7HSPACING,34X,7HN BAR Y // 7X,1P4E20.4,1PE40.4)
  GO TO 299
250 WRITE (6,251) RADIUS,LENGTH,STRSP,FRAMSP,NXORNY
251 FORMAT (/// 18X,8HCYLINDER,12X,7HOVERALL,13X,8HSTRINGER,13X,
1          5HFRAME / 19X,6HRADIUS,14X,6HLENGTH,14X,7HSPACING,12X,
2          7HSPACING,34X,7HN BAR X // 7X,1P4E20.4,1PE40.4)
299 WRITE (6,300) EX,EY,DX,DY,GXY,DX,Y
300 FORMAT (/// 18X,7HE SUB X,13X,7HE SUB Y,13X,7HD SUB X,13X,
1          7HD SUB Y,13X,8HG SUB XY,12X,8HD SUB XY // 7X,1P6E20.4 )
  WRITE (6,400) PRIMMX,PRIMMY,MX,MY
400 FORMAT (/// 18X,8HMU PRIME,12X,8HMU PRIME / 19X,5HSUB X,15X,
1          5HSUB Y,14X,8HMU SUB X,12X,8HMU SUB Y // 7X,1P6E20.4 )
  WRITE (6,500) ES,GS,AS,IOS,JS,ZBARS
500 FORMAT (/// 119X,5HZ BAR / 18X,7HE SUB S,13X,7HG SUB S,13X,
1          7HA SUB S,13X,8HI SUB OS,12X,7HJ SUB S,14X,5HSUB S //
2          7X,1P6E20.4 )
  WRITE (6,600) ER,GR,AR,IOR,JR,ZBARR
600 FORMAT (/// 119X,5HZ BAR / 18X,7HE SUB R,13X,7HG SUB R,13X,
1          7HA SUB R,13X,8HI SUB OR,12X,7HJ SUB R,14X,5HSUB R //
2          7X,1P6E20.4 )

```

TABLE XXVII - Fortran Listing - Program 3962 (Cont'd)

```

WRITE (6,700)
700 FORMAT (6(/),51X,29HCRITICAL COMBINED LOAD VALUES ///
```

*	12X, 9HNUMBER OF, 24X,
1	9HNUMBER OF,24X,10HAXIAL LOAD,23X,9HHOOP LOAD / 11X,
2	12HLONGITUDINAL,19X,15HCIRCUMFERENTIAL,21X,9HBAR SUB X,
3	24X,9HBAR SUB Y / 12X,10HHALF WAVES,23X,10HHALF WAVES,
4	23X,10HLBS PER IN,23X,10HLBS PER IN // )

```

GO TO (1000,2000),NOP
1000 WRITE (6,1001) (X(I),Y(I),VALUE(I),NXORNY,I=1,5)
1001 FORMAT (1X,1PE21.4,1P3E33.4// (1X,1PE21.4,1P3E33.4))
RETURN
2000 WRITE (6,1001) (X(I),Y(I),NXORNY,VALUE(I),I=1,5)
3000 RETURN
END
@ END OF FILE OR DATA CARD DATA FOLLOWS
EXAMPLE 4 OF SECTION 8.3, HOOP COMPRESSIVE RUNNING LOAD = 799.1 LBS/IN.

```

1	0						
1COMBY	1.0	12.0	1.0	0.0	12.0	1.0	799.1
38.6	7.2	2.48	6.0				
1.5 +06	2.0 +06	2.5 +02	3.0 +02	2.0 +05	2.0 +02		
2.5 -01	3.5 -01	3.0 -01	4.0 -01				
3.0 +07	1.2 +07	2.0 -02	5.0 -03	4.0 -03	5.0 -01		
2.5 +07	1.0 +07	4.0 -02	1.0 -02	6.0 -03	7.5 -01		

18.3.3 C<sub>R</sub> Correction Factor - The program developed to evaluate a correction factor  $C_R$  which accounts for the effects of finite stiffener spacing on the ring requirement to prevent general instability is designated as General Dynamics Program No. 3942I. This program was used to generate data from which the design curves for the correction factor  $C_R$  shown in Figure 35 of Section 13.1 were obtained. It is expected that the design curves will be adequate for their purposes; however, the computer program can be employed to advantage if a number of points are to be obtained, investigations are to be made beyond the range plotted, or increased accuracy is desired. This program is convenient for preparation of values for input to program 3942 discussed in Section 18.3.1. The applications for  $C_R$  values obtained from program 3942I are discussed in Sections 7.2.1 and 13.1.

The input format is shown in Figure 63. Symbols used are shown in Table XXVIII. Detailed discussion of input, card by card, follows. Runs may be stacked.

CARD TYPE 1: One card per case.

Enter CASENO (case number) as right adjusted integer(s) in columns 1-5.

Enter NS (number of longitudinal stiffeners) as right adjusted integer(s) in columns 6-10.

Enter R2AI (circumferential stiffener parameter,  $R^2 A_r / I_r$ ) in columns 11-30 (D20.5).



A sample input coding form is shown in Figure 64. Note that double precision is employed for input of R2AI.

The execution time is very small and at least 50 cases may be run per second of 7094 DCS time at General Dynamics Convair.

The program output consists of a listing of the input values and the quantities K, K1, K2, and CSUBR as defined in Table XXVIII. The values K, K1, and K2 are printed to 17 significant figures while CSUBR is printed to 5 significant figures. A sample output listing is shown in Figure 65 for the sample input data of Figure 64.

Notation used in Program 3942I vs. the symbols employed in Parts I and II of the report appears in Table XXVIII. A basic flow diagram for the program is given in Figure 66 and a Fortran listing of the program, including input data from the sample coding form (Figure 64), is given in Table XXIX.

80 COLUMN-GENERAL PURPOSE-WORKSHEET

LINE	PROGRAM	PROGRAMMER	WORK ORDER
1	3942 I	L.S. FØSSUM	
2	98 .34751		
3	98 .34668		
4	98 .35059		
5	60 .11354		
6	60 .11392		
7	60 .11492		
8	60 .11589		
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			

Figure 64 - Sample Input Data - Program 3942I

CR CORRECTION FACTOR

CASE NO	NS	(R A / I) <sup>2</sup>	K	K1	K2	C SUB P
1	98	3.4751D 04	7.3222389573857060D-07	3.1194369578235380D 01	3.1194368846011490D 01	9.9918D-01
2	98	3.4668D 04	7.3222389573857060D-07	3.1194369578235380D 01	3.1194368846011490D 01	9.9719D-01
3	98	3.5059D 04	7.3222389573857060D-07	3.1194369578235380D 01	3.1194368846011490D 01	9.9318D-01
4	60	1.1354D 05	3.1916126399522680D-06	1.9098596362640080D 01	1.9098593171027440D 01	9.8138D-01
5	60	1.1392D 05	3.1916126399522680D-06	1.9098596362640080D 01	1.9098593171027440D 01	9.8132D-01
6	60	1.1492D 05	3.1916126399522680D-06	1.9098596362640080D 01	1.9098593171027440D 01	9.8116D-01
7	60	1.1589D 05	3.1916126399522680D-06	1.9098596362640080D 01	1.9098593171027440D 01	9.8107D-01

GENERAL DYNAMICS  
Convair Division

Figure 65 - Sample Output Listing - Program 3942I

TABLE XXVIII - Program 3942I Notation

<u>PROGRAM NOTATION</u>	<u>REPORT NOTATION</u>	<u>DESCRIPTION</u>
CASENO	-	Case number.
CR	$C_R$	Correction factor; $CR = 1 / \left[ 1 + \pi \left( \frac{K}{N_s} \right) \left( R^2 A_r / I_r \right) \right]$
K1	-	$K1 = \frac{1}{\sin^2 \theta} \left( \frac{\theta}{2} + \frac{\sin \theta \cos \theta}{2} \right) .$
K2	-	$K2 = \frac{1}{\theta} .$
K	-	$K = K1 - K2 .$
NS	$N_s$	Number of stringers
R2AI	$R^2 A_r / I_r$	Circumferential stiffener parameter.
THETA	$\theta$	$2\theta =$ angle between stringers.

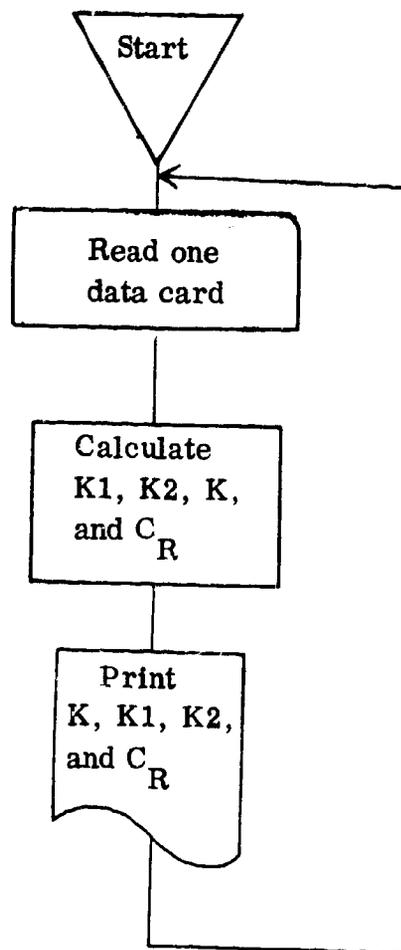


Figure 66 - Flow Diagram - Program 3942I

TABLE XXIX - Fortran Listing - Program 3942I

```

$EXECUTE      1BJOB
$1BJOB
$1BFTC KCR
      DOUBLE PRECISION R2AI,PI,THETA,K,K1,K2,CR,DNS
      PI = 3.141592653589793
C
C      WRITE HEADING
      WRITE (6,25)
25  FORMAT (1H1,56X,2UHCR CORRECTION FACTOR ///)
      WRITE (6,50)
50  FORMAT (/// 23X,1H2 / 8H CASE NO 4X,2HNS 7X,9H(R A / I) 17X,1HK
50*      27X,2HK1,26X,2HK2,17X,7HC SUB R /// )
C
C      READ A CASE
100 READ (5,101) CASENO,NS,R2AI
101 FORMAT (A5,I5,D20.5 )
C
C      CALCULATE
      FNS = FLOAT('S')
      DNS = DBLE(FNS)
      THETA = PI/DNS
      K1 = (1.0 / (DSIN(THETA)**2) * (THETA/ 2.0 + DSIN(THETA)*
1      DCOS(THETA) / 2.0)
      K2 = 1.0 / THETA
      K = K1 - K2
      CR = 1.0 / (1.0 + (PI*K/DNS)*R2AI)
C
C      PRINT
      WRITE (6,300) CASENO,NS,R2AI,K,K1,K2,CR
300 FORMAT (1X,A5,4X,I5,1PD16.4,3D28.16,3D16.4 / )
      GO TO 100
      END

```

END OF FILE OR DATA CARD DATA FOLLOWS

1	98	.34751	+05
2	98	.34068	+05
3	98	.35059	+05
4	00	.11354	+06
5	00	.11392	+06
6	00	.11492	+06
7	00	.11589	+06

19.0 REFERENCES

1. DeLuzio, A. and Lunsford, L. R., "Theoretical and Experimental Analysis of Orthotropic-Shell Stability," Contract NAS 8-9500, Lockheed Missiles and Space Company Report LMSC-A701014, 11 September 1964.
2. Anonymous, "Structural Panel Stability Development Program, Final Report, Phase I," Contract NAS 8-9500, Lockheed Missiles and Space Company Report LMSC-A746888, 30 April 1965.
3. Anonymous, "Structural Panel Stability Development Program," Contract NAS 8-9500, Lockheed Missiles and Space Company Report LMSC-A770422, 30 October 1965.
4. Card, M. F., "Preliminary Results of Compression Tests on Cylinders with Eccentric Longitudinal Stiffeners," NASA TMX-1004, September 1964.
5. Block, D. L., Card, M. F. and Mikulas, M. M., Jr., "Buckling of Eccentrically Stiffened Orthotropic Cylinders," NASA TN D-2960, August 1965.
6. Schapitz, E., Festigkeitslehre für den Leichtbau, 2 Aufl., VDI-Verlag GmbH, Düsseldorf, 1963.
7. Shanley, F. R., "Simplified Analysis of General Instability of Stiffened Shells in Pure Bending," Journal of the Aeronautical Sciences, October 1949.
8. Cheatham, J. F., "Buckling Characteristics of Corrugated Cylinders," Part II - Summary, Watertown Arsenal Technical Report No. WAL TR 715/3-Pt. 2, July 1960.
9. Zahn, J. J. and Kuenzi, E. W., "Classical Buckling of Cylinders of Sandwich Construction in Axial Compression-Orthotropic Cores," Forest Products Laboratory Report FPL-018, November 1963.
10. Stuhlman, C. E., "Computer Program for Integrally Stiffened Cylinders Under Axial Compression," Contract NAS 8-9500, Lockheed Missiles and Space Company Report LMSC-A740914, Rev. 1, 23 October 1965.
11. Anonymous, "Buckling of Thin-Walled Circular Cylinders," NASA Space Vehicle Design Criteria, NASA SP-8007, September 1965.

12. Timoshenko, S. P. and Gere, J. M., Theory of Elastic Stability, McGraw-Hill Book Company, Inc., New York, 1961.
13. Seide, P., Weingarten, V. I., and Morgan, E. J., "Final Report on the Development of Design Criteria for Elastic Stability of Thin Shell Structures," STL-TR-60-0000-19425, 31 December 1960.
14. Schumacher, J. G., "Development of Design Curves for the Stability of Thin Pressurized and Unpressurized Circular Cylinders," General Dynamics Convair Report No. AZS-27-275, Revision B, 22 July 1960.
15. Gerard, G. and Becker, H., "Handbook of Structural Stability, Part I - Buckling of Flat Plates," NACA TN 3781, July 1957.
16. Peterson, J. P., Whitely, R. O., and Deaton, J. W., "Structural Behavior and Compressive Strength of Circular Cylinders with Longitudinal Stiffening," NASA TN D-1251, May 1962.
17. Cox, H. L. and Clenshaw, W. J., "Compression Tests on Curved Plates of Thin Sheet Duralumin," British A.R.C. Technical Report R. & M. No. 1894, November 1941.
18. Langhaar, H. L., "General Theory of Buckling," Applied Mechanics Reviews, Vol. 11, No. 11, November 1958, p.585.
19. Thielemann, W. F., "New Developments in the Nonlinear Theories of the Buckling of Thin Cylindrical Shells," Proceedings of the Durand Centennial Conference, Held at Stanford University, 5-8 August 1959, Pergamon Press, New York, 1960.
20. Almroth, B. O., "Buckling of Orthotropic Cylinders Under Axial Compression," Lockheed Missiles and Space Company Report LMSC-6-90-63-65, June 1963.
21. Almroth, B. O., "Postbuckling Behavior of Orthotropic Cylinders Under Axial Compression," AIAA Journal, Vol. 2, No. 10, October 1964, pp 1795-1799.
22. Peterson, J. P. and Dow, M. B., "Compression Tests on Circular Cylinders Stiffened Longitudinally by Closely Spaced Z-Section Stringers," NASA Memo 2-12-59L, March 1959.

GENERAL DYNAMICS  
Convair Division

23. DeLuzio, A. J., Stuhlman, C. E., and Almroth, B. O., "Influence of Stiffener Eccentricity and End Moment on the Stability of Cylinders in Compression," Paper Presented at AIAA 6th Structures and Materials Conference, Palm Springs, Calif., April 5-7, 1965.
24. Ramberg, W. and Osgood, W. R., "Description of Stress-Strain Curves by Three Parameters," NACA TN 902, July 1943.
25. Budiansky, B., Seide, P., and Weinberger, R. A., "The Buckling of a Column on Equally Spaced Deflectional and Rotational Springs", NACA TN 1519, March 1948.
26. Smith, G. W., "Analysis of Multiple Discontinuities in Shells of Revolution Including Coupled Effects of Meridional Load", General Dynamics Convair Report Number GD/A 63-0044, 31 July 1963.
27. Anon., NASA (MSFC) Structures Manual, Page 17, Section C1, 25 May 1961.
28. Balog, E. M., "Subscale Intertank Structural Test Program", Boeing Document No. T5-6029, Vol's. 1 and 2, 12 August 1965.
29. Card, M. F., "Bending Tests of Large-Diameter Stiffened Cylinders Susceptible to General Instability", NASA TN D-2200, April 1964.
30. Dunn, L. G., "Some Investigations of the General Instability of Stiffened Metal Cylinders", NACA Technical Note No. 1198, November 1947.
31. Hoff, N. J., Boley, B. A., and Nardo, S. V., "The Inward Bulge Type Buckling of Monocoque Cylinders", NACA Technical Note No. 1499, September 1948.
32. Becker, H., "Handbook of Structural Stability, Part VI - Strength of Stiffened Curved Plates and Shells", NACA Technical Note 3786, July 1958.
33. Stein, M. and Mayers, J., "Compressive Buckling of Simply Supported Curved Plates and Cylinders of Sandwich Construction", NACA Technical Note 2601, January 1952.
34. Flügge, W., Stresses in Shells, Springer-Verlag, Berlin, 1962.

35. Hetenyi, M., Beams on Elastic Foundation, University of Michigan Press, Ann Arbor, 1955.
36. Roark, R. J., Formulas for Stress and Strain, Third Edition, McGraw-Hill Book Company, Inc., New York, 1954.
37. Boley, B. A., "The Shearing Rigidity of Buckled Sheet Panels", *Journal of the Aeronautical Sciences*, June 1950.
38. Lakshmikantham, C., Gerard, G., and Milligan, R., "General Instability of Orthotropically Stiffened Cylinders, Part II, Bending and Combined Compression and Bending", Air Force Flight Dynamics Laboratory Technical Report AFFDL TR 65 161, Part II.
39. Hedgepeth, J. M. and Hall, D. B., "Stability of Stiffened Cylinders", *AIAA Journal*, Vol. 3, No. 12, December 1965.
40. Flügge, W., "Die Stabilität der Kreiszyllinderschale", *Ingenieur-Archiv*, Vol. 3, 1932, pp. 463-506.
41. Timoshenko, S., Theory of Elastic Stability, McGraw-Hill Book Company, Inc., New York, N.Y., 1932, pp 463-467.
42. Seide, P. and Weingarten, V. I., "On the Buckling of Circular Cylindrical Shells Under Pure Bending", *Trans. of the ASME, Journal of Applied Mechanics*, March 1961.
43. Horton, W. H. and Cox, J. W., "The Stability of Thin-Walled Unstiffened Circular Cylindrical Shells Under Nonuniformly Distributed Axial Load", Stanford University, Department of Aeronautics and Astronautics, Report SUDAER No. 220, February 1965.
44. Anon., "Some Investigations of the General Instability of Stiffened Metal Cylinders, VII - Stiffened Metal Cylinders Subjected to Combined Bending and Torsion", NACA Technical Note No. 911, November 1943.
45. Peterson, J. P., "Weight-Strength Studies of Structures Representative of Fuselage Construction", NACA Technical Note 4114, October 1957.
46. Almroth, B. O., "Postbuckling Behavior of Axially Compressed Circular Cylinders", *AIAA Journal*, Vol. 1, No. 3, March 1963.
47. Hoff, N. J., Madsen, W. A., and Mayers, J., "The Postbuckling Equilibrium of Axially Compressed Circular Cylindrical Shells", Stanford University Department of Aeronautics and Astronautics, Report SUDAER No. 221, February 1965.

APPENDIX A - SUPPLEMENTARY OPTIONS FOR ANALYSIS OF  
BUCKLING OF ISOTROPIC SKIN PANELS  
SUBJECTED TO EDGE COMPRESSION

The design curves of Section 11.2 for buckling of isotropic skin panels under edge compression were obtained using the Schapitz [6] criterion for buckling and the lower bound curve of Seide, et al. [13] for  $\sigma_R$ . This analysis approach was discussed in Section 5.2 and designated OPTION 1. Also presented in Section 5.2 were the mean expected and 90% probability analyses for  $\sigma_R$  from reference 14. The use of these latter analyses for  $\sigma_R$  with the Schapitz criterion was designated OPTION 2 and OPTION 3, respectively. As supplementary information which could occasionally be of value, plotted output from the digital computer program of Section 18.1 is presented here for OPTIONS 2 and 3 using Poisson's ratio as 0.30.

A.1 OPTION 2

A.1.1 General - The mean expected (50% probability) analysis of reference 14 is used in OPTION 2 for determining  $\sigma_R$ . The applicable expressions for  $\sigma_R$  are given in Section 5 as equations (5-8) and (5-9). Values of  $\sigma_R$  so obtained are used with equations (5-1) through (5-5) to obtain  $\sigma_{cr}$ , the critical buckling stress for the skin panel. These equations may be solved by hand or the buckling curves of Section A.1.2 may be employed. The digital computer program of Section 18.1 may also be used. OPTION 2 may be of value in predicting average expected buckling results and, when used with OPTIONS 1 or 3, can give an indication of the dispersion of data to be expected. OPTION 2 should not be considered

a design level and it must be expected that approximately one-half of all panels tested in the range where  $\sigma_{cr} = \sigma_R$  will buckle at stresses less than the OPTION 2 level.

For OPTION 2, the procedures of Section 11.1 may be used with the curves of A.1.2 substituted for those of 11.2.

A.1.2 Buckling Curves - Supplementary buckling curves generated by a S.C.4020 plotter using the digital computer program of Section 18.1 are presented in this section for OPTION 2. Table XXX lists the families provided here.

TABLE XXX - Table of Contents for the Supplementary Curves "Buckling of Isotropic Panels"(OPTION 2).

<u>Figure Number</u>	<u>Ordinate</u>	<u>Abscissa</u>	<u>a/b</u>	<u>K</u>	<u>Page</u>
67(a)	$\left( \frac{\text{Buckling Stress}}{\text{Elastic Modulus}} \right)$	$\frac{R}{t}$	.4	7.00	A-3
67(b)	"	"	.6	4.00	A-4
67(c)	"	"	.6	5.70	A-5
67(d)	"	"	.8	3.29	A-6
67(e)	"	"	.8	5.70	A-7
67(f)	"	"	1.0	3.29	A-8
67(g)	"	"	1.0	5.70	A-9
67(h)	"	"	2.0	3.29	A-10
67(i)	"	"	2.0	5.70	A-11
67(j)	"	"	4.0	3.29	A-12
67(k)	"	"	4.0	5.70	A-13
67(l)	"	"	10.0	3.29	A-14
67(m)	"	"	10.0	5.70	A-15
67(n)	"	"	100.	3.29	A-16
67(o)	"	"	100.	5.70	A-17

# BUCKLING OF ISOTROPIC PANELS

$A/B = 9.400$

$K = 7.000$

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

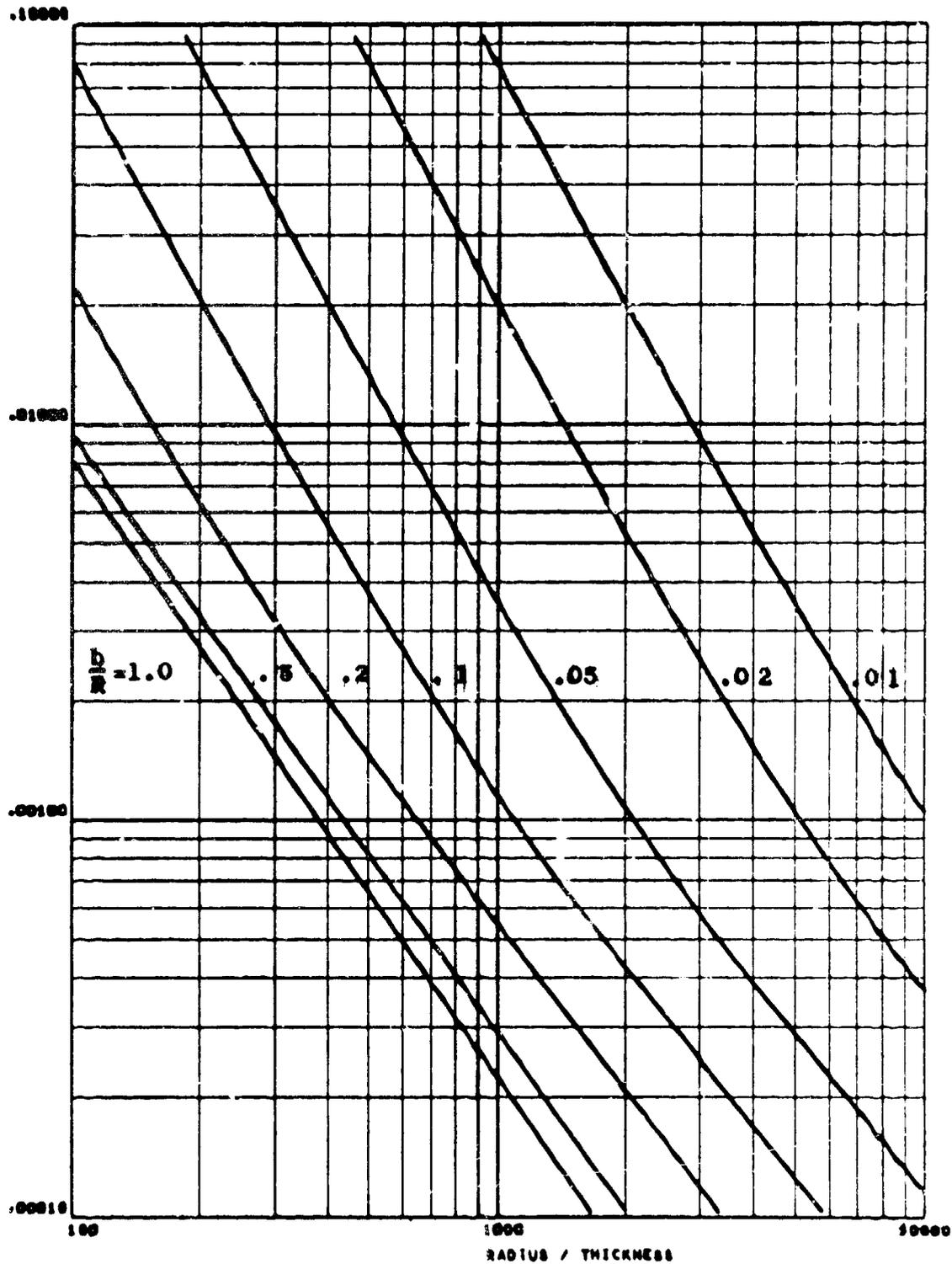


Figure 67(a) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.000

K = 4.000

OPTION 2

BUCKLING STRESS

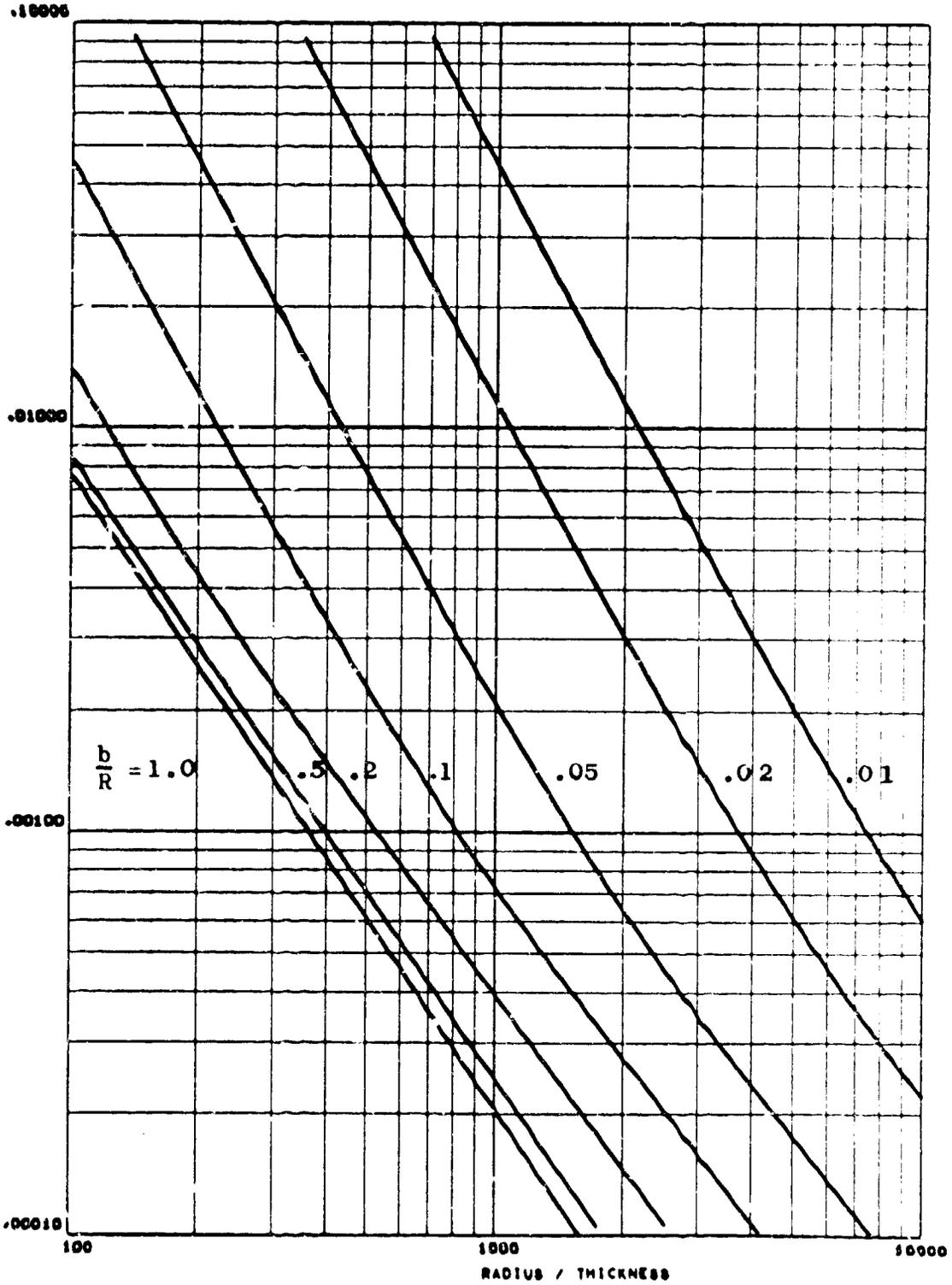


Figure 67(b) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

$\lambda/B = 0.600$

$\nu = 0.700$

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

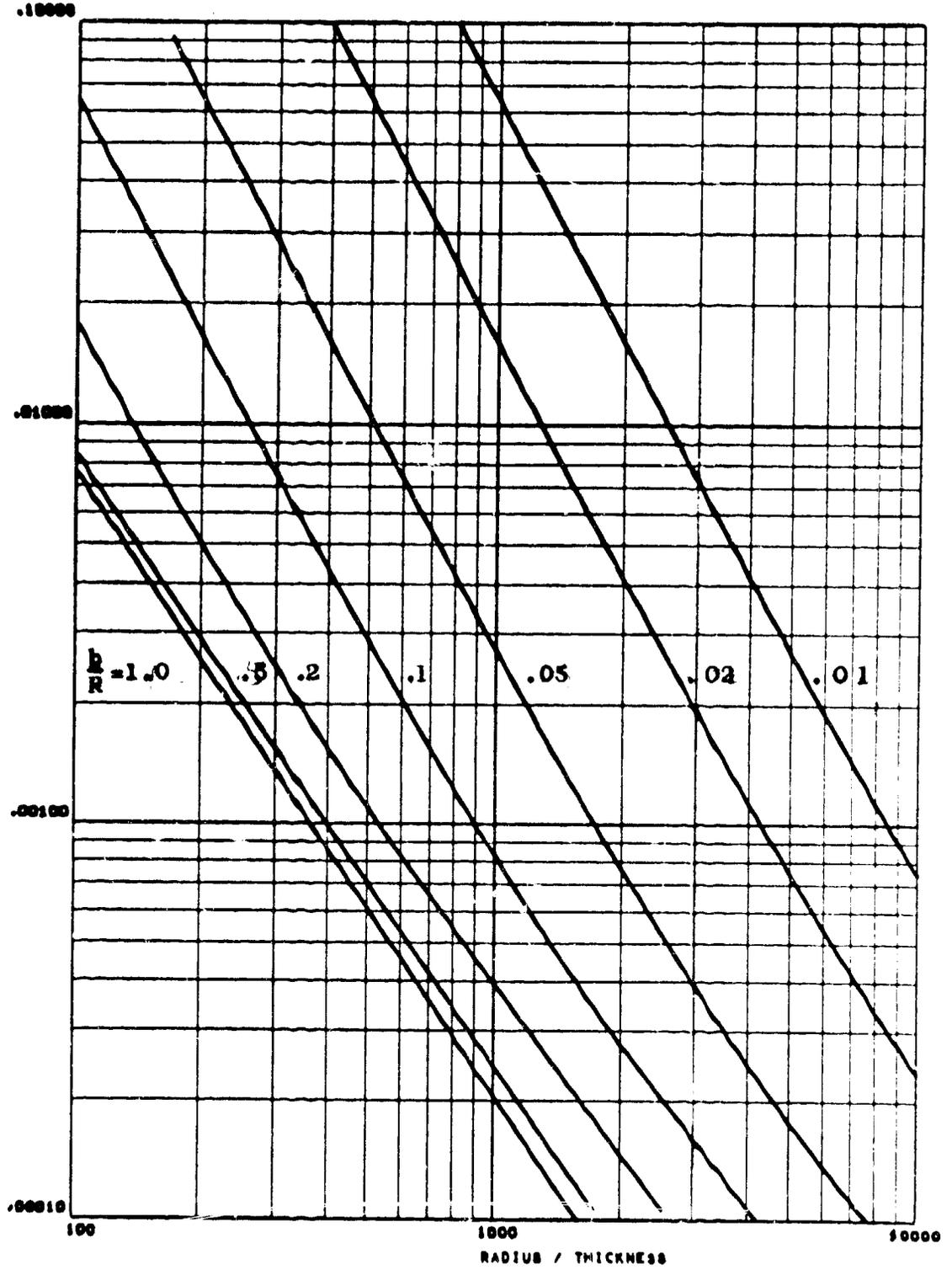


Figure 67(c) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.800

K = 3.200

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

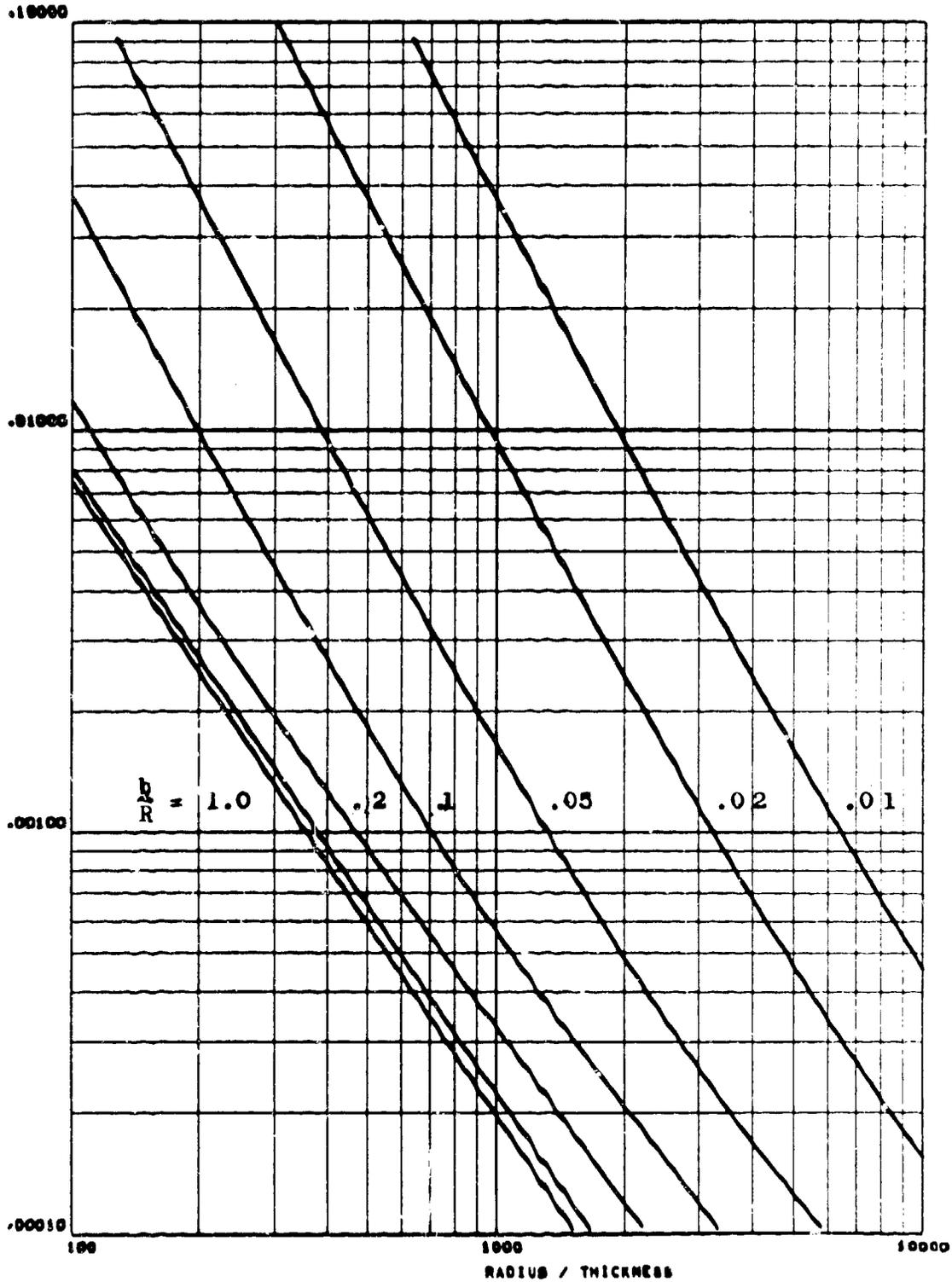


Figure 67(d) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.80

K = 9.700

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

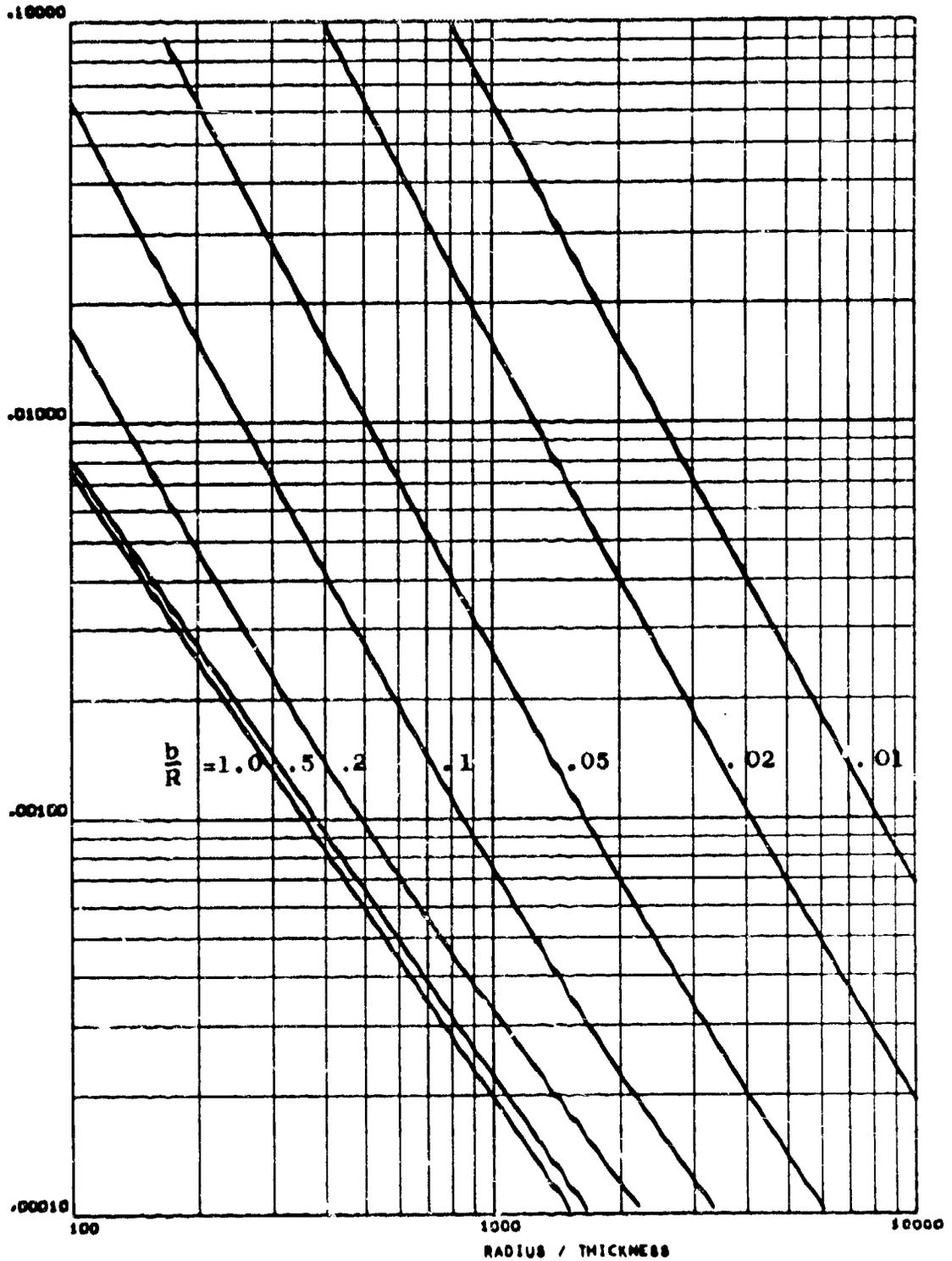


Figure 67(e) - (See Table XXV)

# BUCKLING OF ISOTROPIC PANELS

$A/B = 1.000$

$K = 3.200$

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

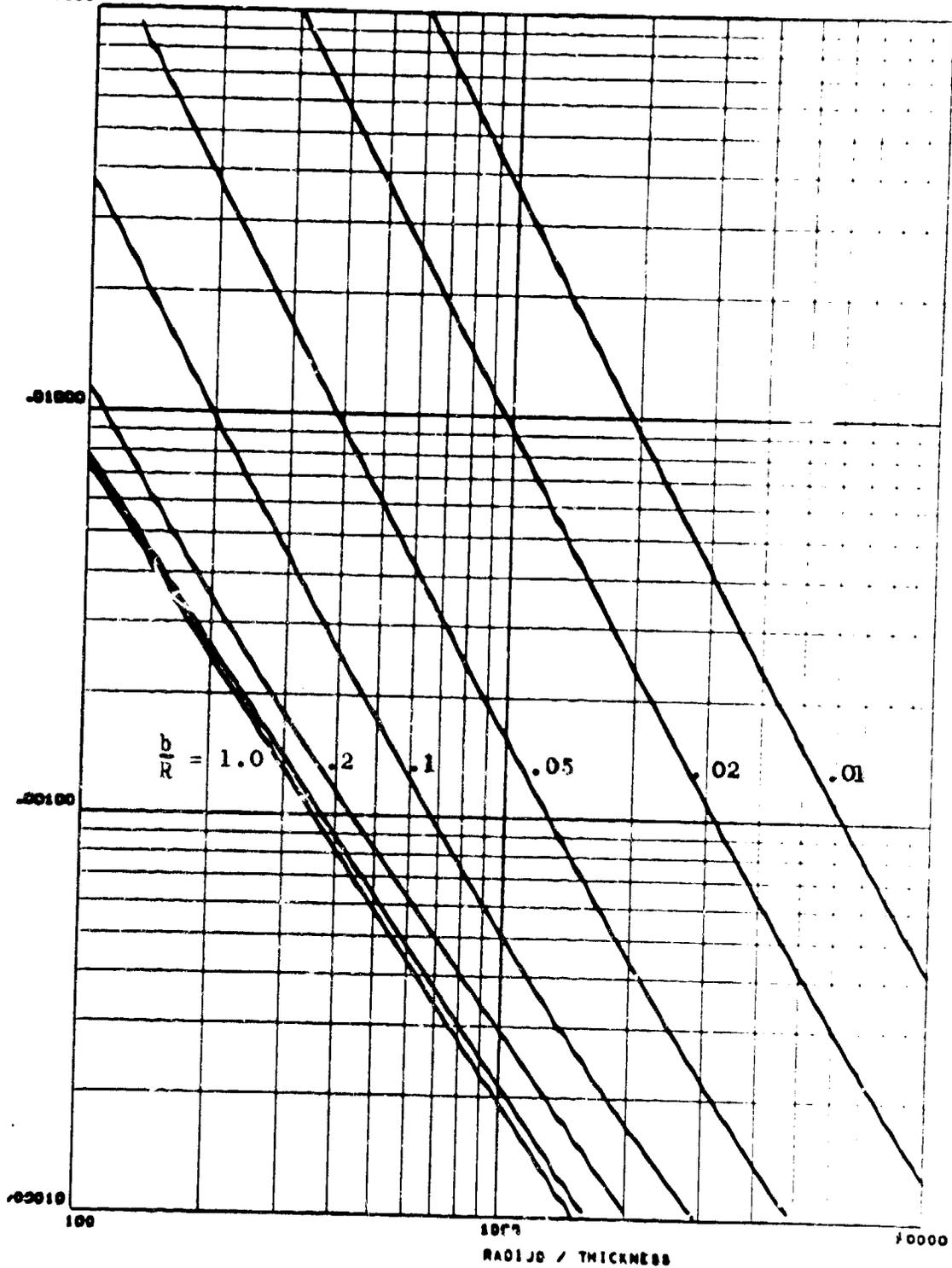


Figure 67(f) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

A/B = 1.000

K = 9.700

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

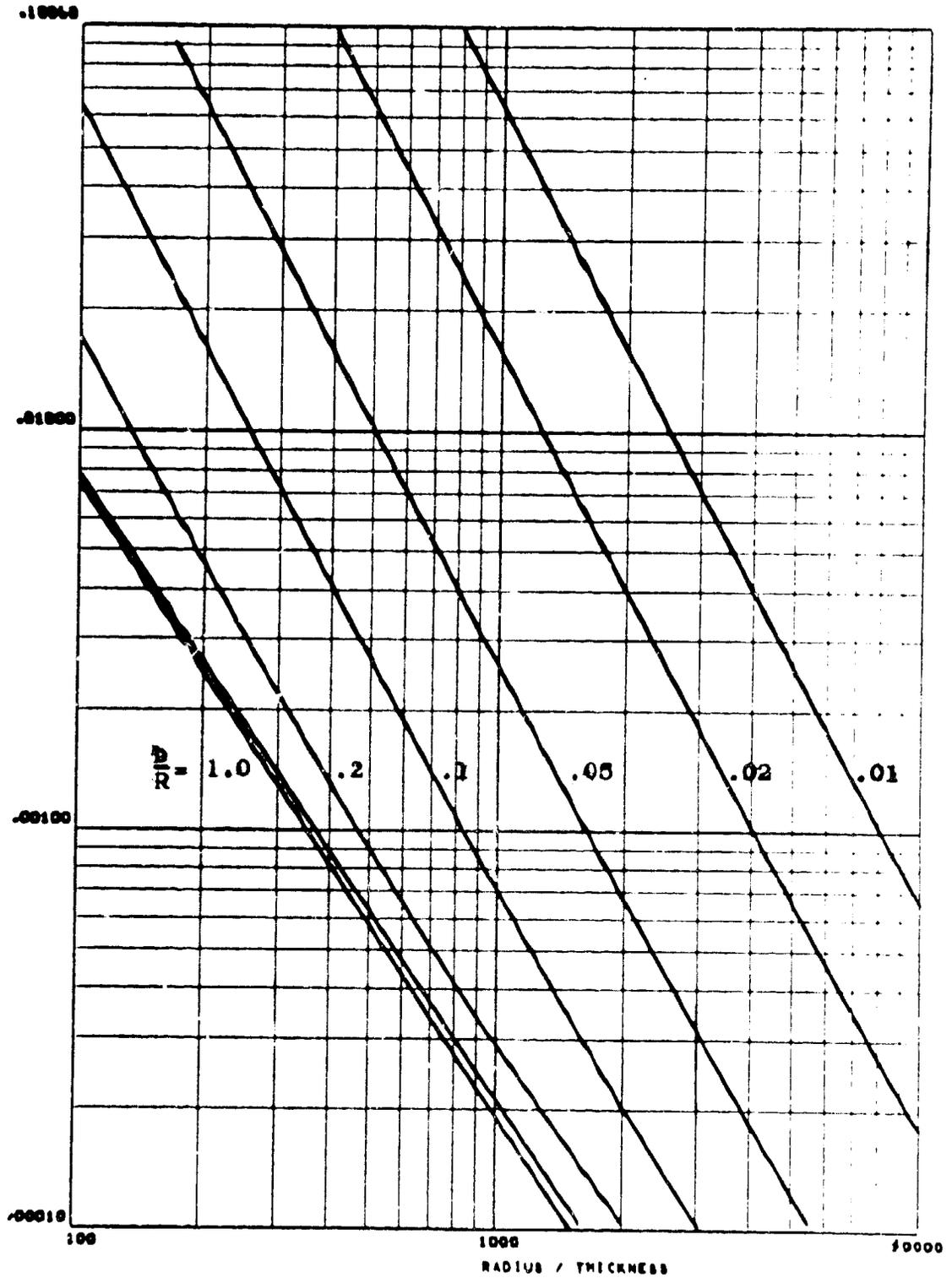


Figure 67(g) - (See Table XXX)

BUCKLING OF ISOTROPIC PANELS

A/B = 2.000

K = 3.800

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

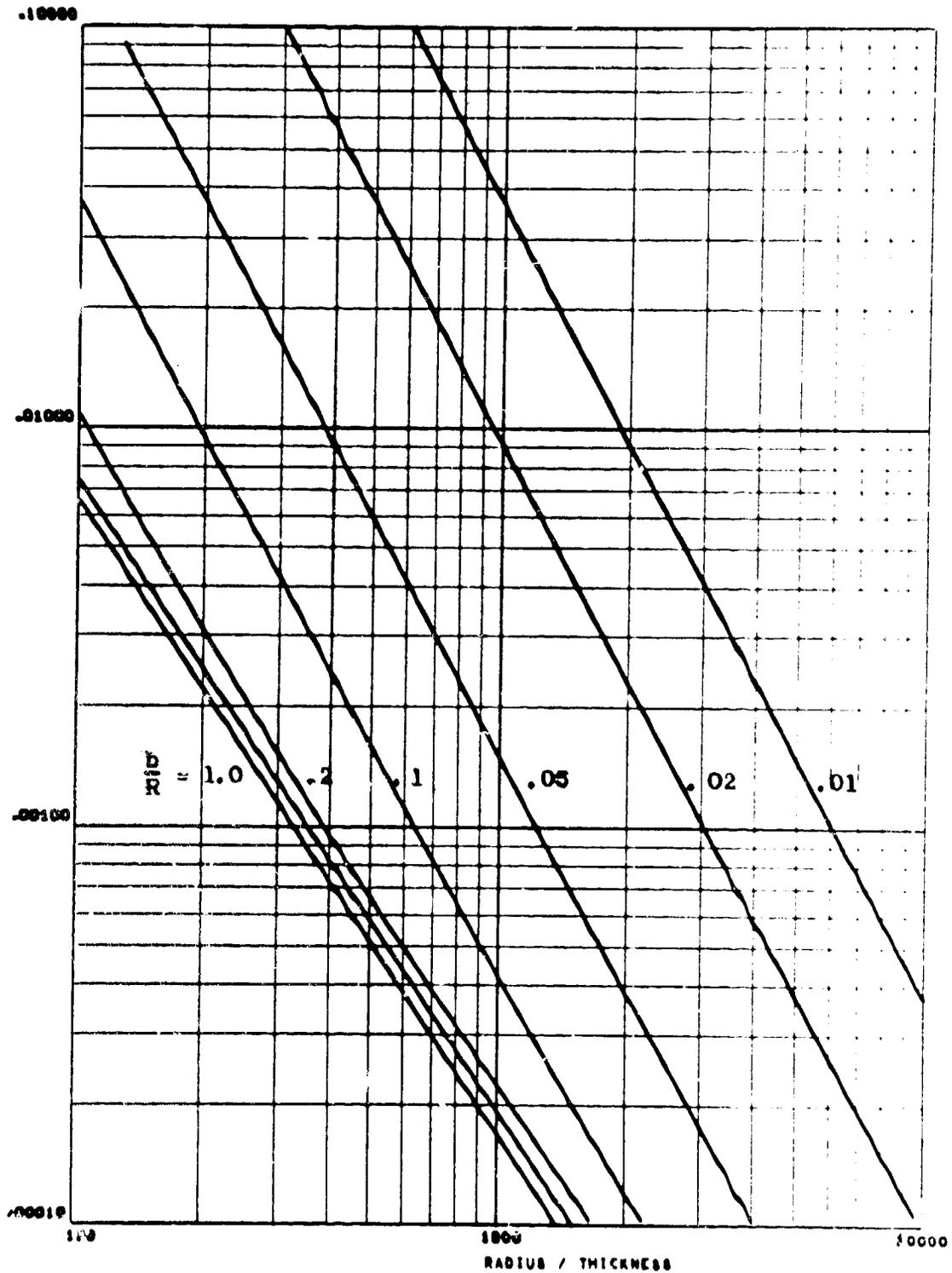


Figure 67(h) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

$A/B = 2.000$

$\nu = 0.300$

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

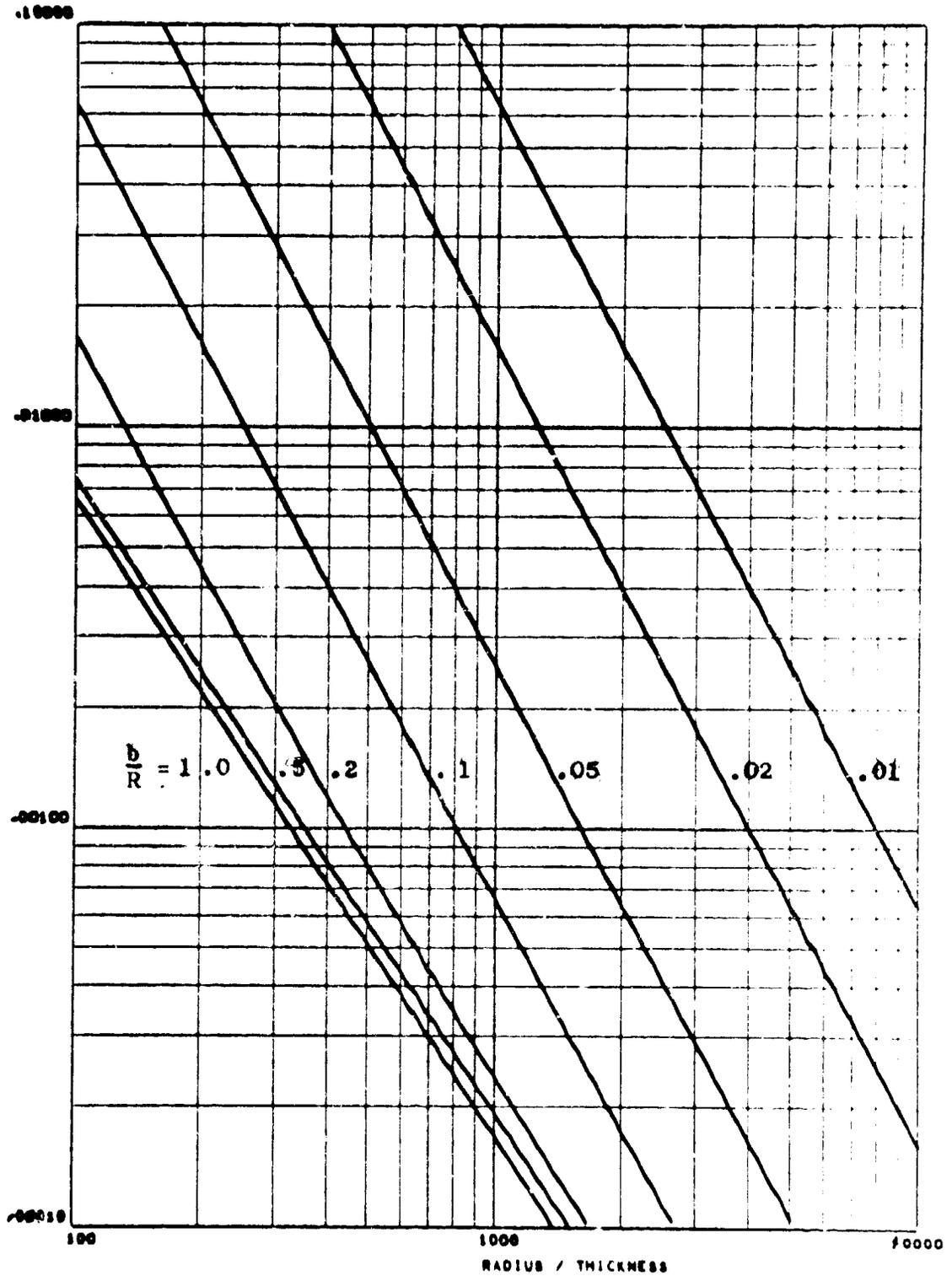


Figure 67(i) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

A/B = 4.000

K = 3.200

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

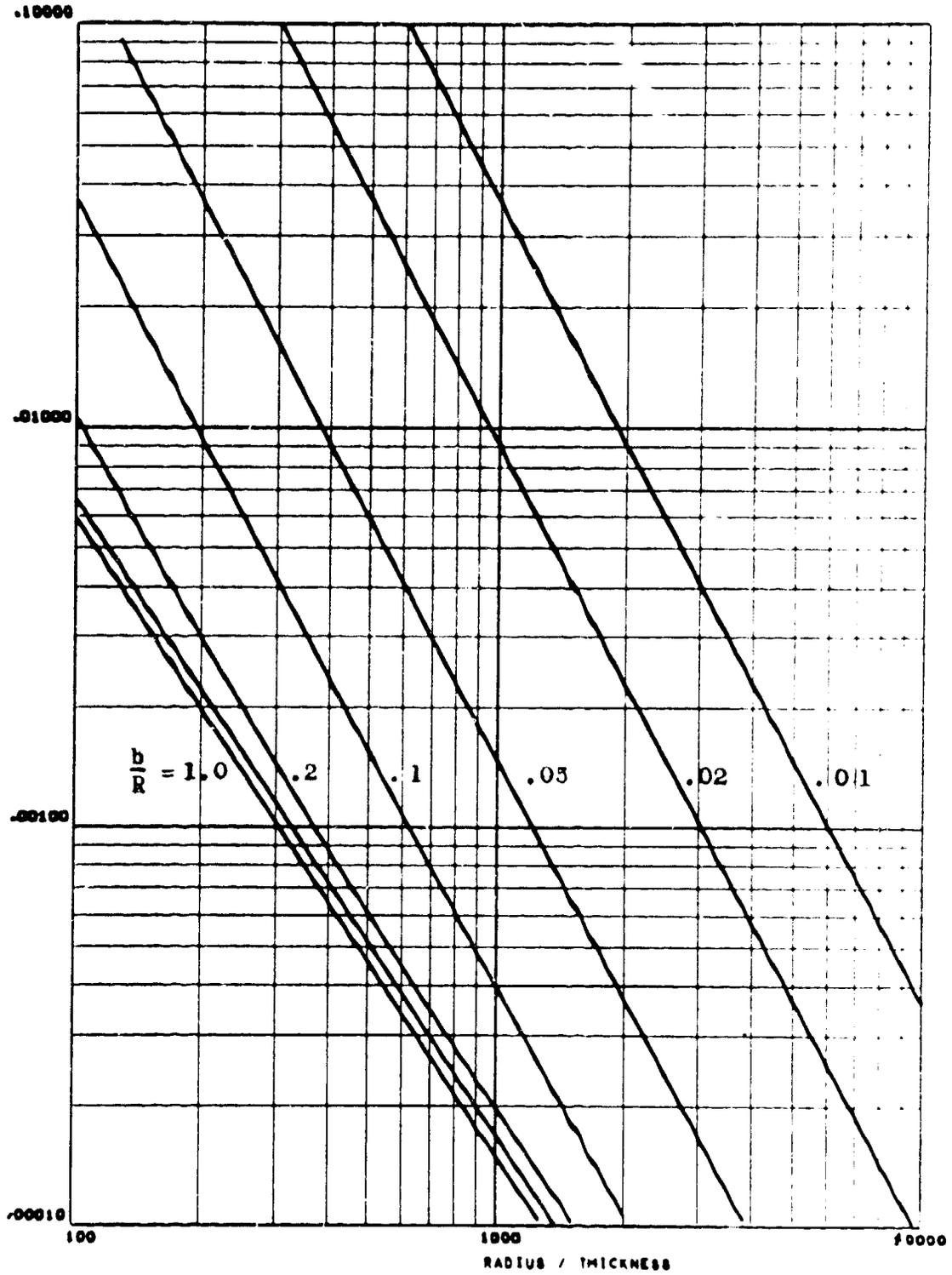


Figure 67(j) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

A/B = 4.000

K = 9.700

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

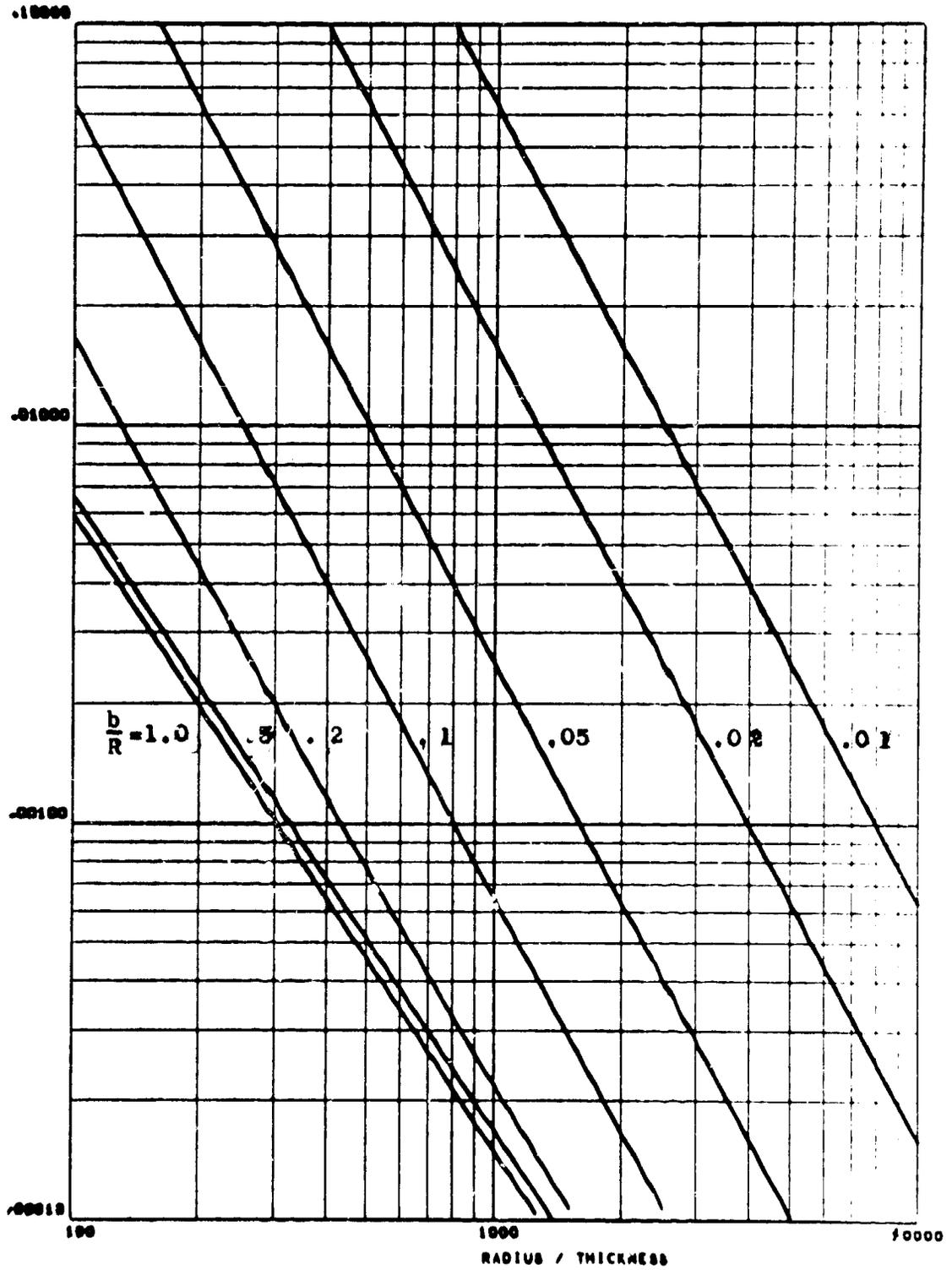


Figure 67(k) - (See Table XXX)

# BUCKLING OF ISOTROPIC PANELS

$\lambda/B = 10.000$

$\nu = 0.300$

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

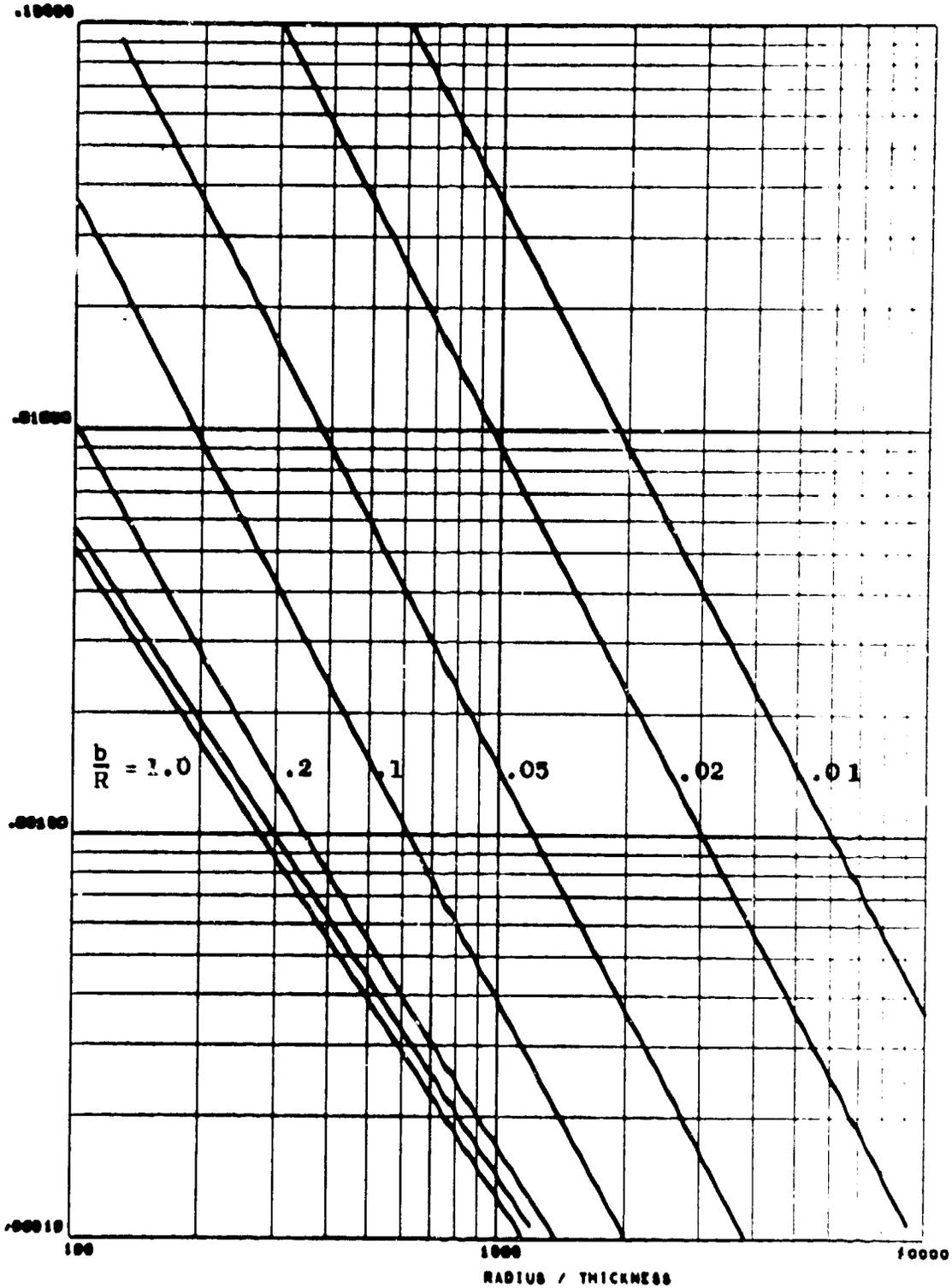


Figure 67(1) - (See Table XXX)

### BUCKLING OF ISOTROPIC PANELS

$\lambda/B = 10.000$

$K = 9.700$

OPTION 2

BUCKLING STRESS / ELASTIC MODULUS

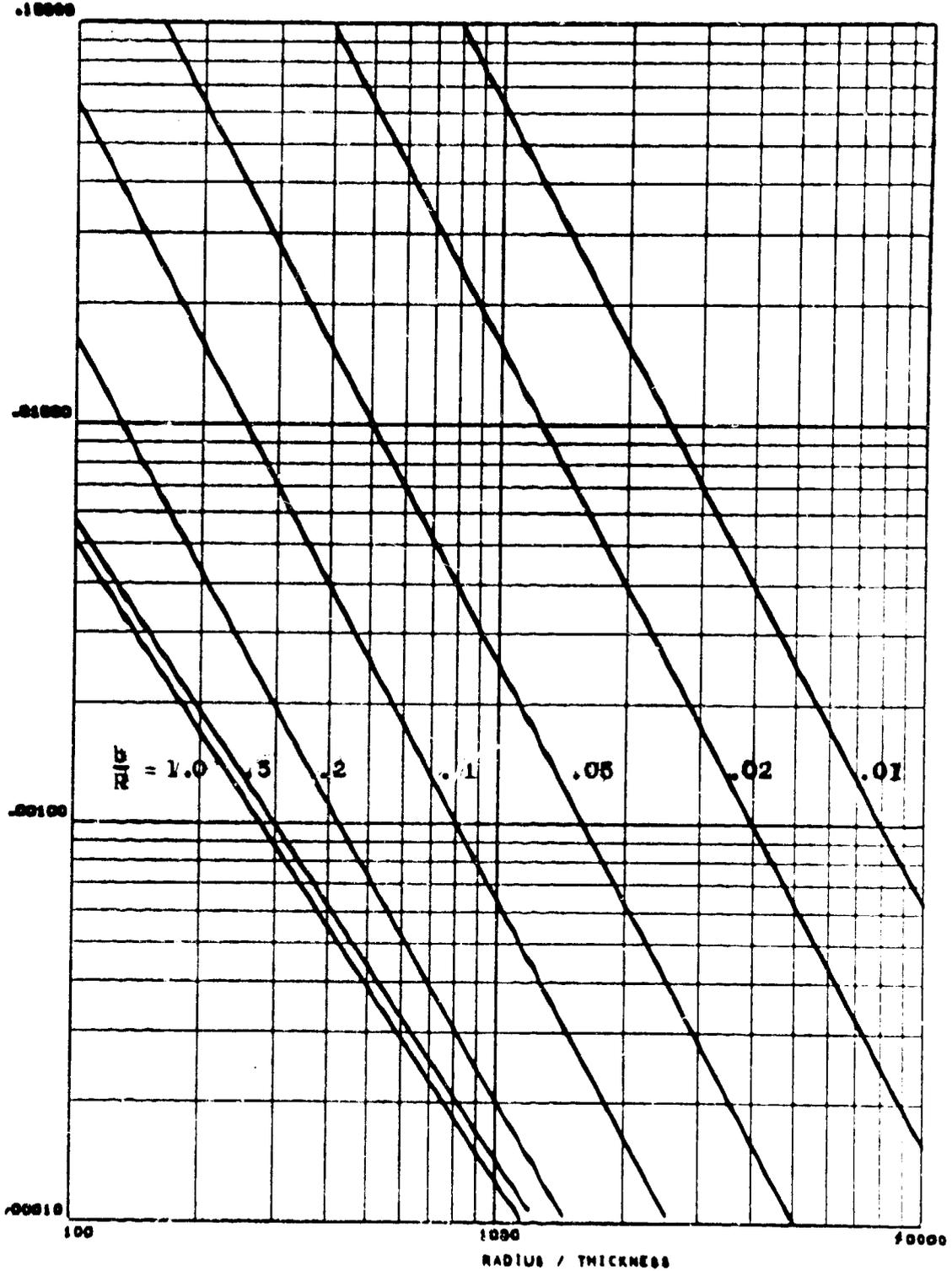


Figure 67(m) - (See Table XXX)

### BUCKLING OF ISOTROPIC PANELS

$\mu/B = 100$

$\kappa = 3.896$

OPTION 2

BUCKLING OF ISOTROPIC PANELS

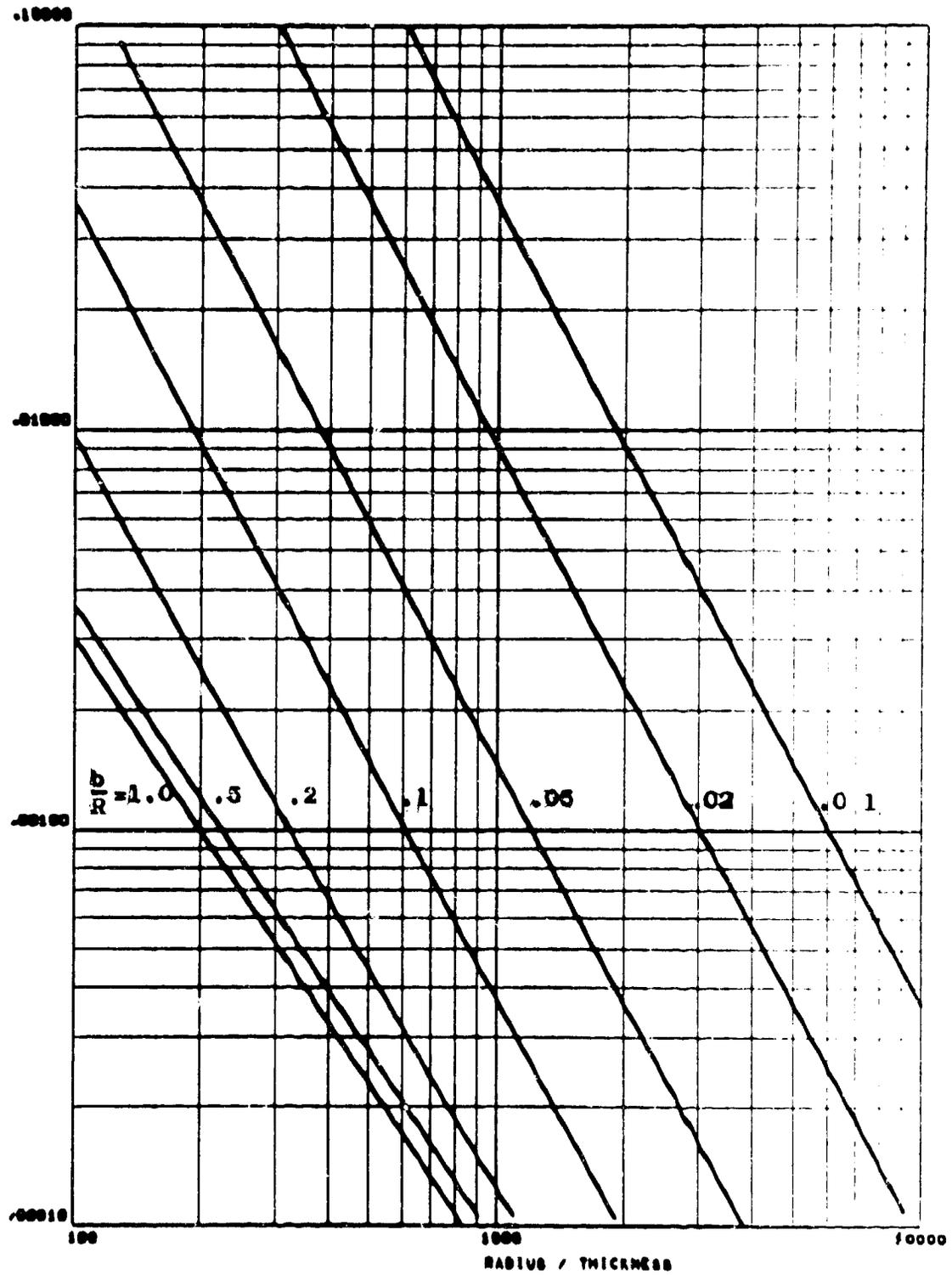


Figure 67(n) - (See Table XXX)

### BUCKLING OF ISOTROPIC PANELS

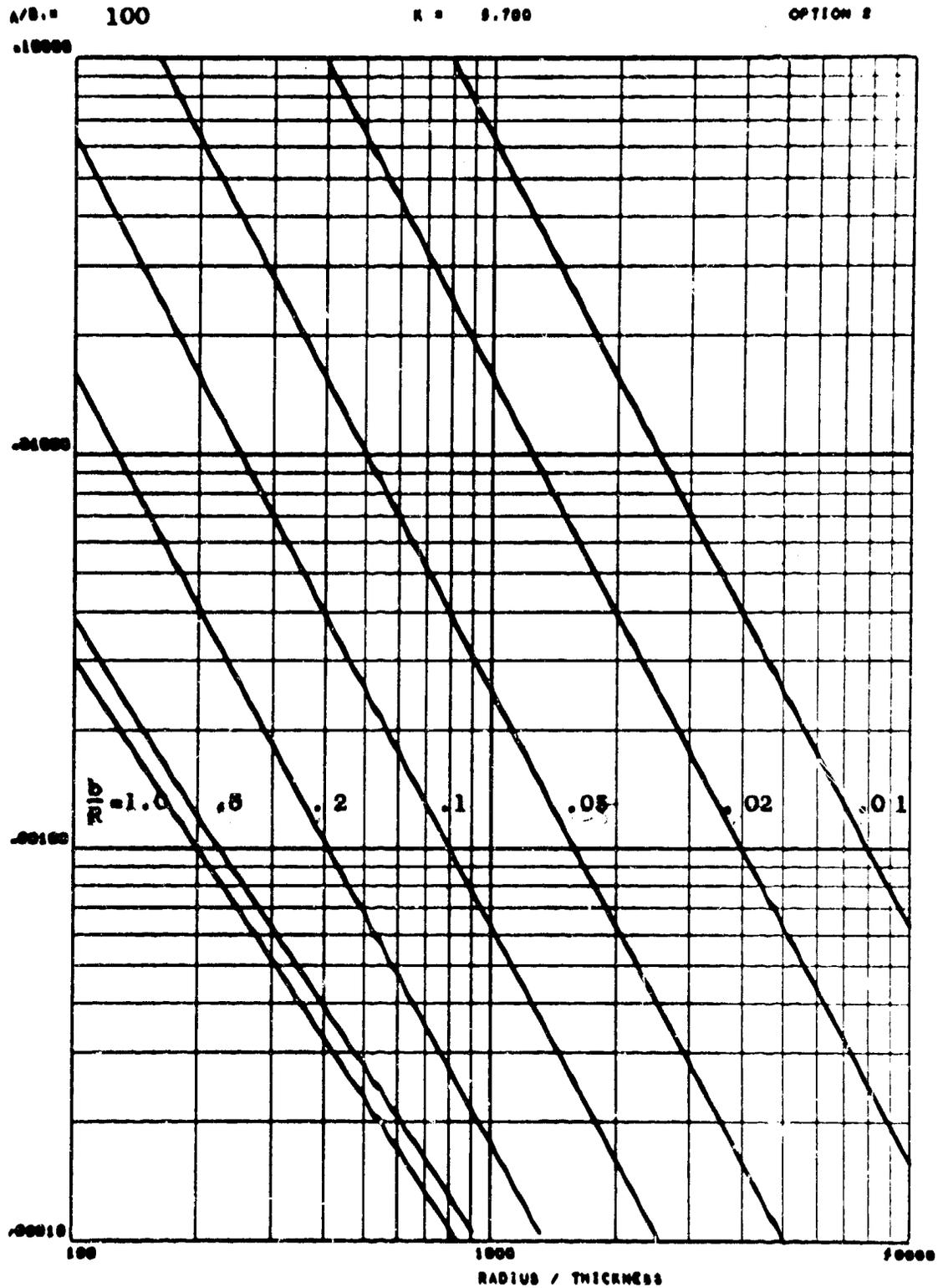


Figure 67(o) - (See Table XXX)

## A.2 OPTION 3

A.2.1 General - The 90% probability, 95% confidence analysis of reference 14 is used in OPTION 3 for determining  $\sigma_R$ . The applicable expressions for  $\sigma_R$  are given in Section 5 as equations (5-10) and (5-11). Values of  $\sigma_R$  so obtained are used with equations (5-1) through (5-5) to obtain  $\sigma_{cr}$ , the critical buckling stress for the skin panels. These equations may be solved by hand or the buckling curves of Section A.2.2 may be employed. The digital computer program of Section 18.1 may also be used. OPTION 3 may be of value when high reliability  $\sigma_R$  values are needed for unusual panel a/b ratios since, as discussed in Section 5.2, the analysis includes statistically inferred length effects. For most practical cases, however, the OPTION 1 analysis recommended for design gives essentially the same results.

For OPTION 3, the procedures of Section 11.1 may be used with the curves of A.2.2 substituted for those of 11.2.

A.2.2 Buckling Curves - Supplementary buckling curves generated by a S.C.4020 plotter using the digital computer program of Section 18.1 are presented in this section for OPTION 3. Table XXXI lists the families provided here.

TABLE XXXI - Table of Contents for the  
Supplementary Curves "Buckling  
of Isotropic Panels" (OPTION 3).

<u>Figure Number</u>	<u>Ordinate</u>	<u>Abscissa</u>	<u>a/b</u>	<u>K</u>	<u>Page</u>
68(a)	[ <u>Buckling Stress</u> <u>Elastic Modulus</u> ]	$\frac{R}{t}$	.4	7.00	A-20
68(b)	"	"	.6	4.00	A-21
68(c)	"	"	.6	5.70	A-22
68(d)	"	"	.8	3.29	A-23
68(e)	"	"	.8	5.70	A-24
68(f)	"	"	1.0	3.29	A-25
68(g)	"	"	1.0	5.70	A-26
68(h)	"	"	2.0	3.29	A-27
68(i)	"	"	2.0	5.70	A-28
68(j)	"	"	4.0	3.29	A-29
68(k)	"	"	4.0	5.70	A-30
68(l)	"	"	10.0	3.29	A-31
68(m)	"	"	10.0	5.70	A-32
68(n)	"	"	100.	3.29	A-33
68(o)	"	"	100.	5.70	A-34

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.400

K = 7.000

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

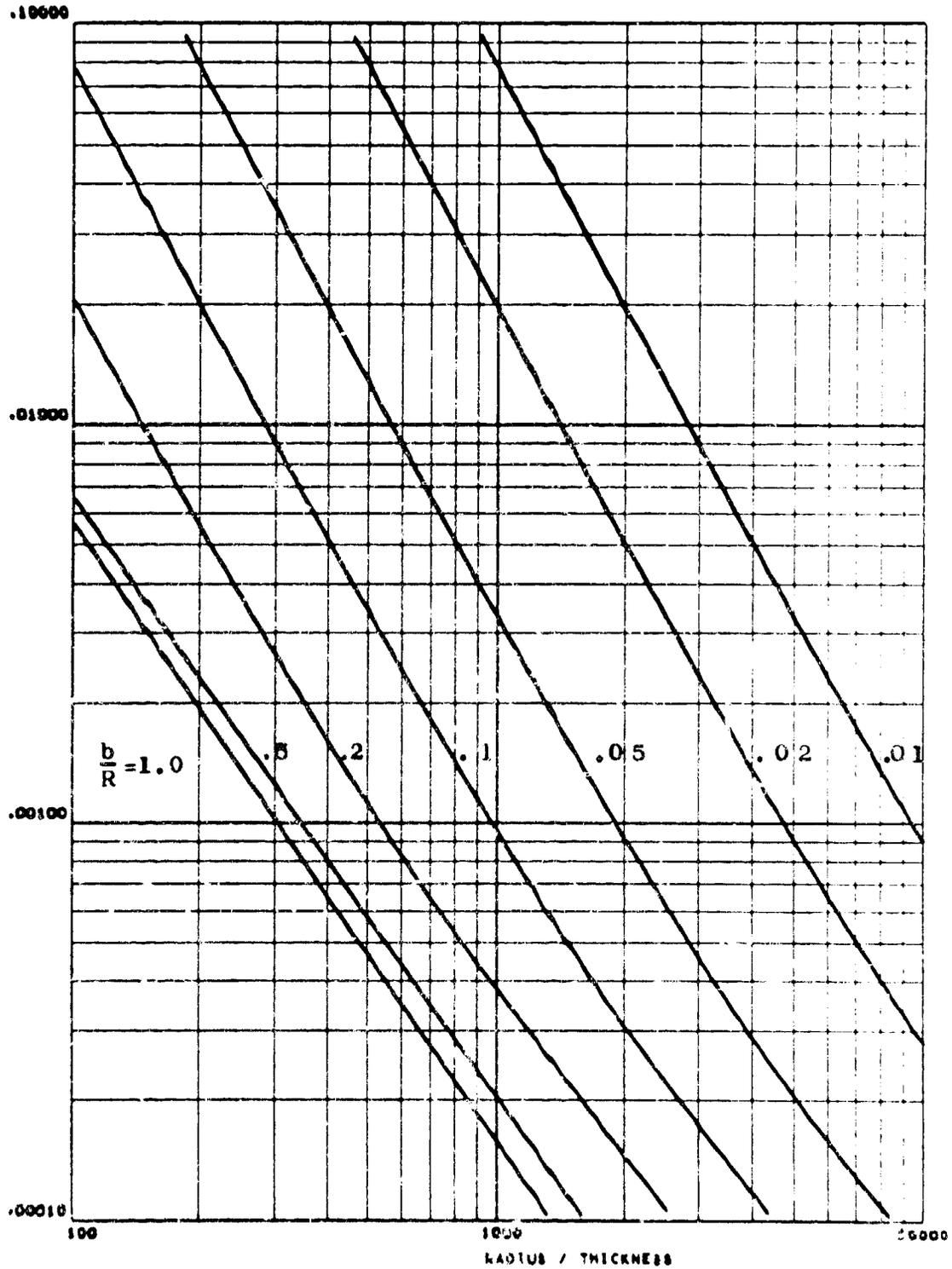


Figure 68(a) - (See Table XXXI)

# BUCKLING OF ISOTROPIC PANELS

$A/B = 0.600$

$K = 4.000$

OPTION 1

BUCKLING STRESS / ELASTIC MODULUS

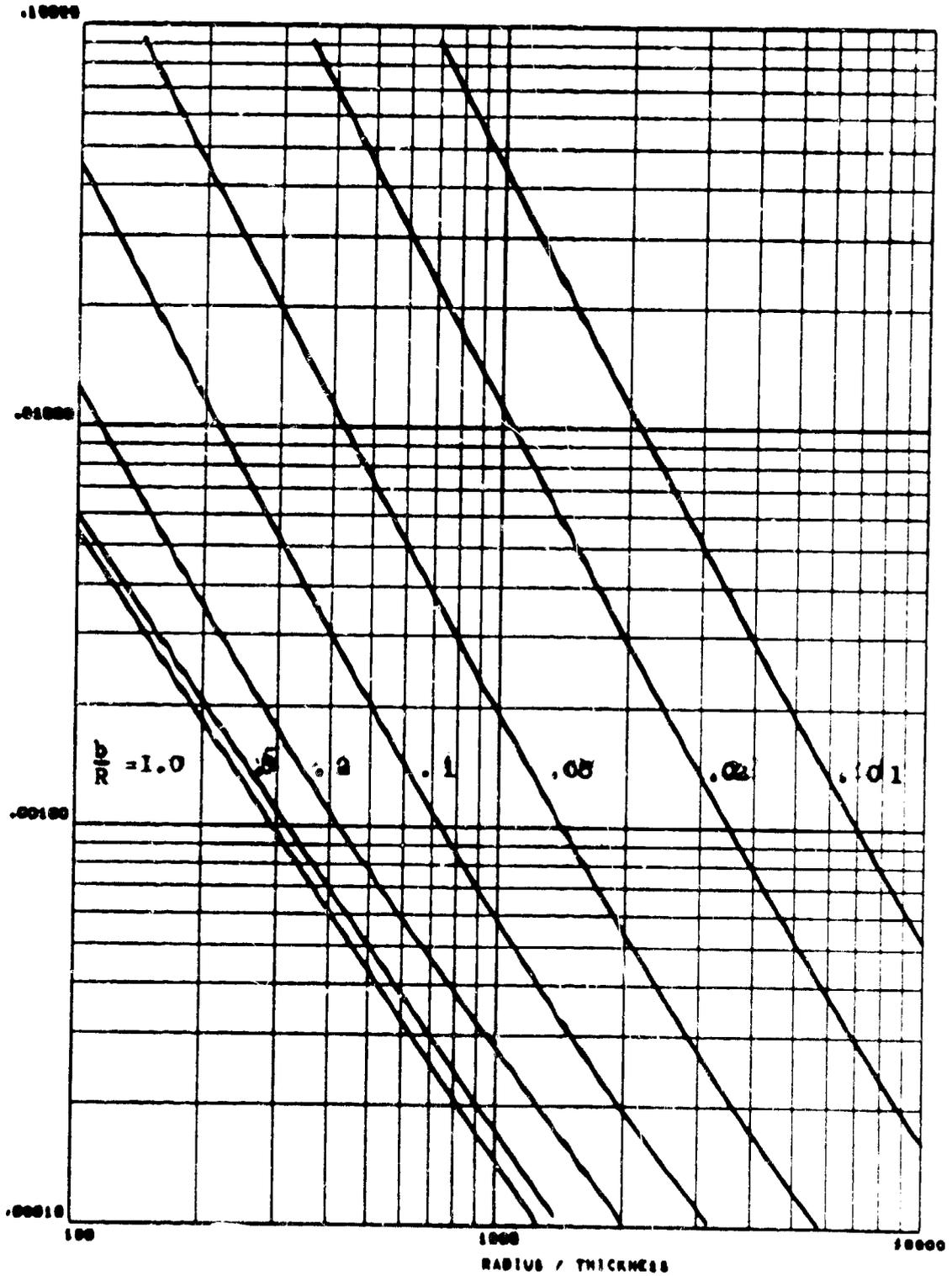


Figure 68(b) - (See Table XXXI)

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.600

K = 9.700

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

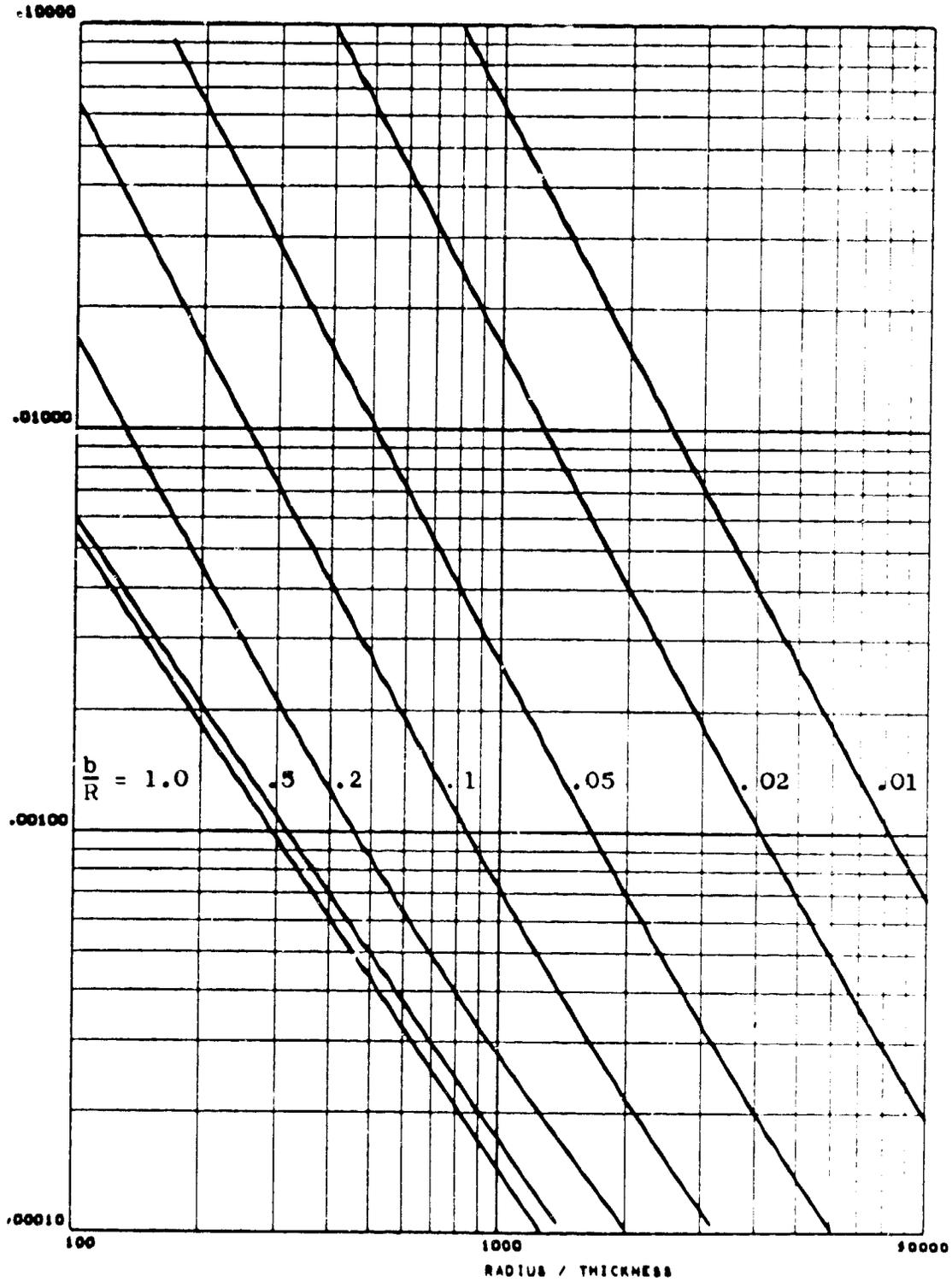


Figure 68(c) - (See Table XXXI)

# BUCKLING OF ISOTROPIC PANELS

$\mu = 0.476$

$K = 3.828$

OPTION 3

BUCKLING OF ISOTROPIC PANELS

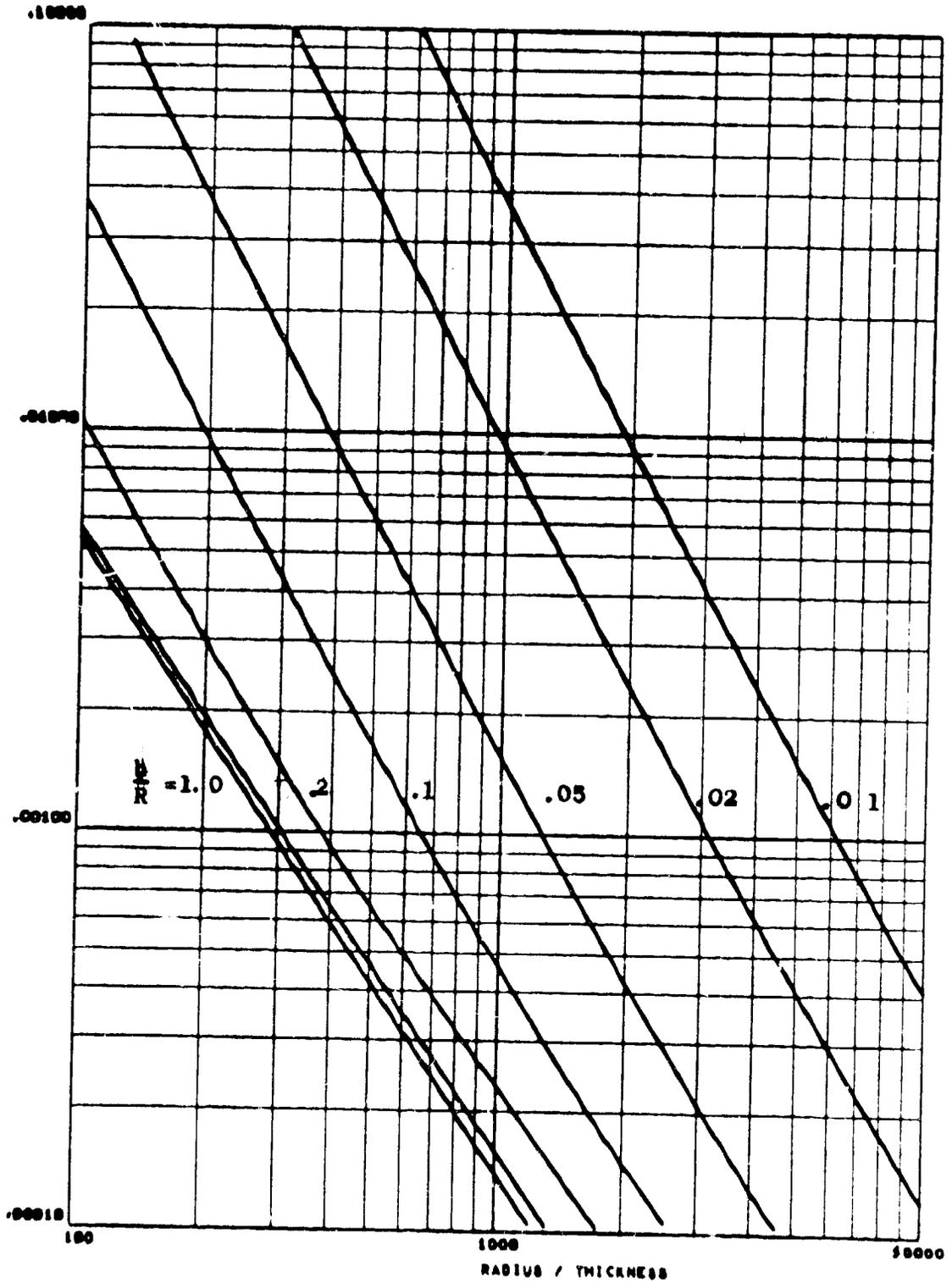


Figure 68(d) - (See Table XXXI)

# BUCKLING OF ISOTROPIC PANELS

A/B = 0.80

$\mu = 0.700$

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

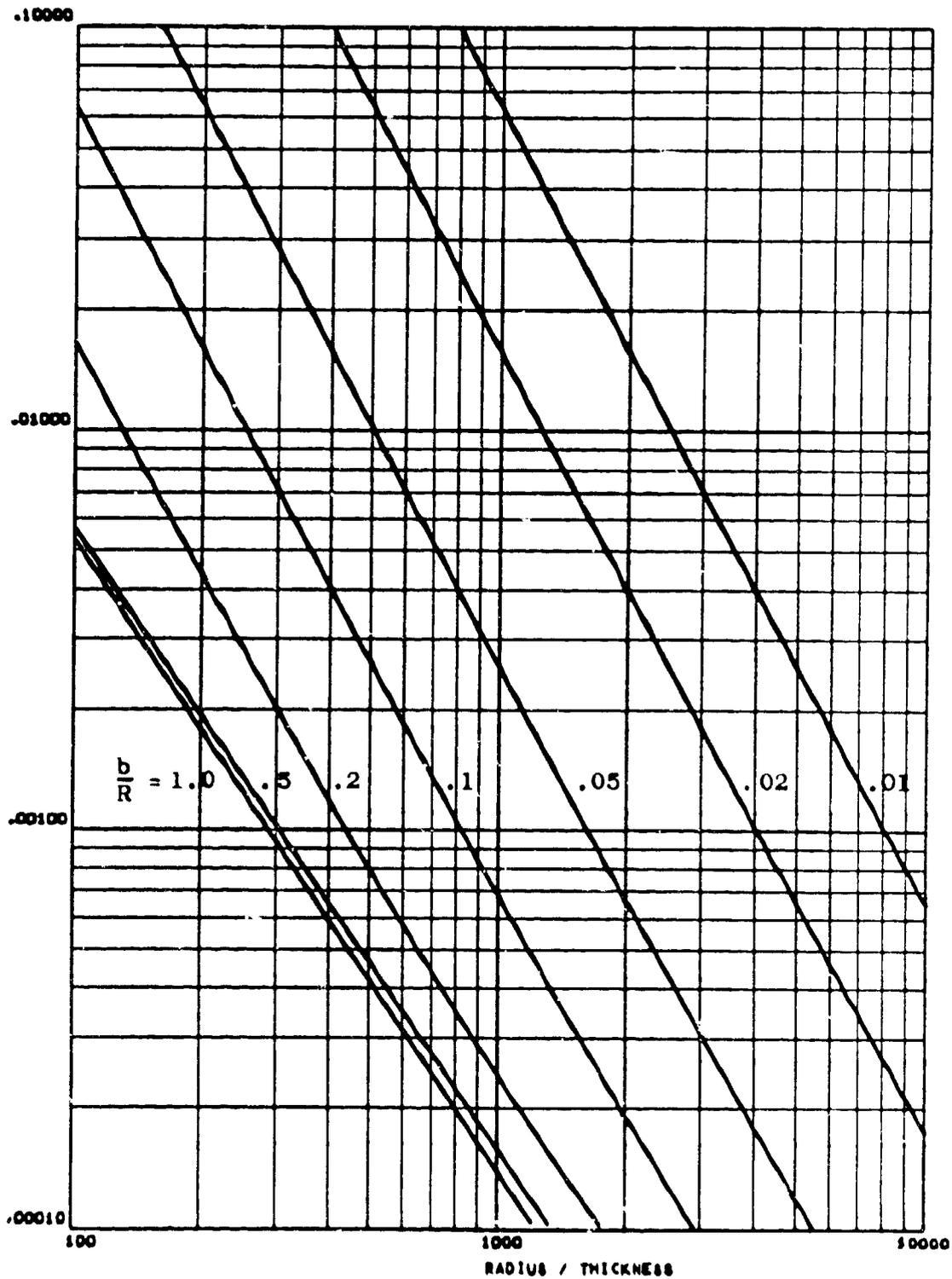


Figure 68(e) - (See Table XXXI)

### BUCKLING OF ISOTROPIC PANELS

$\mu/B = 1.000$

$K = 3.800$

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

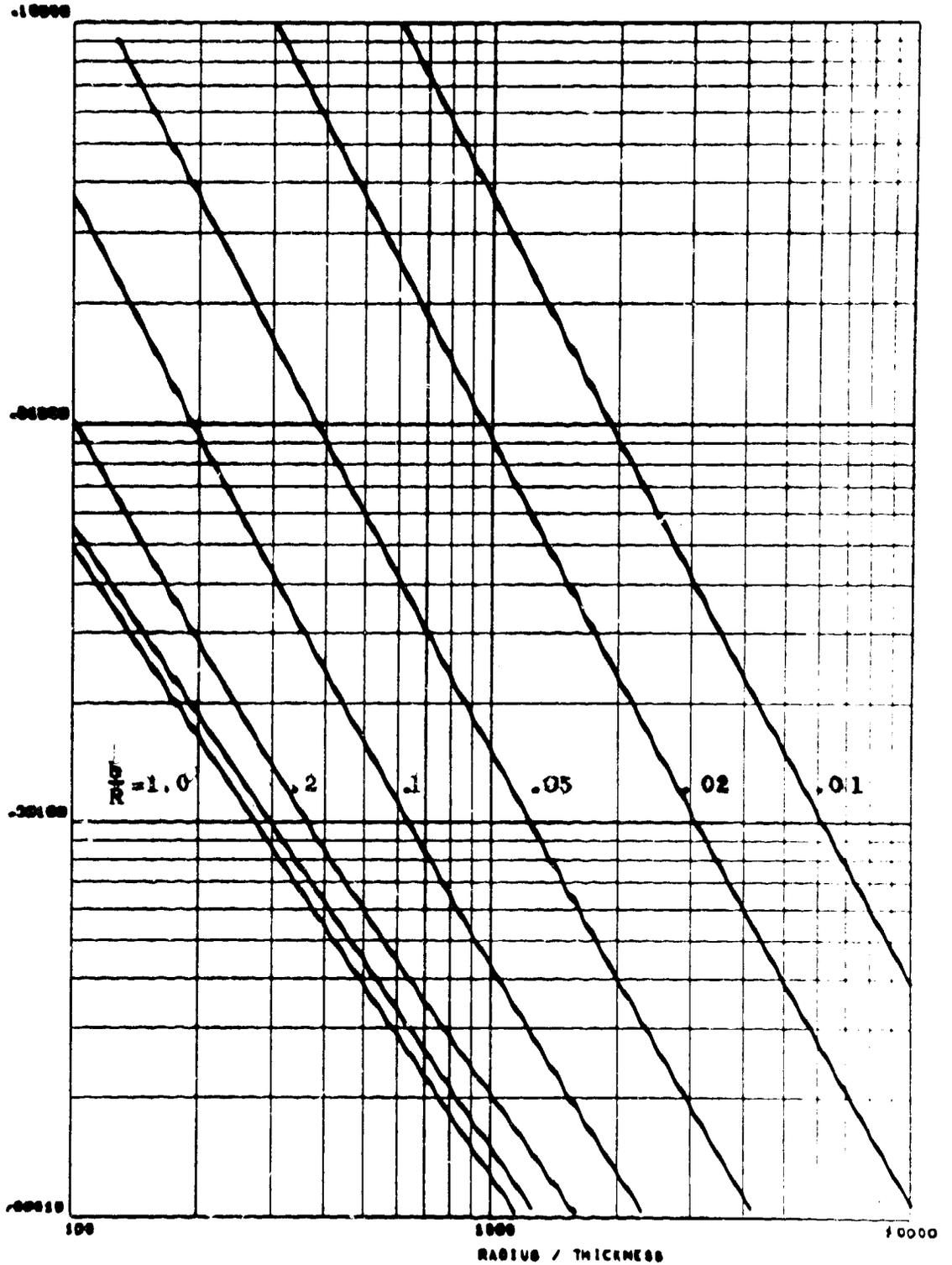


Figure 68(f) - (See Table XXXI)

### BUCKLING OF ISOTROPIC PANELS

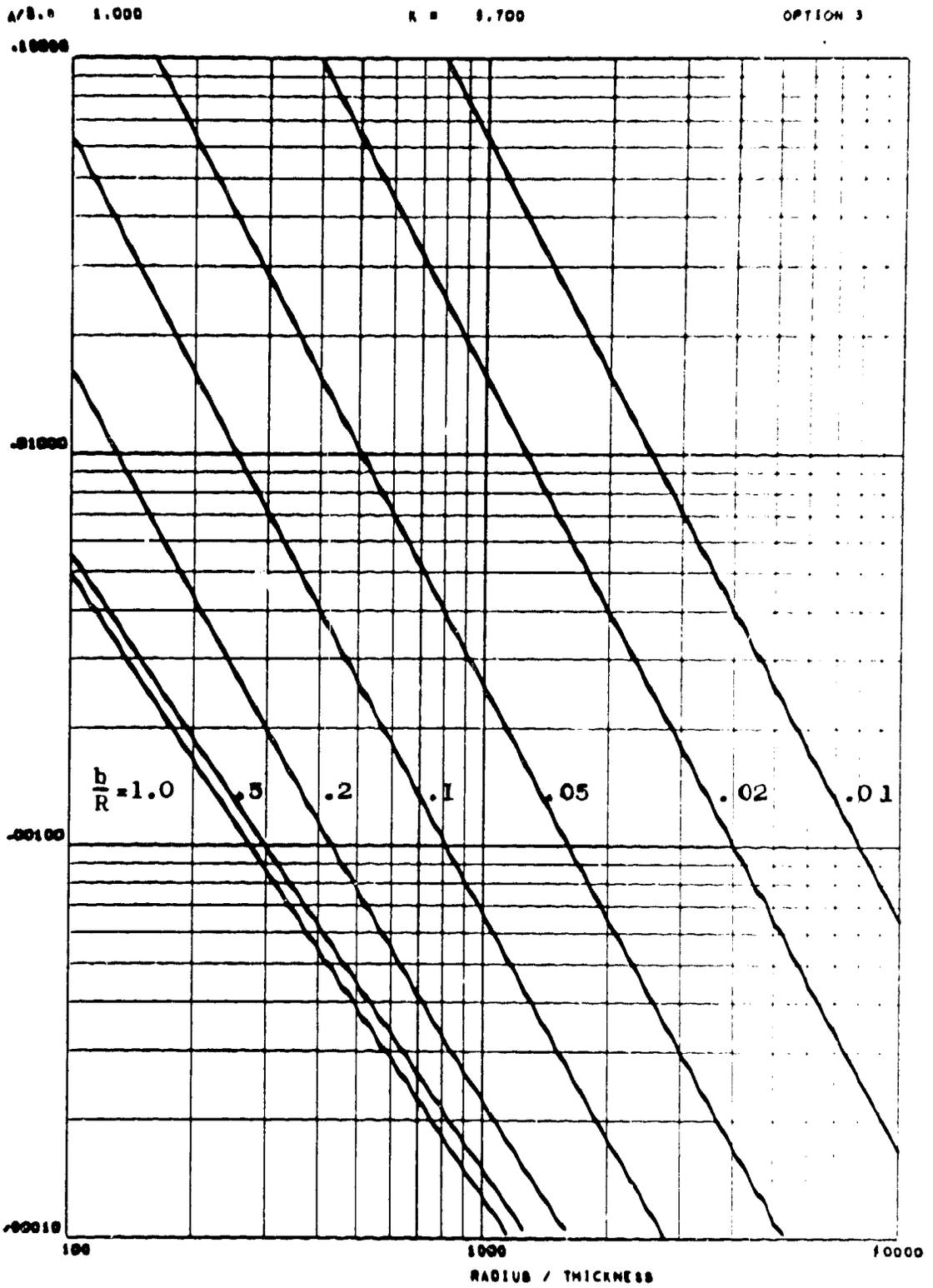


Figure 68(g) - (See Table XXXI)

# BUCKLING OF ISOTROPIC PANELS

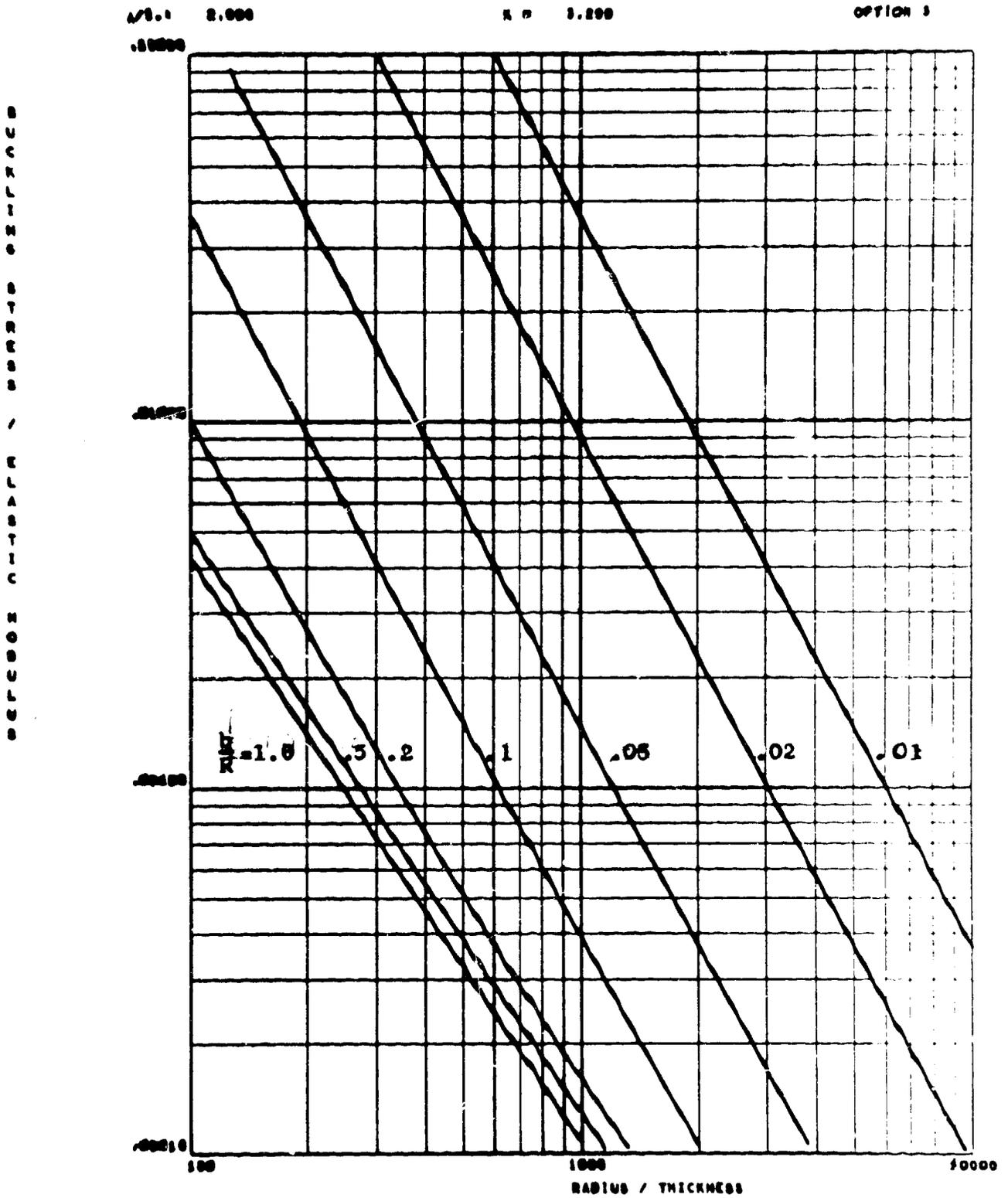


Figure 68(h) - (See Table XXXI)

### BUCKLING OF ISOTROPIC PANELS

$A/B = 2.000$

$\nu = 0.700$

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

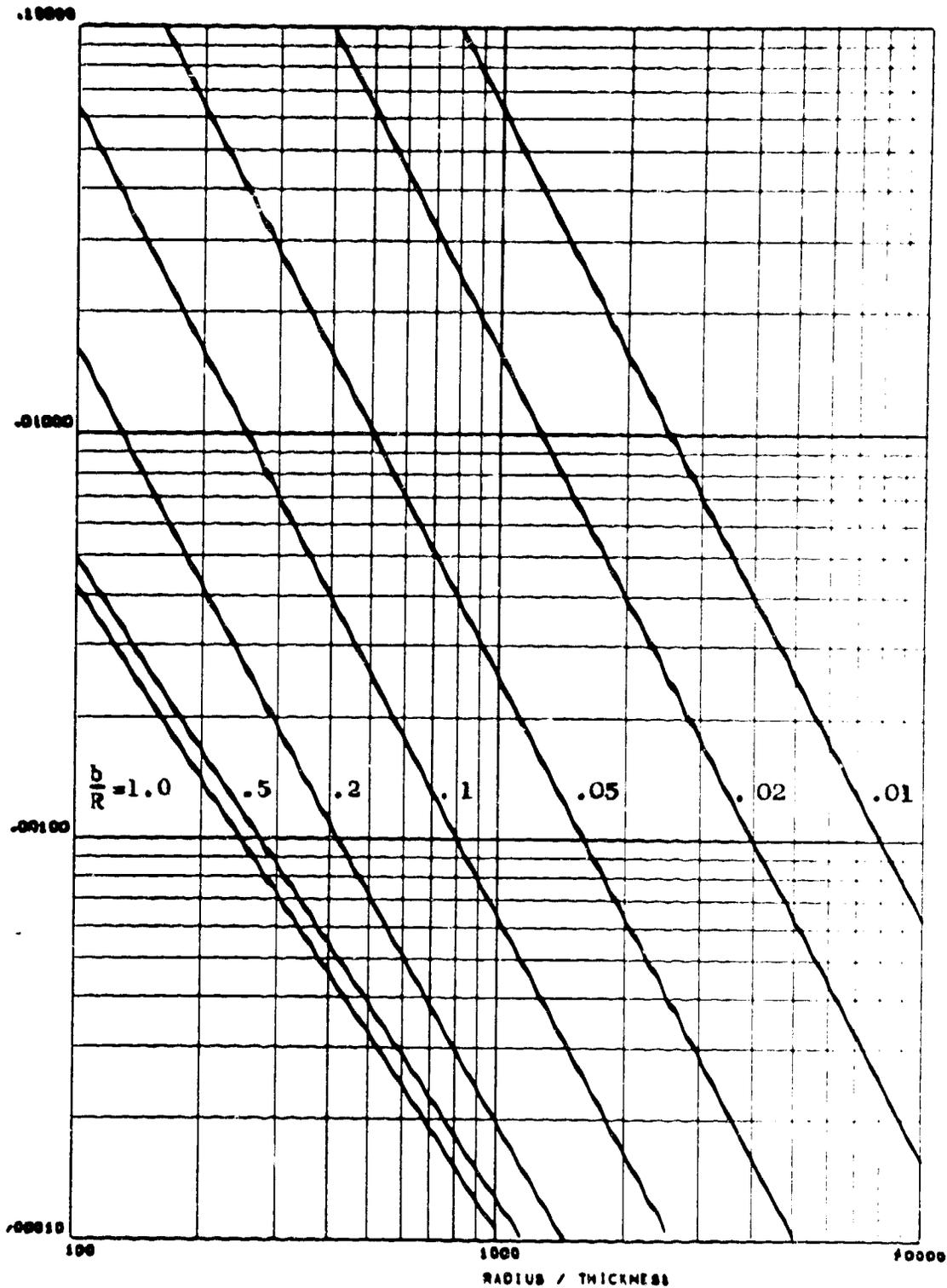


Figure 68(i) - (See Table XXXI)

# BUCKLING OF ISOTROPIC PANELS

$\lambda/B = 4.000$

$K = 3.800$

OPTION 3

BUCKLING OF ISOTROPIC PANELS

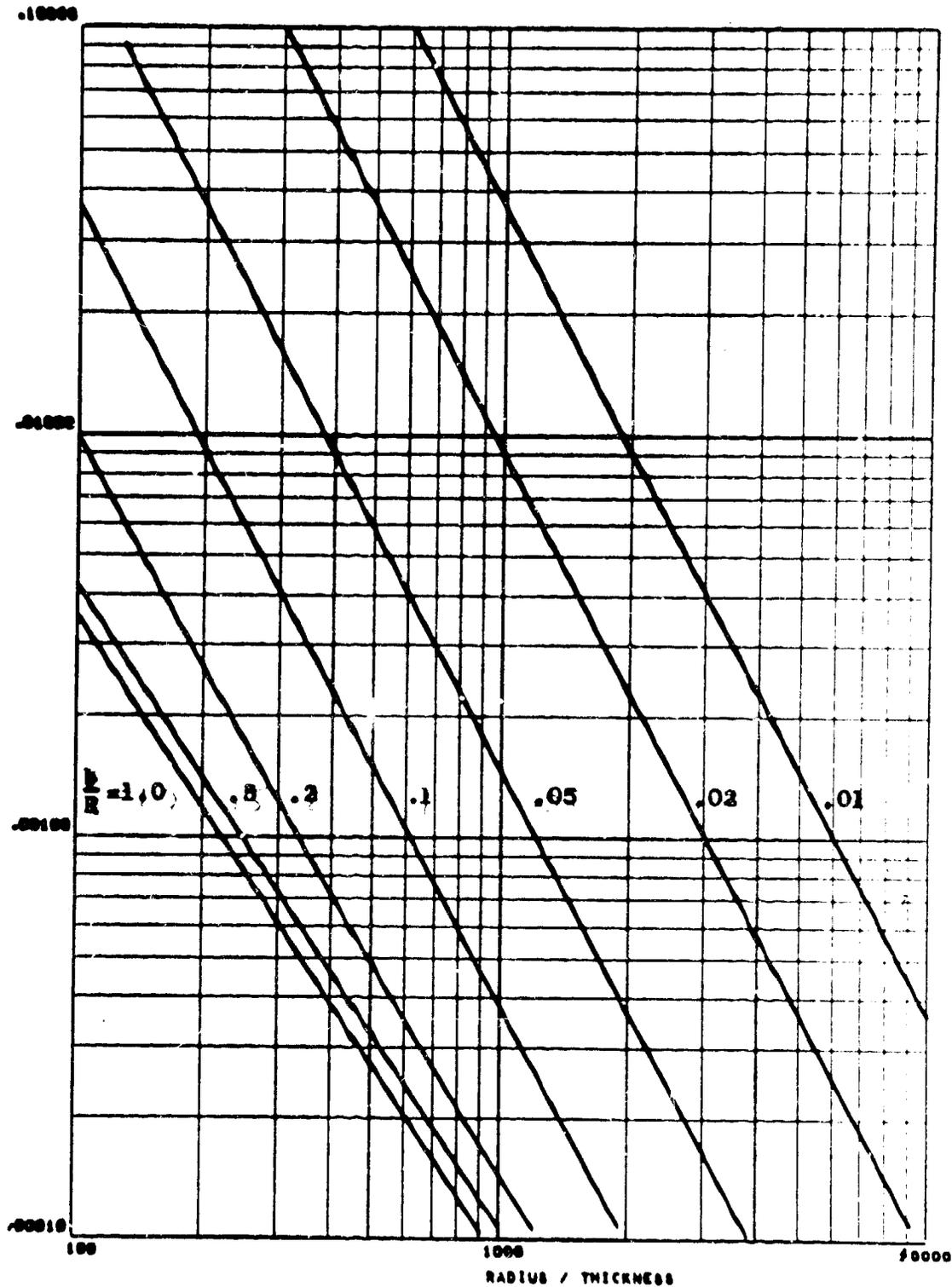


Figure 68(j) - (See Table XXXI)

BUCKLING OF ISOTROPIC PANELS

$A/B = 4.000$

$K = 9.700$

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

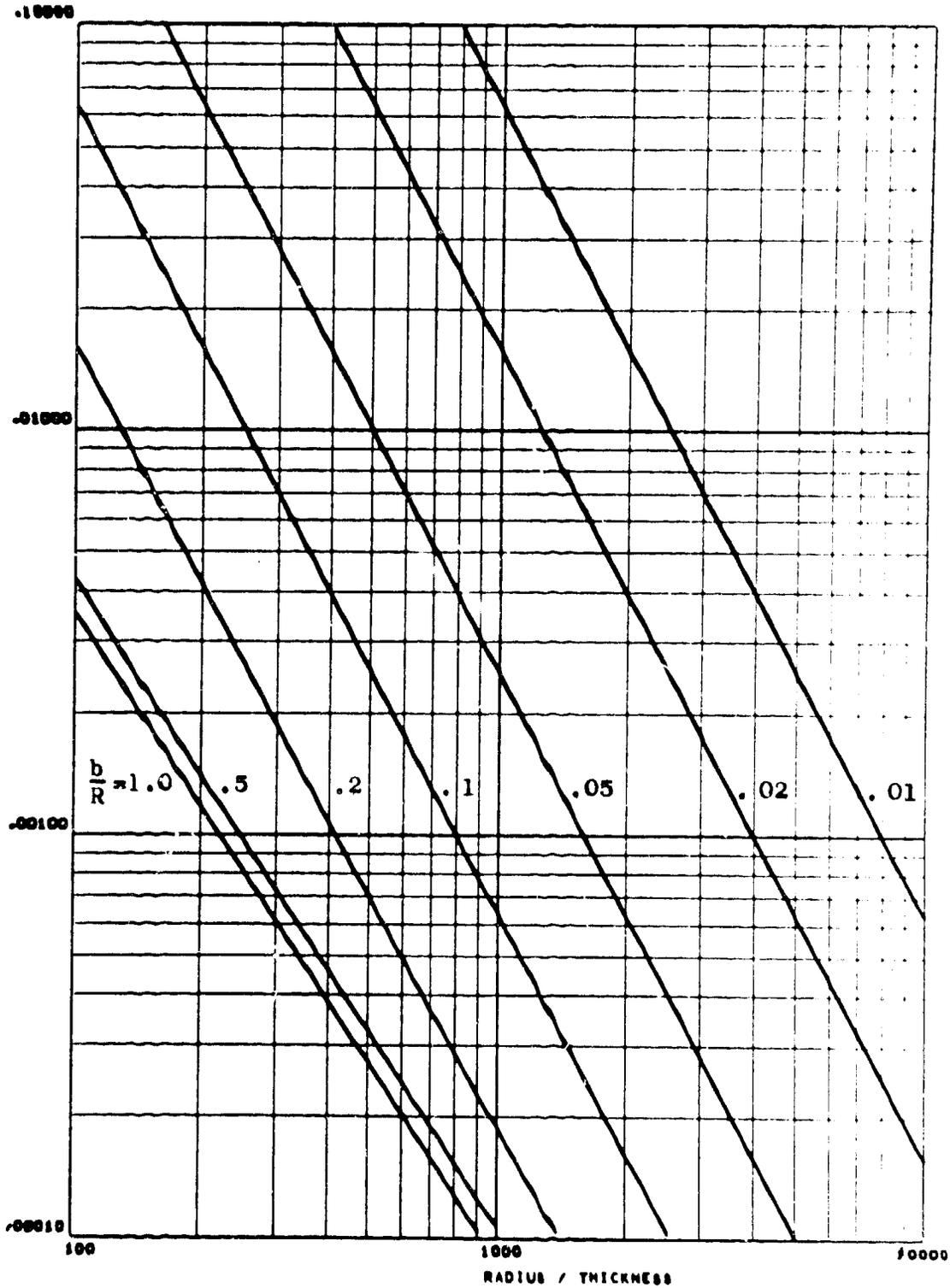


Figure 68(k) - (See Table XXXI)

# BUCKLING OF ISOTROPIC PANELS

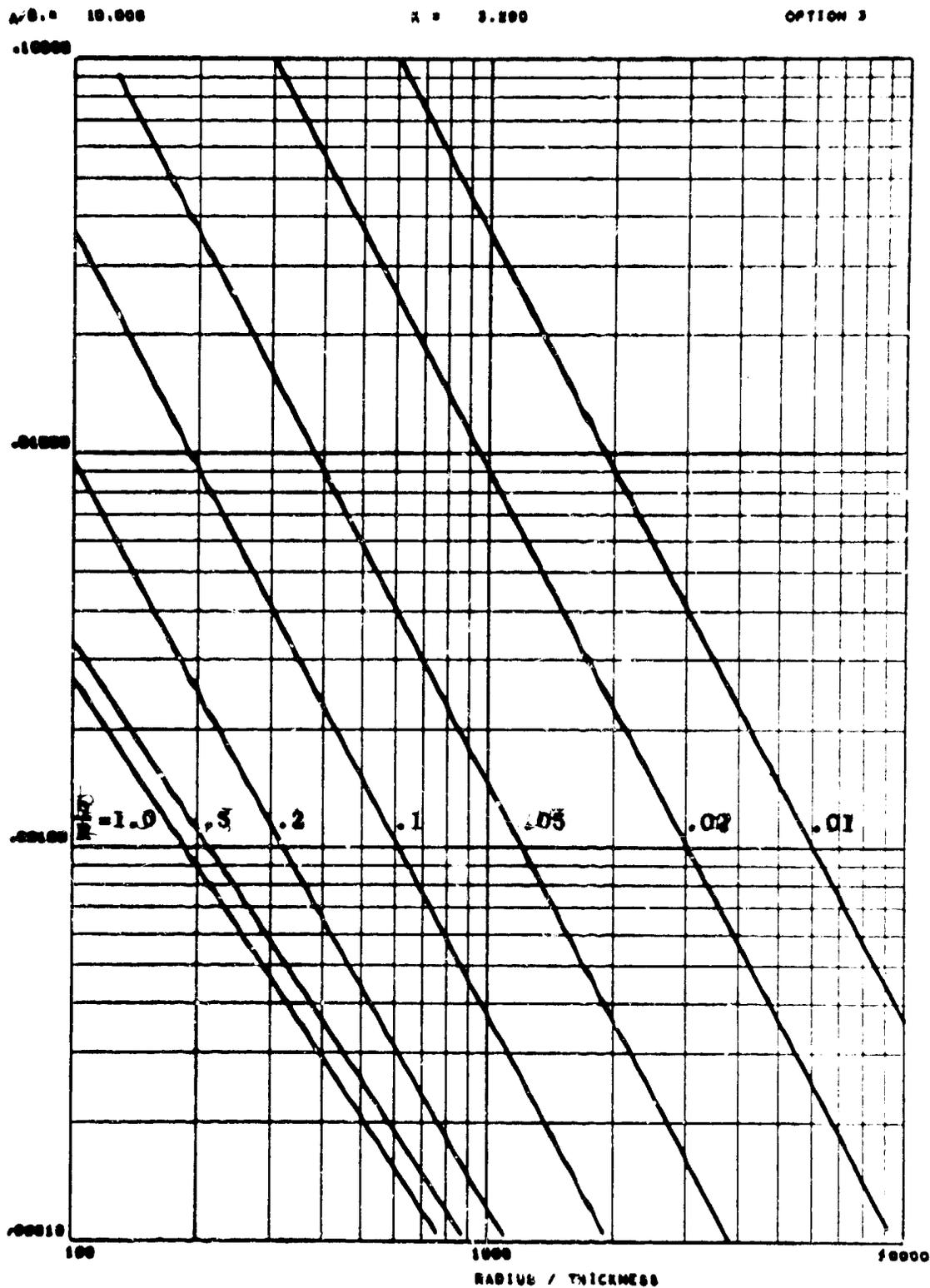


Figure 68(1) - (See Table XXXI)

### BUCKLING OF ISOTROPIC PANELS

$A/B = 10.000$

$\nu = 0.700$

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

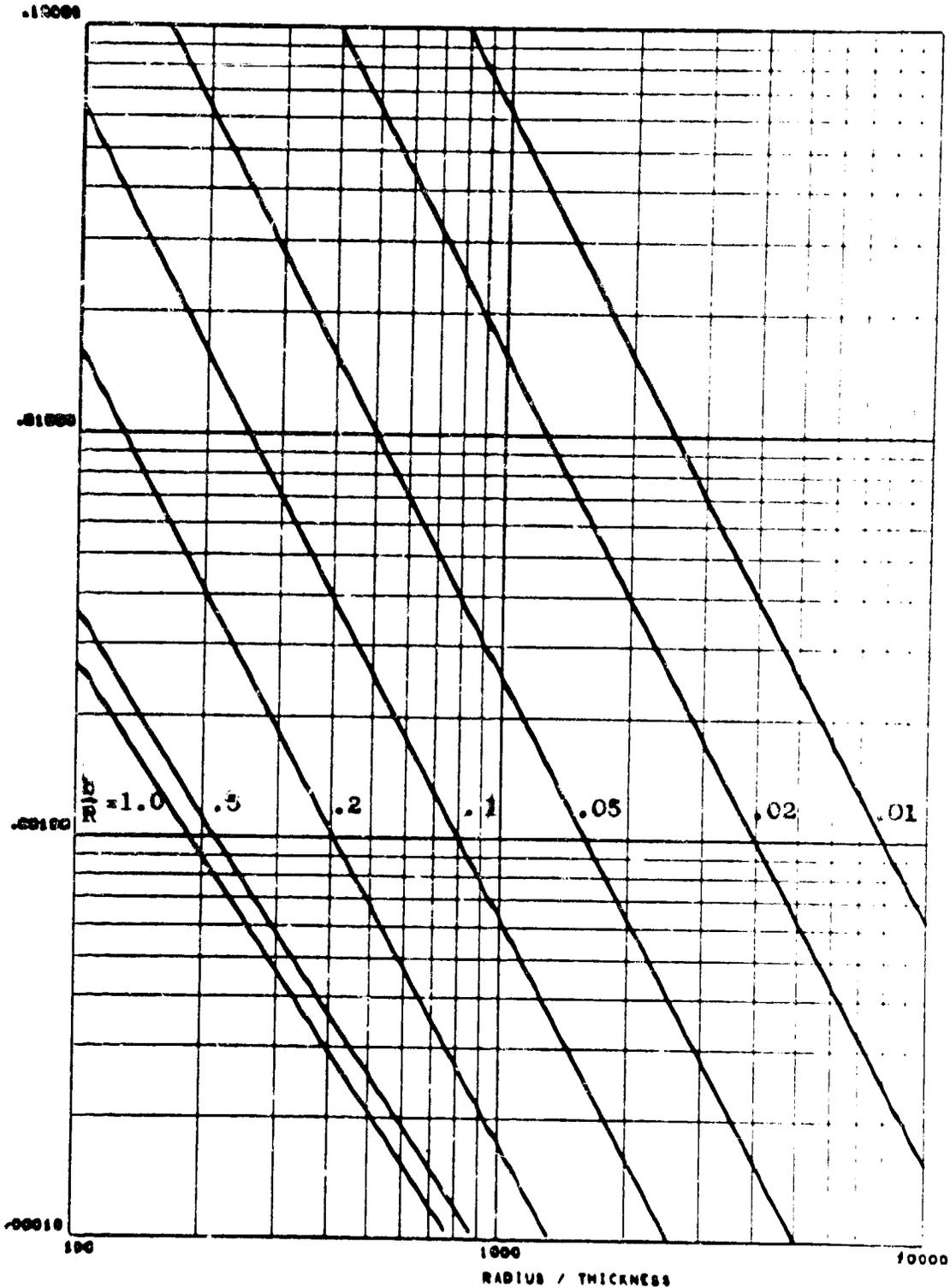


Figure 68(m) - (See Table XXXI)

BUCKLING OF ISOTROPIC PANELS

$\lambda/B = 100$

$\mu = 3.290$

OPTION 3

BUCKLING OF ISOTROPIC PANELS

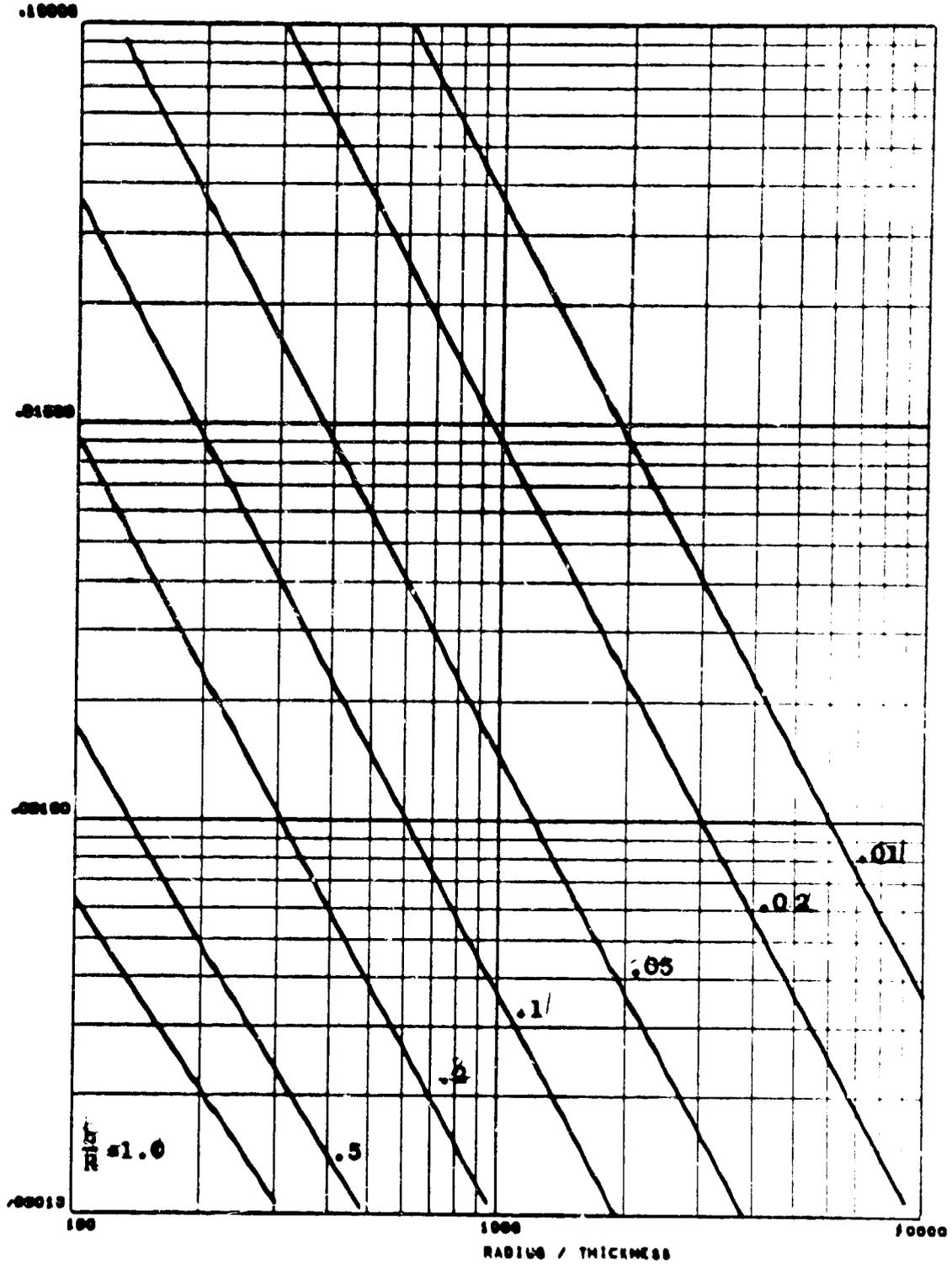


Figure 68(n) - (See Table XXXI)

BUCKLING OF ISOTROPIC PANELS

$\mu/B = 100$

$K = 9.700$

OPTION 3

BUCKLING STRESS / ELASTIC MODULUS

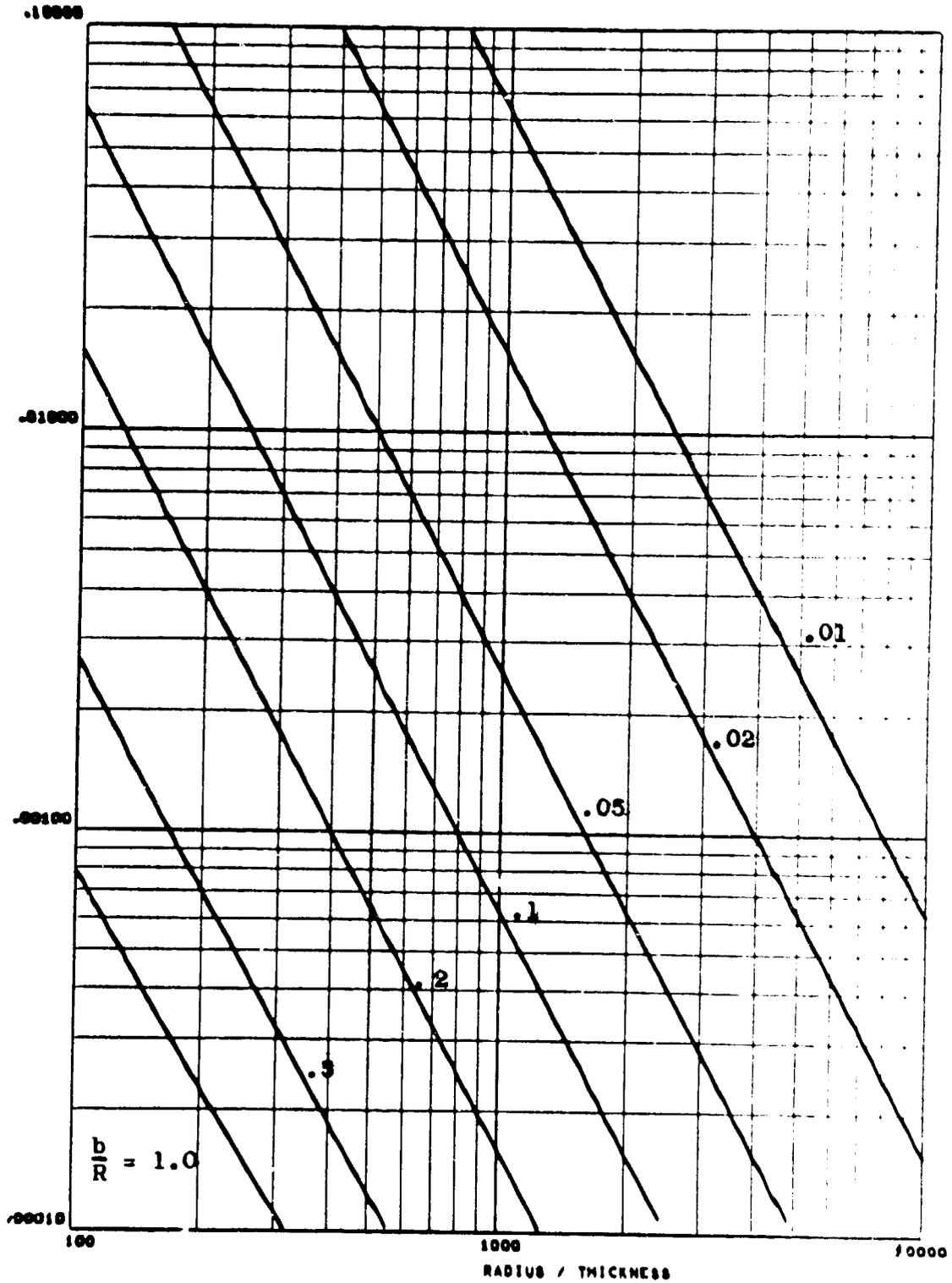


Figure 68(o) - (See Table XXXI)

END

DATE

FILMED

AUG 24 1966